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Electromagnetic scattering and emission by a fixed multi-particle object in local thermal equilibrium: General formalism

Michael I. Mishchenko

NASA Goddard Institute for Space Studies, 2880 Broadway, New York, NY 10025, USA

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ABSTRACT

The majority of previous studies of the interaction of individual particles and multi-particle groups with electromagnetic field have focused on either elastic scattering in the presence of an external field or self-emission of electromagnetic radiation. In this paper we apply semi-classical fluctuational electrodynamics to address the ubiquitous scenario wherein a fixed particle or a fixed multi-particle group is exposed to an external quasi-polychromatic electromagnetic field as well as thermally emits its own electromagnetic radiation. We summarize the main relevant axioms of fluctuational electrodynamics, formulate in maximally rigorous mathematical terms the general scattering–emission problem for a fixed object, and derive such fundamental corollaries as the scattering–emission volume integral equation, the Lippmann–Schwinger equation for the dyadic transition operator, the multi-particle scattering–emission equations, and the far-field limit. We show that in the framework of fluctuational electrodynamics, the computation of the self-emitted component of the total field is completely separated from that of the elastically scattered field. The same is true of the computation of the emitted and elastically scattered components of quadratic/bilinear forms in the total electromagnetic field. These results pave the way to the practical computation of relevant optical observables.

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1. Introduction

The standard treatment of the scattering of electromagnetic waves by particles and multi-particle groups [1–20] has traditionally been based on "deterministic" macroscopic electromagnetics [21–24] and as such has not included explicitly the "stochastic" phenomenon of thermal emission by bodies having non-zero absolute temperatures. In contrast, a large body of recent publications (see, e.g., Refs. [25–39] and references therein) have focused on the study of (near-field) energy transfer in thermally emitting physical systems (including many-particle groups) using the semi-classical "fluctuational" electrodynamics (FED) [40–44]. However, there are practical situations wherein thermal emission processes are accompanied by elastic scattering of external electromagnetic radiation [36]. An important example of such mixed scenario is a cloud of particles in a planetary atmosphere which can both scatter the incident stellar light at near-infrared wavelengths as well as emit its own near-infrared radiation (see, e.g., Refs. [45–49] and references therein). Fortunately, by its very construct, FED is ideally suited to address such situations.

Indeed, FED amounts to a reformulation of the macroscopic Maxwell equations (MMEs) wherein the usual "deterministic" volume charge density is supplemented by the volume density of the "stochastic" thermal electric current. The latter is caused by randomly fluctuating positions of elementary charges constituting a body at a non-zero absolute temperature. As a consequence, the modified MMEs describe simultaneously the processes of thermal emission as well as elastic scattering, albeit at the price of added mathematical complexity.

The classical formalism based on the MMEs and developed for the study of elastic electromagnetic scattering by single- and multi-particle objects is well developed [1–10,12–20]. The next obvious step is to analyze in a systematic way how the main aspects of this elastic-scattering formalism are modified by the inclusion of thermal emission effects. This paper is intended to facilitate this analysis by summarizing the main relevant axioms of FED, formulating in maximally rigorous mathematical terms the general scattering–emission problem for a fixed (multi-particle) object exposed to a quasi-polychromatic external field, and generalizing such fundamental corollaries of the MMEs as the volume integral equation, the Lippmann–Schwinger equation for the dyadic transition operator, the Foldy equations, and the far-zone approximation. Fundamentally, we show that the FED framework allows one to split the problem of finding the total electromagnetic field into the computation of the self-emitted field and the calculation of the elastically scattered field. Furthermore, we demonstrate that the same is true of the problem of computing second moments of the

E-mail address: michael.i.mishchenko@nasa.gov

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total electromagnetic field. These results are expected to pave the way to the calculation of optical observables encountered in actual practical applications.

2. Stochastic macroscopic Maxwell equations, constitutive relations, and boundary conditions

Under the assumption that all media involved are nonmagnetic, the system of four stochastic MMEs for the instantaneous macroscopic electromagnetic field at an arbitrary observation point $\mathbf{r}$ is as follows [44]:

$$\nabla \cdot \mathbf{D}(\mathbf{r}, t) = \rho(\mathbf{r}, t), \tag{1}$$

$$\nabla \times \mathbf{E}(\mathbf{r}, t) = -\mu_0 \frac{\partial \mathbf{H}(\mathbf{r}, t)}{\partial t}, \tag{2}$$

$$\nabla \cdot \mathbf{H}(\mathbf{r}, t) = 0, \tag{3}$$

$$\nabla \times \mathbf{H}(\mathbf{r}, t) = \mathbf{J}(\mathbf{r}, t) + \mathbf{J}^{\mathrm{f}}(\mathbf{r}, t) + \frac{\partial \mathbf{D}(\mathbf{r}, t)}{\partial t}, \tag{4}$$

where we use the SI units, $\mathbf{E}(\mathbf{r},t)$ is the electric and $\mathbf{H}(\mathbf{r},t)$ the magnetic field, $\mathbf{D}(\mathbf{r},t)$ is the electric displacement, $\rho(\mathbf{r},t)$ and $\mathbf{J}(\mathbf{r},t)$ are the macroscopic (free) volume charge density and current density, respectively, $\mathbf{J}^{\mathrm{f}}(\mathbf{r},t)$ is the volume density of the *fluctuating* electric current, and μ_0 is the magnetic permeability of a vacuum. All quantities entering Eqs. (1)–(4) are real-valued functions of time t as well as of spatial coordinates. Implicit in the stochastic MMEs is the continuity equation

$$\frac{\partial \rho(\mathbf{r}, t)}{\partial t} + \nabla \cdot \mathbf{J}(\mathbf{r}, t) + \nabla \cdot \mathbf{J}^{\mathrm{f}}(\mathbf{r}, t) = 0, \tag{5}$$

which is obtained by combining the time derivative of Eq. (1) with the divergence of Eq. (4) and making use of the vector identity

$$\nabla \cdot (\nabla \times \mathbf{a}) \equiv 0. \tag{6}$$

Typically Eqs. (1)–(4) must be supplemented by appropriate constitutive relations. In the case of a time-dispersive medium, we have

$$\mathbf{D}(\mathbf{r}, t) = \int_{-\infty}^{t} \mathrm{d}t' \, \varepsilon\big(\mathbf{r}, t - t'\big)\mathbf{E}\big(\mathbf{r}, t'\big), \tag{7}$$

$$\mathbf{J}(\mathbf{r}, t) = \int_{-\infty}^{t} \mathrm{d}t' \, \sigma(\mathbf{r}, t - t')\mathbf{E}(\mathbf{r}, t'), \tag{8}$$

where ε is the electric permittivity and σ is the electric conductivity.

If two different continuous media with finite conductivity are separated by an interface S then it is postulated that the tangential components of the electric and magnetic field vectors are continuous across S:

$$\hat{\mathbf{n}} \times [\mathbf{E}_1(\mathbf{r}, t) - \mathbf{E}_2(\mathbf{r}, t)] \equiv \mathbf{0}, \tag{9}$$

$$\hat{\mathbf{n}} \times [\mathbf{H}_1(\mathbf{r}, t) - \mathbf{H}_2(\mathbf{r}, t)] \equiv \mathbf{0}, \tag{10}$$

where $\mathbf{0}$ is a zero vector and $\hat{\mathbf{n}}$ is a unit vector along the local normal to S.

