



# Electromagnetic Scattering by Particles and Particle Groups

An Introduction

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# Preface

The phenomena of scattering and absorption of light and other electromagnetic radiation by small particles and particle groups are central to a great variety of science and engineering fields. Owing to a large body of research, the discipline of studying these phenomena has recently undergone profound and paradigm-shifting developments. Among the most important advances are the following:

- Dramatic improvements in numerical solvers of the Maxwell equations coupled with the ever growing computer capability have enabled direct, numerically exact modeling of electromagnetic scattering by particles and particle groups of unprecedented morphological complexity.
- The rigorous physical basis of monochromatic and polychromatic scattering by random particles and random particle groups has been established.
- Owing to the development of a rigorous microphysical approach, the centuries-old disciplines of directional photometry and radiative transfer have become legitimate branches of physical optics.
- Direct computer solutions of the Maxwell equations have confirmed the mesoscopic origin of radiative transfer and weak localization of electromagnetic waves (also known as coherent backscattering) in sparse particulate media.

The main purpose of this textbook is to provide a self-contained and accessible summary of these developments in the framework of a thorough introduction to the fundamental physical and mathematical principles of the subject. Particular attention is paid to key (and often overlooked) aspects such as time and ensemble averaging at different scales, ergodicity of stochastic scattering objects, and the physical nature of measurements afforded by actual directional photometers and photopolarimeters. Given its subject matter and specific content, it is hoped that this textbook will be useful to graduate students and researchers in terrestrial and planetary remote sensing, physical and biomedical optics, optical communica-



tions, optical particle characterization, atmospheric physics, oceanography, and astrophysics.

Consistent with its overall objective, this textbook features more than 120 end-of-chapter exercises of varying complexity. The simplest ones require the student to complete derivations that are sketched but not detailed in the main text or serve to illustrate certain key concepts and results. Most of these exercises are relatively straightforward, but can sometimes be tedious. Some exercises invite the student to think on issues not specifically covered by the textbook but nonetheless useful for understanding the main text. A few exercises border on research projects and may have been the subject of recent publications. The more challenging exercises are supplemented by hints or full solutions collected in Appendix H.

Several appendices provide the requisite mathematical background beyond the standard calculus and vector calculus material and help making this textbook a one-stop resource suitable for self-study or classroom use. The reference list is intended to be representative but not necessarily comprehensive. Preference is given to classical papers, monographs, and state-of-the-art reviews. Most chapters are concluded by notes briefly outlining the history of the subject and helping the reader navigate through more advanced literature.

In many respects this textbook is an outgrowth of the previous monographs by Mishchenko *et al.* (2002, 2006) in which the reader can find further technical details as well as a plethora of specific examples and applications. Both monographs are available on-line and will hereinafter be referred to as MTL1 and MTL2, respectively.

For consistency, I closely follow the notation adopted in MTL1 and MTL2 and denote vectors using the Times New Roman Bold font and matrices using the Arial bold font. Unit vectors are denoted by a caret, whereas dyads and dyadics are denoted by the symbol  $\leftrightarrow$ . The Times New Roman Italic font is reserved for scalar quantities, important exceptions being the square root of negative one, the differential sign, and the base of natural logarithms, which are denoted by Times roman characters  $i$ ,  $d$ , and  $e$ . Another exception is the relative refractive index, which is denoted by a sloping sans serif  $m$ . Simple angular brackets  $\langle \dots \rangle$  are used to denote an average over a time interval much longer than the period of time-harmonic oscillations of a monochromatic electromagnetic field. Double brackets  $\langle\langle \dots \rangle\rangle$  denote an average over a “sufficiently long” time interval  $T$ , the actual value of  $T$  being defined by the specific context. Averaging over a parameter  $X$  other than time is denoted by  $\langle \dots \rangle_X$ . All color plates are grouped together into the “Color plates” section and are numbered decimally per chapter as Plate M.N, where N is the plate number in Chapter M.