3. The Poynting theorem

The system of axioms (1)–(4) and (7)–(10) of fluctuational electromagnetics must provide a link to other physical quantities, including those directly measurable with suitable instrumentation. This is accomplished in part by using the Lorentz force postulate which states that if a differential volume element $\mathrm{d}V$ contains a total charge $\rho(\mathbf{r},t)\mathrm{d}V$ moving at a velocity $\mathbf{v}(\mathbf{r},t)$ then the force exerted by the electromagnetic field on that charge is

$$\mathrm{d}\mathbf{F} = \rho(\mathbf{r}, t)\mathbf{E}(\mathbf{r}, t)\mathrm{d}V + \mu_0 \rho(\mathbf{r}, t)\mathbf{v}(\mathbf{r}, t) \times \mathbf{H}(\mathbf{r}, t)\mathrm{d}V. \tag{11}$$

Upon scalar multiplying $\mathrm{d}\mathbf{F}$ by $\mathbf{v}(\mathbf{r},t)$, we see that the magnetic field does no work, while for the local charge $\rho(\mathbf{r},t)\mathrm{d}V$ the rate of doing work by the electric field is $\rho(\mathbf{r},t)\mathbf{v}(\mathbf{r},t) \cdot \mathbf{E}(\mathbf{r},t)\mathrm{d}V$. Thus the total rate of work done by the electromagnetic field inside a finite volume V is given by

$$Q = \int_V \mathrm{d}^3\mathbf{r} \, [\mathbf{J}(\mathbf{r}, t) + \mathbf{J}^{\mathrm{f}}(\mathbf{r}, t)] \cdot \mathbf{E}(\mathbf{r}, t). \tag{12}$$

We now make use of Eqs. (2) and (4), the vector identity

$$\nabla \cdot (\mathbf{a} \times \mathbf{b}) = \mathbf{b} \cdot (\nabla \times \mathbf{a}) - \mathbf{a} \cdot (\nabla \times \mathbf{b}) \tag{13}$$

with $\mathbf{a}=\mathbf{E}$ and $\mathbf{b}=\mathbf{H}$, and the Gauss theorem

$$\int_V \mathrm{d}^3\mathbf{r} \, \nabla \cdot \mathbf{A}(\mathbf{r}) = \int_S \mathrm{d}^2\mathbf{r} \, \mathbf{A}(\mathbf{r}) \cdot \hat{\mathbf{n}}(\mathbf{r}), \tag{14}$$

where S is the closed surface bounding V and $\hat{\mathbf{n}}(\mathbf{r})$ is a unit vector in the direction of the local outward normal to S. The result is the so-called Poynting theorem quantifying the energy budget of the volume V:

$$-\int_S \mathrm{d}^2\mathbf{r} \, \mathbf{S}(\mathbf{r}, t) \cdot \hat{\mathbf{n}}(\mathbf{r}) = Q + \frac{\mathrm{d}U}{\mathrm{d}t}, \tag{15}$$

where

$$\mathbf{S}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r}, t) \times \mathbf{H}(\mathbf{r}, t) \tag{16}$$

is the Poynting vector and the term

$$\frac{\mathrm{d}U}{\mathrm{d}t} = \int_V \mathrm{d}^3\mathbf{r} \left[\mathbf{E}(\mathbf{r}, t) \cdot \frac{\partial \mathbf{D}(\mathbf{r}, t)}{\partial t} + \mu_0 \mathbf{H}(\mathbf{r}, t) \cdot \frac{\partial \mathbf{H}(\mathbf{r}, t)}{\partial t} \right] \tag{17}$$

accounts for both the rate of change of the stored electromagnetic energy in V and the rate of energy dissipated by the material in V [24]. It is postulated that the left-hand side of Eq. (15) represents the net flow of electromagnetic energy entering V.

4. Fourier decomposition

Let us express all time-varying fields entering the stochastic MMEs in terms of time-harmonic components using the Fourier analysis:

$$\mathbf{E}(\mathbf{r}, t) = \int_{-\infty}^{\infty} \mathrm{d}\omega \, \mathbf{E}(\mathbf{r}, \omega) \exp(-\mathrm{i}\omega t) \tag{18}$$

and similarly for $\mathbf{H}(\mathbf{r},t)$, $\mathbf{D}(\mathbf{r},t)$, $\rho(\mathbf{r},t)$, $\mathbf{J}(\mathbf{r},t)$, and $\mathbf{J}^{\mathrm{f}}(\mathbf{r},t)$, where $\mathrm{i} = (-1)^{1/2}$. The respective frequency spectra are given by the Fourier transforms

$$\mathbf{E}(\mathbf{r}, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathrm{d}t \, \mathbf{E}(\mathbf{r}, t) \exp(\mathrm{i}\omega t), \text{ etc.} \tag{19}$$

It is straightforward to verify that since the actual physical fields are real-valued, the frequency spectra satisfy the symmetry relations

$$\mathbf{E}(\mathbf{r}, -\omega) = [\mathbf{E}(\mathbf{r}, \omega)]^*, \text{ etc.}, \tag{20}$$

where the asterisk denotes the complex-conjugate value.

By virtue of the Fourier integral theorem, the frequency-domain system of the stochastic Maxwell equations and boundary conditions takes the form

$$\nabla \cdot [\varepsilon(\mathbf{r}, \omega)\mathbf{E}(\mathbf{r}, \omega)] = -\frac{\mathrm{i}}{\omega} \nabla \cdot \mathbf{J}^{\mathrm{f}}(\mathbf{r}, \omega), \tag{21}$$

$$\nabla \times \mathbf{E}(\mathbf{r}, \omega) = \mathrm{i}\omega\mu_0 \mathbf{H}(\mathbf{r}, \omega), \tag{22}$$

$$\nabla \cdot \mathbf{H}(\mathbf{r}, \omega) = 0, \tag{23}$$

$$\nabla \times \mathbf{H}(\mathbf{r},\omega) = -\mathrm{i}\omega\varepsilon(\mathbf{r},\omega)\mathbf{E}(\mathbf{r},\omega) + \mathbf{J}^{\mathrm{f}}(\mathbf{r},\omega), \tag{24}$$

$$\nabla \cdot \mathbf{J}(\mathbf{r},\omega) + \nabla \cdot \mathbf{J}^{\mathrm{f}}(\mathbf{r},\omega) - \mathrm{i}\omega\rho(\mathbf{r},\omega) = 0, \tag{25}$$

$$\hat{\mathbf{n}} \times [\mathbf{E}_1(\mathbf{r},\omega) - \mathbf{E}_2(\mathbf{r},\omega)] \equiv \mathbf{0}, \tag{26}$$

$$\hat{\mathbf{n}} \times [\mathbf{H}_1(\mathbf{r},\omega) - \mathbf{H}_2(\mathbf{r},\omega)] \equiv \mathbf{0}, \tag{27}$$

where

$$\varepsilon(\mathbf{r},\omega) = \int_0^\infty \mathrm{d}\tau \left[\varepsilon(\mathbf{r},\tau) + \frac{\mathrm{i}}{\omega}\sigma(\mathbf{r},\tau)\right]\exp(\mathrm{i}\omega\tau) \tag{28}$$

is the so-called complex permittivity. Obviously,

$$\varepsilon(\mathbf{r},\ -\omega) = [\varepsilon(\mathbf{r},\omega)]^*. \tag{29}$$

Materials with $\mathrm{Im}[\varepsilon(\mathbf{r},\omega)] \equiv 0$ are traditionally called lossless, where "Im" stands for "the imaginary part of". It is postulated in the framework of FED that such materials are non-emitting: $\mathbf{J}^{\mathrm{f}}(\mathbf{r},t) \equiv \mathbf{0}$ and hence $\mathbf{J}^{\mathrm{f}}(\mathbf{r},\omega) \equiv \mathbf{0}$. Using the approach outlined in Section 4.5.1 of Ref. [24], it is straightforward to show that such materials are also nonabsorbing. This means that the Poynting theorem (15) takes the form

$$\int_{-\infty}^{\infty} \mathrm{d}t \int_S \mathrm{d}^2\mathbf{r}\,\mathbf{S}(\mathbf{r},t)\cdot\hat{\mathbf{n}}(\mathbf{r}) = 0 \tag{30}$$

provided that the total electromagnetic field builds from zero starting at $t=-\infty$ and then decays back to zero at $t=\infty$.

Note that in the frequency domain Eqs. (21)–(24) are no longer independent. Indeed, taking the divergence of both sides of Eq. (22) and accounting for Eq. (6) yields Eq. (23), while taking the divergence of both sides of Eq. (24) yields Eq. (21). Therefore, of the four Maxwell equations (21)–(24) we will consider in what follows only the curl equations (22) and (24).

5. Standard scattering–emission problem

Consider a fixed finite object embedded in an infinite medium that is assumed to be homogeneous, linear, isotropic, and nonabsorbing. Accordingly, the complex permittivity of the host medium ε_1 is assumed to be real-valued: $\mathrm{Im}\,\varepsilon_1 = 0$. The object can be either a single body or a cluster consisting of a finite number $N \geq 1$ of separated or touching components; it occupies collectively a finite "interior" region V_{INT} given by

$$V_{\mathrm{INT}} = \bigcup_{i=1}^{N} V_i, \tag{31}$$

where V_i is the volume occupied by the ith component (see Fig. 1). The object is surrounded by the infinite exterior region V_{EXT} such that $V_{\mathrm{INT}} \cup V_{\mathrm{EXT}} = \Re^3$, where $\Re^3$ denotes the entire three-dimensional space. The interior region is filled with isotropic, linear, and possibly inhomogeneous material. Point O serves as the common origin of all position vectors and as the origin of the laboratory coordinate system.

Since the host medium is assumed to be non-absorbing, it is also non-emitting. Therefore, the Maxwell curl equations (22) and (24) can now be re-written as follows:

$$\left.\begin{aligned} \nabla \times \mathbf{E}(\mathbf{r},\omega) &= \mathrm{i}\omega\mu_0\mathbf{H}(\mathbf{r},\omega) \\ \nabla \times \mathbf{H}(\mathbf{r},\omega) &= -\mathrm{i}\omega\varepsilon_1\mathbf{E}(\mathbf{r},\omega) \end{aligned}\right\} \mathbf{r} \in V_{\mathrm{EXT}}, \tag{32}$$

$$\left.\begin{aligned} \nabla \times \mathbf{E}(\mathbf{r},\omega) &= \mathrm{i}\omega\mu_0\mathbf{H}(\mathbf{r},\omega) \\ \nabla \times \mathbf{H}(\mathbf{r},\omega) &= -\mathrm{i}\omega\varepsilon_2(\mathbf{r},\omega)\mathbf{E}(\mathbf{r},\omega) + \mathbf{J}^{\mathrm{f}}(\mathbf{r},\omega) \end{aligned}\right\} \mathbf{r} \in V_{\mathrm{INT}}, \tag{33}$$

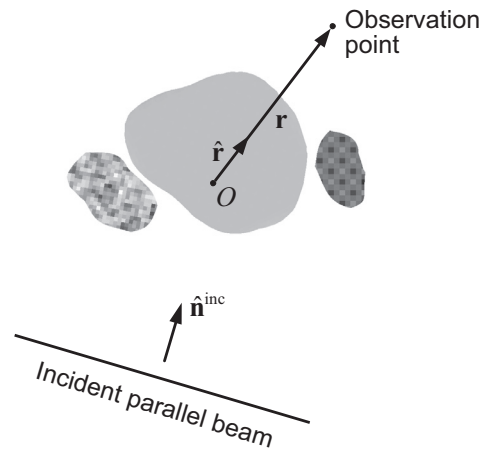

Fig. 1. Schematic representation of the standard scattering–emission problem. The unshaded exterior region V_{EXT} is unbounded in all directions, whereas the shaded areas collectively represent the interior region of the object V_{INT}.

where $\varepsilon_2(\mathbf{r},\omega)$ is the (potentially coordinate-dependent) complex permittivity of the object. The corresponding boundary conditions now read:

$$\left.\begin{aligned} \hat{\mathbf{n}} \times [\mathbf{E}_1(\mathbf{r},\omega) - \mathbf{E}_2(\mathbf{r},\omega)] &= \mathbf{0} \\ \hat{\mathbf{n}} \times [\mathbf{H}_1(\mathbf{r},\omega) - \mathbf{H}_2(\mathbf{r},\omega)] &= \mathbf{0} \end{aligned}\right\} \mathbf{r} \in S_{\mathrm{INT}}, \tag{34}$$

where the subscripts 1 and 2 correspond to the exterior and interior sides of the boundary S_{INT} of the object, respectively, and $\hat{\mathbf{n}}$ is the local outward normal to S_{INT}. According to Eq. (31), S_{INT} is the union of the closed surfaces of the N components of the object:

$$S_{\mathrm{INT}} = \bigcup_{i=1}^{N} S_i. \tag{35}$$

Let us assume that the total field $\{\mathbf{E}(\mathbf{r},t), \mathbf{H}(\mathbf{r},t)\}$ everywhere in space can be represented by a vector superposition of two components: the *incident field* (superscript "inc") and the total *induced field* (superscript "ind") representing cumulatively the field scattered and emitted by the object:

$$\mathbf{E}(\mathbf{r},\omega) = \mathbf{E}^{\mathrm{inc}}(\mathbf{r},\omega) + \mathbf{E}^{\mathrm{ind}}(\mathbf{r},\omega), \tag{36a}$$

$$\mathbf{H}(\mathbf{r},\omega) = \mathbf{H}^{\mathrm{inc}}(\mathbf{r},\omega) + \mathbf{H}^{\mathrm{ind}}(\mathbf{r},\omega). \tag{36b}$$

The incident field is assumed to be a solution of Eq. (32) in the absence of the object, i.e., when $V_{\mathrm{EXT}} = \Re^3$. For example, if the incident field represents a polychromatic parallel beam of light propagating in the direction of the unit vector $\hat{\mathbf{n}}^{\mathrm{inc}}$ then

$$\mathbf{E}^{\mathrm{inc}}(\mathbf{r},\omega) = \mathbf{E}_0^{\mathrm{inc}}(\omega)\exp[\mathrm{i}k_1(\omega)\hat{\mathbf{n}}^{\mathrm{inc}}\cdot\mathbf{r}], \tag{37a}$$

$$\mathbf{H}^{\mathrm{inc}}(\mathbf{r},\omega) = \sqrt{\frac{\varepsilon_1}{\mu_0}}\,\hat{\mathbf{n}}^{\mathrm{inc}} \times \mathbf{E}_0^{\mathrm{inc}}(\omega)\exp[\mathrm{i}k_1(\omega)\hat{\mathbf{n}}^{\mathrm{inc}}\cdot\mathbf{r}] \cancel{- \frac{\mathrm{i}}{\omega\mu_0}\nabla \times \mathbf{E}^{\mathrm{ind}}(\mathbf{r},\omega)}, \tag{37b}$$

where

$$k_1(\omega) = \omega\sqrt{\varepsilon_1\mu_0} \tag{38}$$

is the wave number in the host medium. Note that $k_1(-\omega) = -k_1(\omega)$ and

$$\mathbf{E}_0^{\mathrm{inc}}(-\omega) = [\mathbf{E}_0^{\mathrm{inc}}(\omega)]^*. \tag{39}$$

To ensure the uniqueness of solution of the standard scattering–emission problem, we postulate that the induced field satisfies the following condition at infinity:

$$\lim_{r\to\infty}\{\sqrt{\mu_0}\,\mathbf{r} \times \mathbf{H}^{\mathrm{ind}}(\mathbf{r},\omega) + r\sqrt{\varepsilon_1}\,\mathbf{E}^{\mathrm{ind}}(\mathbf{r},\omega)\} = \mathbf{0}, \tag{40}$$

where $r=|\mathbf{r}|$ is the distance from the origin to the observation point (Fig. 1). The limit (40) holds uniformly over all outgoing directions $\hat{\mathbf{r}}=\mathbf{r}/r$ and is traditionally called the Sommerfeld (or Silver–Müller) radiation condition [2,50–53].

6. Stationarity of the random incident field and fluctuating sources

Let us assume that owing to relatively slow random variations in time of the Fourier harmonics $\mathbf{E}^{\rm inc}(\mathbf{r},\omega)$, the incident field

$$\mathbf{E}^{\rm inc}(\mathbf{r},t)=\int_{-\infty}^{\infty}{\rm d}\omega\,\mathbf{E}^{\rm inc}(\mathbf{r},\omega)\exp(-{\rm i}\omega t) \tag{41}$$

is quasi-polychromatic (this term is introduced by analogy with the term "quasi-monochromatic" typically used to characterize a monochromatic field with a slowly fluctuating complex amplitude [17,20]) and as such is a stationary random process. Then, by definition, correlation functions of the form $\langle\mathbf{E}^{\rm inc}(\mathbf{r},t)\otimes\mathbf{E}^{\rm inc}(\mathbf{r}',t+\tau)\rangle_\xi$ must be functions of τ only, where $\otimes$ denotes the dyadic product of two vectors and $\langle\cdots\rangle_\xi$ hereinafter denotes ensemble averaging (in this case the average over the ensemble of realizations of the random incident field). We thus have

$$\begin{aligned}&\langle\mathbf{E}^{\rm inc}(\mathbf{r},t)\otimes\mathbf{E}^{\rm inc}(\mathbf{r}',t+\tau)\rangle_\xi\\&=\int_{-\infty}^{\infty}{\rm d}\omega\int_{-\infty}^{\infty}{\rm d}\omega'\,\langle\mathbf{E}^{\rm inc}(\mathbf{r},\omega)\otimes\mathbf{E}^{\rm inc}(\mathbf{r}',\omega')\rangle_\xi,\\&\quad\times\exp(-{\rm i}\omega t)\exp[-{\rm i}\omega'(t+\tau)].\end{aligned} \tag{42}$$

For the right-hand side to depend on τ only, the Fourier components of the incident field must be delta-correlated:

$$\langle\mathbf{E}^{\rm inc}(\mathbf{r},\omega)\otimes\mathbf{E}^{\rm inc}(\mathbf{r}',\omega')\rangle_\xi$$

$$=\delta(\omega+\omega')\langle\mathbf{E}^{\rm inc}(\mathbf{r},\omega)\otimes[\mathbf{E}^{\rm inc}(\mathbf{r}',\omega)]^*\rangle_\xi \tag{43a}$$

$$\begin{aligned}&=\delta(\omega+\omega')\exp[{\rm i}k_1(\omega)\hat{\mathbf{n}}^{\rm inc}\cdot(\mathbf{r}-\mathbf{r}')]\\&\quad\times\langle\mathbf{E}^{\rm inc}(\omega)\otimes[\mathbf{E}^{\rm inc}(\omega)]^*\rangle_\xi,\end{aligned} \tag{43b}$$

where $\delta(\omega)$ is the delta function, the first equality is a manifestation of the Khinchin (or Wiener–Khinchin) theorem [54], and the second equality applies specifically to a quasi-polychromatic parallel beam.

Let us also assume that the object is in local thermal equilibrium at a temperature $T(\mathbf{r})$, $\mathbf{r}\in V_{\rm INT}$, and, consequently, the volume density of the fluctuating electric current inside $V_{\rm INT}$ is a stationary random process. Thus the Fourier components of $\mathbf{J}^{\rm f}(\mathbf{r},t)$ must also be delta-correlated in angular frequency. Furthermore, invoking the fluctuation–dissipation theorem for an isotropic medium [40–44], we have for $\mathbf{r}$, $\mathbf{r}'\in V_{\rm INT}$:

$$\begin{aligned}&\langle\mathbf{J}^{\rm f}(\mathbf{r},\omega)\otimes\mathbf{J}^{\rm f}(\mathbf{r}',\omega')\rangle_\xi\\&=\delta(\omega+\omega')\langle\mathbf{J}^{\rm f}(\mathbf{r},\omega)\otimes[\mathbf{J}^{\rm f}(\mathbf{r}',\omega)]^*\rangle_\xi\end{aligned} \tag{44a}$$

$$=\frac{\omega}{\pi}{\rm Im}[\varepsilon_2(\mathbf{r},\omega)]\Theta[|\omega|,T(\mathbf{r})]\delta(\mathbf{r}-\mathbf{r}')\delta(\omega+\omega')\overleftrightarrow{I}, \tag{44b}$$

where $\overleftrightarrow{I}$ is the identity dyadic, $\delta(\mathbf{r})$ is the three-dimensional delta function,

$$\Theta[\omega,T]=\frac{\hbar\omega}{\exp\left(\frac{\hbar\omega}{k_{\rm B}T}\right)-1} \tag{45}$$

is the mean energy of the quantum harmonic oscillator devoid of the zero-point energy term [44], $\hbar$ is the reduced Planck constant, and $k_{\rm B}$ is the Boltzmann constant. Note that consistent with the previous discussion, the object does not emit electromagnetic radiation if ${\rm Im}[\varepsilon_2(\mathbf{r},\omega)]\equiv 0$.

It is further postulated that for any t,

$$\langle\mathbf{J}^{\rm f}(\mathbf{r},t)\rangle_\xi=\mathbf{0}, \tag{46}$$

which implies that

$$\int_{-\infty}^{\infty}{\rm d}\omega\,\langle\mathbf{J}^{\rm f}(\mathbf{r},\omega)\rangle_\xi\exp(-{\rm i}\omega t)\equiv\mathbf{0} \tag{47}$$

and hence

$$\langle\mathbf{J}^{\rm f}(\mathbf{r},\omega)\rangle_\xi\equiv\mathbf{0}. \tag{48}$$

Finally, we assume that the incident field and the fluctuating sources are *independent* random processes. This implies, in particular, that

$$\begin{aligned}&\langle\mathbf{E}^{\rm inc}(\mathbf{r},t)\otimes\mathbf{J}^{\rm f}(\mathbf{r}',t')\rangle_\xi\\&=\langle\mathbf{E}^{\rm inc}(\mathbf{r},t)\rangle_\xi\otimes\langle\mathbf{J}^{\rm f}(\mathbf{r}',t')\rangle_\xi=\overleftrightarrow{0}\end{aligned} \tag{49}$$

and hence

$$\begin{aligned}&\langle\mathbf{E}^{\rm inc}(\mathbf{r},\omega)\otimes\mathbf{J}^{\rm f}(\mathbf{r}',\omega')\rangle_\xi\\&=\langle\mathbf{E}^{\rm inc}(\mathbf{r},\omega)\rangle_\xi\otimes\langle\mathbf{J}^{\rm f}(\mathbf{r}',\omega')\rangle_\xi=\overleftrightarrow{0},\end{aligned} \tag{50}$$

where $\overleftrightarrow{0}$ is a zero dyad.

Eqs. (43a), (44), (46), and (49) supplement the set of axioms that are used to describe electromagnetic scattering and emission in the framework of FED.

7. Scattering–emission volume integral equation

In Sections 7–10 we will assume that ω is non-negative. The corresponding results for negative angular frequencies follow from Eq. (20) and its analogues.

Although the standard scattering–emission problem has been formulated above as a boundary-value problem for the differential stochastic MMEs, it is more convenient in many cases to deal with an equivalent integral-equation formulation. Eqs. (32) and (33) imply that if $\mathbf{E}(\mathbf{r},\omega)$ is known everywhere in space then $\mathbf{H}(\mathbf{r},\omega)$ can also be determined everywhere in space. From these equations, we easily derive the following vector wave equations for $\mathbf{E}(\mathbf{r},\omega)$:

$$\nabla\times\nabla\times\mathbf{E}(\mathbf{r},\omega)-k_1^2(\omega)\mathbf{E}(\mathbf{r},\omega)=\mathbf{0},\quad\mathbf{r}\in V_{\rm EXT}, \tag{51}$$

$$\begin{aligned}&\nabla\times\nabla\times\mathbf{E}(\mathbf{r},\omega)-\omega^2\varepsilon_2(\mathbf{r},\omega)\mu_0\mathbf{E}(\mathbf{r},\omega)\\&=-{\rm i}\omega\mu_0\mathbf{J}^{\rm f}(\mathbf{r},\omega),\quad\mathbf{r}\in V_{\rm INT}.\end{aligned} \tag{52}$$

Eqs. (51) and (52) can be rewritten as a single inhomogeneous differential equation

$$\nabla\times\nabla\times\mathbf{E}(\mathbf{r},\omega)-k_1^2(\omega)\mathbf{E}(\mathbf{r},\omega)=\mathbf{j}(\mathbf{r},\omega),\quad\mathbf{r}\in\Re^3, \tag{53}$$

where

$$\mathbf{j}(\mathbf{r},\omega)=U(\mathbf{r},\omega)\mathbf{E}(\mathbf{r},\omega)+{\rm i}\omega\mu_0\mathbf{J}^{\rm f}(\mathbf{r},\omega) \tag{54}$$

is the forcing function and

$$U(\mathbf{r},\omega)=\begin{cases}0,&\mathbf{r}\in V_{\rm EXT},\\ \omega^2\varepsilon_2(\mathbf{r},\omega)\mu_0-k_1^2(\omega),&\mathbf{r}\in V_{\rm INT}\end{cases} \tag{55}$$

is the potential function. It is evident that $\mathbf{j}(\mathbf{r},\omega)$ vanishes everywhere outside the interior region.

Any solution of an inhomogeneous linear differential equation can be expressed as a sum of two parts: (i) a solution of the respective homogeneous equation with the right-hand side identically equal to zero, and (ii) a particular solution of the inhomogeneous equation. In the case of Eq. (53), the first part satisfies the equation

$$\nabla\times\nabla\times\mathbf{E}^{\rm inc}(\mathbf{r},\omega)-k_1^2(\omega)\mathbf{E}^{\rm inc}(\mathbf{r},\omega)=\mathbf{0},\quad\mathbf{r}\in\Re^3 \tag{56}$$

and describes the field that would exist in free space in the absence of the object, i.e., the *incident field*. The physically appropriate particular solution of Eq. (53) must give the *induced field* $\mathbf{E}^{\rm ind}(\mathbf{r},\omega)$ corresponding to the forcing function $\mathbf{j}(\mathbf{r},\omega)$. Obviously, of all possible particular solutions of Eq. (53), we must choose the one that satisfies the boundary conditions (34) and the radiation condition (40).

Paralleling the derivation detailed in Section 4.3 of [17] and explicitly based on the boundary conditions as well as on the radiation condition at infinity, we obtain:

$$\mathbf{E}^{\rm ind}(\mathbf{r},\omega)=\int_{V_{\rm INT}}{\rm d}^3\mathbf{r}'\overleftrightarrow{G}(\mathbf{r},\mathbf{r}',\omega)\cdot\mathbf{j}(\mathbf{r}',\omega),\quad \mathbf{r}\in\Re^3, \tag{57}$$

where

$$\overleftrightarrow{G}(\mathbf{r},\mathbf{r}',\omega)=\left(\overleftrightarrow{I}+\frac{1}{k_1^2(\omega)}\nabla\otimes\nabla\right)g(\mathbf{r},\mathbf{r}',\omega) \tag{58}$$

is the free-space dyadic Green's function and

$$g(\mathbf{r},\mathbf{r}',\omega)=\frac{\exp[{\rm i}k_1(\omega)|\mathbf{r}-\mathbf{r}'|]}{4\pi|\mathbf{r}-\mathbf{r}'|} \tag{59}$$

is the scalar Green's function. The latter satisfies the three-dimensional Helmholtz equation

$$[\nabla^2+k_1^2(\omega)]g(\mathbf{r},\mathbf{r}',\omega)=-\delta(\mathbf{r}-\mathbf{r}'). \tag{60}$$

The final step is to substitute Eq. (54) in Eq. (57), which yields

$$\mathbf{E}^{\rm ind}(\mathbf{r},\omega)=\mathbf{E}^{\rm sca}(\mathbf{r},\omega)+\mathbf{E}^{\rm f}(\mathbf{r},\omega), \tag{61}$$

where

$$\mathbf{E}^{\rm sca}(\mathbf{r},\omega)=\int_{V_{\rm INT}}{\rm d}^3\mathbf{r}'\,U(\mathbf{r}',\omega)\overleftrightarrow{G}(\mathbf{r},\mathbf{r}',\omega)\cdot\mathbf{E}(\mathbf{r}',\omega) \tag{62}$$

can be called the *scattered field* since it does not vanish in the absence of emission, and

$$\mathbf{E}^{\rm f}(\mathbf{r},\omega)={\rm i}\omega\mu_0\int_{V_{\rm INT}}{\rm d}^3\mathbf{r}'\overleftrightarrow{G}(\mathbf{r},\mathbf{r}',\omega)\cdot\mathbf{J}^{\rm f}(\mathbf{r}',\omega) \tag{63}$$

is the *fluctuating self-emitted field*. Eqs. (36a) and (61)–(63) represent collectively the scattering–emission volume integral equation (SEVIE).

Note that owing to the short-hand way Eqs. (57), (62), and (63) are written, they contain a strong singularity (strictly speaking, a non-integrable one) when $\mathbf{r}\in V_{\rm INT}$. As discussed in Refs. [55,56], this implies that the integration must be carried in the following specific principal-value sense:

$$\int_{V_{\rm INT}}{\rm d}^3\mathbf{r}'\overleftrightarrow{G}(\mathbf{r},\mathbf{r}',\omega)F(\mathbf{r}')$$
$$=\lim_{V_0\to 0}\int_{V_{\rm INT}\setminus V_0}{\rm d}^3\mathbf{r}'\overleftrightarrow{G}(\mathbf{r},\mathbf{r}',\omega)F(\mathbf{r}')-\frac{1}{3k_1^2(\omega)}F(\mathbf{r}), \tag{64}$$

where V_0 is a spherical exclusion volume around $\mathbf{r}$. In what follows, we always imply the abbreviation (64).

Let us now re-write the total field as follows:

$$\mathbf{E}(\mathbf{r},\omega)=\mathbf{E}^{\rm exc}(\mathbf{r},\omega)+\mathbf{E}^{\rm sca}(\mathbf{r},\omega),\quad \mathbf{r}\in\Re^3, \tag{65}$$

where

$$\mathbf{E}^{\rm exc}(\mathbf{r},\omega)=\mathbf{E}^{\rm inc}(\mathbf{r},\omega)+\mathbf{E}^{\rm f}(\mathbf{r},\omega) \tag{66}$$

is the *exciting field*. Both $\mathbf{E}^{\rm inc}(\mathbf{r},\omega)$, defined by Eq. (36a), and $\mathbf{E}^{\rm f}(\mathbf{r},\omega)$, given by Eq. (63), are assumed to be known. The SEVIE can now be cast in the following compact form:

$$\mathbf{E}(\mathbf{r},\omega)=\mathbf{E}^{\rm exc}(\mathbf{r},\omega)+\int_{V_{\rm INT}}{\rm d}^3\mathbf{r}'\,U(\mathbf{r}',\omega)\overleftrightarrow{G}(\mathbf{r},\mathbf{r}',\omega)$$
$$\cdot\mathbf{E}(\mathbf{r}',\omega),\quad \mathbf{r}\in\Re^3. \tag{67}$$

This equation is essentially the same as Eq. (17) in Ref. [36] but uses notation that is more convenient for the following derivations. In particular, it makes it obvious that the only formal difference of Eqs. (65) and (67) from their classical elastic-scattering counterparts [17] is that $\mathbf{E}^{\rm inc}(\mathbf{r},\omega)$ in the latter has been replaced by $\mathbf{E}^{\rm exc}(\mathbf{r},\omega)$ in the former.

8. Dyadic transition operator

The linearity and the integration domain of the SEVIE imply that it must be possible to express the scattered electric field linearly in terms of the exciting field inside $V_{\rm INT}$. The general expression for such relation, more specifically, the expression of $U(\mathbf{r},\omega)\mathbf{E}(\mathbf{r},\omega)$ in terms of $\mathbf{E}^{\rm exc}(\mathbf{r},\omega)$, is a linear integral operator with the kernel $\overleftrightarrow{T}(\mathbf{r},\mathbf{r}',\omega)$:

$$U(\mathbf{r},\omega)\mathbf{E}(\mathbf{r},\omega)=\int_{V_{\rm INT}}{\rm d}^3\mathbf{r}'\overleftrightarrow{T}(\mathbf{r},\mathbf{r}',\omega)\cdot\mathbf{E}^{\rm exc}(\mathbf{r}',\omega),\quad \mathbf{r}\in V_{\rm INT}, \tag{68}$$

where $\overleftrightarrow{T}(\mathbf{r},\mathbf{r}',\omega)$ is the so-called dyadic transition operator (cf. Refs. [57,58]). The domain of argument values and the integration domain in Eq. (68) can be extended to the whole space $\Re^3$ by setting $\overleftrightarrow{T}(\mathbf{r},\mathbf{r}',\omega)=\overleftrightarrow{0}$ if $\mathbf{r}\notin V_{\rm INT}$ and/or $\mathbf{r}'\notin V_{\rm INT}$.

Substituting this expression into the right-hand side of Eq. (62) results in

$$\mathbf{E}^{\rm sca}(\mathbf{r},\omega)=\int_{V_{\rm INT}}{\rm d}^3\mathbf{r}'\,\overleftrightarrow{G}(\mathbf{r},\mathbf{r}',\omega)$$
$$\cdot\int_{V_{\rm INT}}{\rm d}^3\mathbf{r}''\overleftrightarrow{T}(\mathbf{r}',\mathbf{r}'',\omega)\cdot\mathbf{E}^{\rm exc}(\mathbf{r}'',\omega). \tag{69}$$

Eqs. (62), (65), and (69) then yield the following equation for $\overleftrightarrow{T}$:

$$\overleftrightarrow{T}(\mathbf{r},\mathbf{r}',\omega)=U(\mathbf{r},\omega)\delta(\mathbf{r}-\mathbf{r}')\overleftrightarrow{I}$$
$$+U(\mathbf{r},\omega)\int_{V_{\rm INT}}{\rm d}^3\mathbf{r}''\overleftrightarrow{G}(\mathbf{r},\mathbf{r}'',\omega)\cdot\overleftrightarrow{T}(\mathbf{r}'',\mathbf{r}',\omega). \tag{70}$$

Equations of this type are traditionally called Lippmann–Schwinger equations [59,60]. It is clear that $\overleftrightarrow{T}(\mathbf{r},\mathbf{r}',\omega)$ is exactly the same dyadic operator as that emerging in the classical theory of elastic electromagnetic scattering by a finite object [12,17,57,58].

Iterating Eq. (70) yields

$$\overleftrightarrow{T}(\mathbf{r},\mathbf{r}',\omega)=U(\mathbf{r},\omega)\Big[\delta(\mathbf{r}-\mathbf{r}')\overleftrightarrow{I}+U(\mathbf{r}',\omega)\overleftrightarrow{G}(\mathbf{r},\mathbf{r}',\omega)$$
$$+U(\mathbf{r}',\omega)\int_{V_{\rm INT}}{\rm d}^3\mathbf{r}''U(\mathbf{r}'',\omega)\overleftrightarrow{G}(\mathbf{r},\mathbf{r}'',\omega)\cdot\overleftrightarrow{G}(\mathbf{r}'',\mathbf{r}',\omega)$$
$$+U(\mathbf{r}',\omega)\int_{V_{\rm INT}}{\rm d}^3\mathbf{r}''\int_{V_{\rm INT}}{\rm d}^3\mathbf{r}'''U(\mathbf{r}'',\omega)U(\mathbf{r}''',\omega)$$
$$\times\overleftrightarrow{G}(\mathbf{r},\mathbf{r}'',\omega)\cdot\overleftrightarrow{G}(\mathbf{r}'',\mathbf{r}''',\omega)\cdot\overleftrightarrow{G}(\mathbf{r}''',\mathbf{r}',\omega)$$
$$+\cdots\Big],\quad \mathbf{r},\mathbf{r}'\in V_{\rm INT}. \tag{71}$$

This Neumann expansion and the symmetry properties of the free-space dyadic Green's function (see Appendix B in Ref. [17]) imply the following symmetry relation for the dyadic transition operator:

$$\overleftrightarrow{T}(\mathbf{r},\mathbf{r}',\omega)=[\overleftrightarrow{T}(\mathbf{r}',\mathbf{r},\omega)]^{\rm T}, \tag{72}$$

where T stands for "transposed".

The above derivation of Eq. (72) relies on the presumed convergence of the series (71). It is possible however to give a potentially less restrictive derivation. To this end, let us define the potential dyadic for $\mathbf{r},\mathbf{r}'\in\Re^3$ as

$$\overleftrightarrow{U}(\mathbf{r},\mathbf{r}',\omega)=U(\mathbf{r},\omega)\delta(\mathbf{r}-\mathbf{r}')\overleftrightarrow{I} \tag{73}$$

(implying that $\overleftrightarrow{U}(\mathbf{r},\mathbf{r}',\omega)=\overleftrightarrow{0}$ if $\mathbf{r}\notin V_{\rm INT}$ and/or $\mathbf{r}'\notin V_{\rm INT}$) and introduce short-hand integral-operator notation according to

$$\hat{B}E=\int_{\Re^3}\mathrm{d}^3\mathbf{r}'\,\overleftrightarrow{B}(\mathbf{r},\mathbf{r}')\cdot\mathbf{E}(\mathbf{r}'),\tag{74a}$$

$$E\hat{B}=\int_{\Re^3}\mathrm{d}^3\mathbf{r}'\,\mathbf{E}(\mathbf{r}')\cdot\overleftrightarrow{B}(\mathbf{r}',\mathbf{r}),\tag{74b}$$

$$\hat{B}\hat{C}=\int_{\Re^3}\mathrm{d}^3\mathbf{r}''\,\overleftrightarrow{B}(\mathbf{r},\mathbf{r}'')\cdot\overleftrightarrow{C}(\mathbf{r}'',\mathbf{r}').\tag{74c}$$

Then the above formulas can be re-written as follows:

$$E=E^{\rm exc}+\hat{G}\hat{U}E,\tag{75}$$

$$E=E^{\rm exc}+\hat{G}\hat{T}E^{\rm exc},\tag{76}$$

$$\hat{T}=\hat{U}+\hat{U}\hat{G}\hat{T},\tag{77}$$

where the integration domains can be freely exchanged between $V_{\rm INT}$ and $\Re^3$ owing to the abovementioned properties of $\overleftrightarrow{T}(\mathbf{r},\mathbf{r}',\omega)$ and $\overleftrightarrow{U}(\mathbf{r},\mathbf{r}',\omega)$, while

$$E^{\rm exc}=E^{\rm inc}+\mathrm{i}\omega\mu_0\hat{G}J^{\rm f}.\tag{78}$$

Again, as it was the case with the SEVIE, Eq. (76) differs from its elastic-scattering analogue only in that $\mathbf{E}^{\rm inc}(\mathbf{r},\omega)$ has been replaced by $\mathbf{E}^{\rm exc}(\mathbf{r},\omega)$.

Let us now define

$$\overleftrightarrow{B}^{\rm tr}(\mathbf{r},\mathbf{r}')=[\overleftrightarrow{B}(\mathbf{r}',\mathbf{r})]^{\rm T}.\tag{79}$$

Obviously,

$$\hat{B}E=E\hat{B}^{\rm tr},\tag{80a}$$

$$E\hat{B}=\hat{B}^{\rm tr}E,\tag{80b}$$

$$(\hat{B}\hat{C})^{\rm tr}=\hat{C}^{\rm tr}\hat{B}^{\rm tr}.\tag{80c}$$

A symmetric operator is equal to its own transpose, $\hat{B}=\hat{B}^{\rm tr}$, implying that its kernel satisfies

$$\overleftrightarrow{B}(\mathbf{r},\mathbf{r}')=[\overleftrightarrow{B}(\mathbf{r}',\mathbf{r})]^{\rm T}.\tag{81}$$

The free-space dyadic Green's operator $\hat{G}$ is symmetric, and the same is obviously true of $\hat{U}$. Then, starting from Eq. (77) and using Eq. (80c), we obtain:

$$\hat{T}^{\rm tr}=\hat{U}+\hat{T}^{\rm tr}\hat{G}\hat{U}.\tag{82}$$

Left-multiplying this formula by $(\hat{I}-\hat{U}\hat{G})$ followed by simple algebraic manipulations yields

$$(\hat{T}^{\rm tr}-\hat{U}-\hat{U}\hat{G}\hat{T}^{\rm tr})(\hat{I}-\hat{G}\hat{U})=\hat{0},\tag{83}$$

where $\hat{I}$ and $\hat{0}$ are the unity and zero operators, respectively.

A detailed discussion of the existence and uniqueness of solution of Eqs. (75) and (77) is beyond the scope of this paper (see, e.g., Section 4.2 of Ref. [20]), but we do assume both. Then the operator $\hat{I}-\hat{G}\hat{U}$ is invertible, which, along with Eq. (83), implies that $\hat{T}^{\rm tr}$ satisfies the same Eq. (77) as $\hat{T}$. The presumed uniqueness of solution of the latter implies that $\hat{T}^{\rm tr}=\hat{T}$, i.e., that its kernel satisfies Eq. (72).

9. Multi-particle scattering–emission equations

Let us now make explicit use of the representation of the scattering/emitting object as a collection of non-overlapping distinct components, as shown in Fig. 1. It is convenient to re-write Eq. (73) as follows:

$$\overleftrightarrow{U}(\mathbf{r},\mathbf{r}',\omega)=\sum_{i=1}^{N}\overleftrightarrow{U}_i(\mathbf{r},\mathbf{r}',\omega),\tag{84}$$

where

$$\overleftrightarrow{U}_i(\mathbf{r},\mathbf{r}',\omega)=\begin{cases}\overleftrightarrow{0}, & \mathbf{r}\notin V_i,\\ U(\mathbf{r},\omega)\delta(\mathbf{r}-\mathbf{r}')\overleftrightarrow{I}, & \mathbf{r}\in V_i.\end{cases}\tag{85}$$

Similarly,

$$\mathbf{J}^{\rm f}(\mathbf{r},\omega)=\sum_{i=1}^{N}\mathbf{J}_i^{\rm f}(\mathbf{r},\omega),\tag{86}$$

where

$$\mathbf{J}_i^{\rm f}(\mathbf{r},\omega)=\begin{cases}\mathbf{0}, & \mathbf{r}\notin V_i,\\ \mathbf{J}^{\rm f}(\mathbf{r},\omega), & \mathbf{r}\in V_i.\end{cases}\tag{87}$$

Let us also introduce the ith-component dyadic transition operator $\overleftrightarrow{T}_i(\mathbf{r},\mathbf{r}',\omega)$ with respect to the common laboratory coordinate system as the one satisfying the equation

$$\hat{T}_i=\hat{U}_i+\hat{U}_i\hat{G}\hat{T}_i,\tag{88}$$

where we use the compact notation of Eq. (74). It is obvious that Eq. (88) is the individual Lippmann–Schwinger equation formulated for the ith component of the object as if all the other components did not exist, and so $\overleftrightarrow{T}_i(\mathbf{r},\mathbf{r}',\omega)=\overleftrightarrow{0}$ if $\mathbf{r}\notin V_i$ and/or $\mathbf{r}'\notin V_i$. It is then straightforward to generalize the derivation outlined in Section 4.1 of Ref. [12] or Section 6.1 of Ref. [17] and show that in the presence of the entire N-component object, the total electric field at any point $\mathbf{r}\in\Re^3$ is given by

$$E=E^{\rm exc}+\sum_{i=1}^{N}\hat{G}\hat{T}_iE_i,\tag{89}$$

where

$$E^{\rm exc}=E^{\rm inc}+E^{\rm f},\tag{90}$$

$$E^{\rm f}=\mathrm{i}\omega\mu_0\sum_{i=1}^{N}\hat{G}J_i^{\rm f},\tag{91}$$

and the partial "exciting" fields are found from the closed system of N integral equations

$$E_i=E^{\rm exc}+\sum_{j(\neq i)=1}^{N}\hat{G}\hat{T}_jE_j.\tag{92}$$

Eqs. (88)–(92) can be called scattering–emission equations for a multi-component object and generalize the famous Foldy equations derived for a non-emitting object [57,61,62]. It is easily seen that they imply the following "order-of-scattering" Neumann series:

$$E=E^{\rm exc}+\sum_{i=1}^{N}\hat{G}\hat{T}_iE^{\rm exc}+\sum_{\substack{i=1\\ j(\neq i)=1}}^{N}\hat{G}\hat{T}_i\hat{G}\hat{T}_jE^{\rm exc}+\sum_{\substack{i=1\\ j(\neq i)=1\\ l(\neq j)=1}}^{N}\hat{G}\hat{T}_i\hat{G}\hat{T}_j\hat{G}\hat{T}_lE^{\rm exc}+\cdots.\tag{93}$$

Comparison of Eqs. (76) and (93) shows that

$$\hat{T} = \sum_{i=1}^{N} \hat{T}_i + \sum_{\substack{i=1 \\ j(\neq i)=1}}^{N} \hat{T}_i \hat{G} \hat{T}_j + \sum_{\substack{i=1 \\ j(\neq i)=1 \\ l(\neq j)=1}}^{N} \hat{T}_i \hat{G} \hat{T}_j \hat{G} \hat{T}_l + \cdots . \tag{94}$$

Needless to say, the practical application of Eqs. (93) and (94) relies on their presumed convergence. Again, the only formal difference of Eqs. (89), (92), and (93) from their elastic-scattering counterparts [12,17] is the substitution of $E^{\rm exc}$ for $E^{\rm inc}$.

10. Far-field limit

Let us now assume that the observation point is located in the far zone of the entire object and hence use the asymptotic formula

$$\overleftrightarrow{G}(\mathbf{r}, \mathbf{r}', \omega) \to (\overleftrightarrow{I} - \hat{\mathbf{r}} \otimes \hat{\mathbf{r}}) \frac{\exp[\mathrm{i}k_1(\omega)r]}{4\pi r} \exp[-\mathrm{i}k_1(\omega)\hat{\mathbf{r}} \cdot \mathbf{r}'] \tag{95}$$

valid in the limit $k_1(\omega)r \to \infty$, $r/r' \to \infty$, and $r/[k_1(\omega)r'^2] \to \infty$ [17]. Then Eqs. (61)–(63) yield

$$\mathbf{E}^{\rm ind}(\mathbf{r}, \omega) \underset{\text{far zone}}{\to} \frac{1}{4\pi} \frac{\exp[\mathrm{i}k_1(\omega)r]}{r} (\overleftrightarrow{I} - \hat{\mathbf{r}} \otimes \hat{\mathbf{r}}) \cdot \int_{V_{\rm INT}} \mathrm{d}^3\mathbf{r}' \, [U(\mathbf{r}', \omega)\mathbf{E}(\mathbf{r}', \omega) + \mathrm{i}\omega\mu_0 \mathbf{J}^{\rm f}(\mathbf{r}', \omega)] \exp[-\mathrm{i}k_1(\omega)\hat{\mathbf{r}} \cdot \mathbf{r}'] \tag{96}$$

and hence

$$\mathbf{H}^{\rm ind}(\mathbf{r}, \omega) \underset{\text{far zone}}{\to} \frac{k_1(\omega)}{\omega\mu_0} \hat{\mathbf{r}} \times \mathbf{E}^{\rm ind}(\mathbf{r}, \omega). \tag{97}$$

These formulas imply that at a sufficiently large distance r from the object, the total field induced by the object via elastic scattering and emission becomes an outgoing transverse spherical wave centered at the object, with an amplitude decreasing as $1/r$ and the electric (as well as magnetic) field vector vibrating perpendicularly to the radial direction:

$$\mathbf{E}^{\rm ind}(\mathbf{r}, \omega) \cdot \hat{\mathbf{r}} \underset{\text{far zone}}{\to} 0, \tag{98}$$

$$\mathbf{H}^{\rm ind}(\mathbf{r}, \omega) \cdot \hat{\mathbf{r}} \underset{\text{far zone}}{\to} 0. \tag{99}$$

11. Separability of the elastic-scattering and thermal-emission problems: the total field

Consistent with the linearity of the stochastic MMEs, the boundary conditions, and the radiation condition at infinity, the results of Sections 7–9 imply that the elastic-scattering and thermal-emission parts of the problem can be completely separated. Indeed, let us define the "elastic", $\mathbf{E}^{\rm sca,e}(\mathbf{r},\omega)$, and "fluctuating", $\mathbf{E}^{\rm sca,f}(\mathbf{r},\omega)$, scattered fields as the solutions of the following separate volume integral equations:

$$\mathbf{E}^{\rm sca,e}(\mathbf{r}, \omega) = \int_{V_{\rm INT}} \mathrm{d}^3\mathbf{r}' \, U(\mathbf{r}', \omega) \overleftrightarrow{G}(\mathbf{r}, \mathbf{r}', \omega) \cdot [\mathbf{E}^{\rm inc}(\mathbf{r}', \omega) + \mathbf{E}^{\rm sca,e}(\mathbf{r}', \omega)], \tag{100}$$

$$\mathbf{E}^{\rm sca,f}(\mathbf{r}, \omega) = \int_{V_{\rm INT}} \mathrm{d}^3\mathbf{r}' \, U(\mathbf{r}', \omega) \overleftrightarrow{G}(\mathbf{r}, \mathbf{r}', \omega) \cdot [\mathbf{E}^{\rm f}(\mathbf{r}', \omega) + \mathbf{E}^{\rm sca,f}(\mathbf{r}', \omega)]. \tag{101}$$

It is then obvious that the sum $\mathbf{E}^{\rm sca,e}(\mathbf{r},\omega) + \mathbf{E}^{\rm sca,f}(\mathbf{r},\omega)$ is the solution of the equation

$$\mathbf{E}^{\rm sca,e}(\mathbf{r}, \omega) + \mathbf{E}^{\rm sca,f}(\mathbf{r}, \omega) = \int_{V_{\rm INT}} \mathrm{d}^3\mathbf{r}' \, U(\mathbf{r}', \omega) \overleftrightarrow{G}(\mathbf{r}, \mathbf{r}', \omega) \cdot [\mathbf{E}^{\rm inc}(\mathbf{r}', \omega) + \mathbf{E}^{\rm f}(\mathbf{r}', \omega) + \mathbf{E}^{\rm sca,e}(\mathbf{r}', \omega) + \mathbf{E}^{\rm sca,f}(\mathbf{r}', \omega)], \tag{102}$$

that is, Eq. (62). Hence,

$$\mathbf{E}^{\rm sca}(\mathbf{r}, \omega) = \mathbf{E}^{\rm sca,e}(\mathbf{r}, \omega) + \mathbf{E}^{\rm sca,f}(\mathbf{r}, \omega). \tag{103}$$

The separability of the elastic-scattering and thermal-emission problems is also explicit in Eqs. (66), (69), and (93). In particular, it is easily seen that

$$\mathbf{E}^{\rm sca,e}(\mathbf{r}, \omega) = \int_{V_{\rm INT}} \mathrm{d}^3\mathbf{r}' \, \overleftrightarrow{G}(\mathbf{r}, \mathbf{r}', \omega) \cdot \int_{V_{\rm INT}} \mathrm{d}^3\mathbf{r}'' \, \overleftrightarrow{T}(\mathbf{r}', \mathbf{r}'', \omega) \cdot \mathbf{E}^{\rm inc}(\mathbf{r}'', \omega) \tag{104}$$

and

$$\mathbf{E}^{\rm sca,f}(\mathbf{r}, \omega) = \int_{V_{\rm INT}} \mathrm{d}^3\mathbf{r}' \, \overleftrightarrow{G}(\mathbf{r}, \mathbf{r}', \omega) \cdot \int_{V_{\rm INT}} \mathrm{d}^3\mathbf{r}'' \, \overleftrightarrow{T}(\mathbf{r}', \mathbf{r}'', \omega) \cdot \mathbf{E}^{\rm f}(\mathbf{r}'', \omega). \tag{105}$$

Thus the total electromagnetic field can be represented as the vector superposition of the "total elastic" and "total fluctuating" fields:

$$\mathbf{E}(\mathbf{r}, \omega) = \mathbf{E}^{\rm te}(\mathbf{r}, \omega) + \mathbf{E}^{\rm tf}(\mathbf{r}, \omega), \tag{106}$$

where

$$\mathbf{E}^{\rm te}(\mathbf{r}, \omega) = \mathbf{E}^{\rm inc}(\mathbf{r}, \omega) + \mathbf{E}^{\rm sca,e}(\mathbf{r}, \omega), \tag{107}$$

$$\mathbf{E}^{\rm tf}(\mathbf{r}, \omega) = \mathbf{E}^{\rm f}(\mathbf{r}, \omega) + \mathbf{E}^{\rm sca,f}(\mathbf{r}, \omega). \tag{108}$$

This of course implies that

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}^{\rm te}(\mathbf{r}, t) + \mathbf{E}^{\rm tf}(\mathbf{r}, t), \tag{109}$$

where

$$\mathbf{E}^{\rm te}(\mathbf{r}, t) = \int_{-\infty}^{\infty} \mathrm{d}\omega \, \mathbf{E}^{\rm te}(\mathbf{r}, \omega) \exp(-\mathrm{i}\omega t), \tag{110}$$

$$\mathbf{E}^{\rm tf}(\mathbf{r}, t) = \int_{-\infty}^{\infty} \mathrm{d}\omega \, \mathbf{E}^{\rm tf}(\mathbf{r}, \omega) \exp(-\mathrm{i}\omega t). \tag{111}$$

Formulas analogous to Eqs. (106)–(111) can be written for the total magnetic field.

If the incident field is a polychromatic parallel beam given by Eq. (37a) then the asymptotic formula (95) and Eq. (104) yield

$$\mathbf{E}^{\rm sca,e}(\mathbf{r}, \omega) = \frac{\exp[\mathrm{i}k_1(\omega)r]}{r} \overleftrightarrow{A}(\hat{\mathbf{r}}, \hat{\mathbf{n}}^{\rm inc}, \omega) \cdot \mathbf{E}_0^{\rm inc}(\omega), \tag{112}$$

where $\overleftrightarrow{A}$ is the so-called scattering dyadic. $\overleftrightarrow{A}$ depends on the incidence, $\hat{\mathbf{n}}^{\rm inc}$, and scattering, $\hat{\mathbf{r}}$, directions, but is independent of $\mathbf{E}_0^{\rm inc}(\omega)$. While

$$\hat{\mathbf{r}} \cdot \overleftrightarrow{A}(\hat{\mathbf{r}}, \hat{\mathbf{n}}^{\rm inc}, \omega) = \mathbf{0}, \tag{113}$$

the dot product $\overleftrightarrow{A}(\hat{\mathbf{n}}^{\rm sca}, \hat{\mathbf{n}}^{\rm inc}, \omega) \cdot \hat{\mathbf{n}}^{\rm inc}$ is not defined by Eq. (112). To complete the definition, we take this product to be zero as well:

$$\overleftrightarrow{A}(\hat{\mathbf{r}}, \hat{\mathbf{n}}^{\rm inc}, \omega) \cdot \hat{\mathbf{n}}^{\rm inc} = \mathbf{0}. \tag{114}$$

Thus,

$$\overleftrightarrow{A}(\hat{\mathbf{r}},\hat{\mathbf{n}}^{\rm inc},\omega)=\frac{1}{4\pi}(\overleftrightarrow{I}-\hat{\mathbf{r}}\otimes\hat{\mathbf{r}})\cdot\int_{V_{\rm INT}}{\rm d}^3\mathbf{r}'\exp[-{\rm i}k_1(\omega)\hat{\mathbf{r}}\cdot\mathbf{r}')]\times\int_{V_{\rm INT}}{\rm d}^3\mathbf{r}''\overleftrightarrow{T}(\mathbf{r}',\mathbf{r}'',\omega)\exp[{\rm i}k_1(\omega)\hat{\mathbf{n}}^{\rm inc}\cdot\mathbf{r}'']\cdot(\overleftrightarrow{I}-\hat{\mathbf{n}}^{\rm inc}\otimes\hat{\mathbf{n}}^{\rm inc}). \tag{115}$$

It is then easily verified that the symmetry property (72) implies the following so-called reciprocity relation for the scattering dyadic:

$$\overleftrightarrow{A}(-\hat{\mathbf{n}}^{\rm inc},-\hat{\mathbf{r}},\omega)=[\overleftrightarrow{A}(\hat{\mathbf{r}},\hat{\mathbf{n}}^{\rm inc},\omega)]^{\rm T}. \tag{116}$$

This relation was originally derived by Saxon [63] using a less straightforward approach.

12. Separability of the elastic-scattering and thermal-emission problems: second moments of the total field

In Section 6 we listed the randomness, stationarity, and independence of the incident field and the fluctuating sources as essential postulates of the semi-classical FED. Coupled with the linearity of Eqs. (63), (104), and (105), they imply that the scattered and emitted fields are also stationary random processes such that

$$\langle\mathbf{E}^{\rm inc}(\mathbf{r},\omega)\otimes\mathbf{E}^{\rm f}(\mathbf{r}',\omega')\rangle_\xi=\overleftrightarrow{0}, \tag{117}$$

$$\langle\mathbf{E}^{\rm inc}(\mathbf{r},\omega)\otimes\mathbf{E}^{\rm sca,f}(\mathbf{r}',\omega')\rangle_\xi=\overleftrightarrow{0}, \tag{118}$$

$$\langle\mathbf{E}^{\rm sca,e}(\mathbf{r},\omega)\otimes\mathbf{E}^{\rm f}(\mathbf{r}',\omega')\rangle_\xi=\overleftrightarrow{0}, \tag{119}$$

$$\langle\mathbf{E}^{\rm sca,e}(\mathbf{r},\omega)\otimes\mathbf{E}^{\rm sca,f}(\mathbf{r}',\omega')\rangle_\xi=\overleftrightarrow{0}. \tag{120}$$

As a consequence,

$$\langle\mathbf{E}^{\rm te}(\mathbf{r},\omega)\otimes\mathbf{E}^{\rm tf}(\mathbf{r}',\omega')\rangle_\xi=\overleftrightarrow{0} \tag{121}$$

and hence

$$\langle\mathbf{E}^{\rm te}(\mathbf{r},t)\otimes\mathbf{E}^{\rm tf}(\mathbf{r}',t')\rangle_\xi=\overleftrightarrow{0}. \tag{122}$$

Furthermore, Eqs. (43a), (44a), (104), and (105) imply that

$$\langle\mathbf{E}^{\rm te}(\mathbf{r},\omega)\otimes\mathbf{E}^{\rm te}(\mathbf{r}',\omega')\rangle_\xi=\delta(\omega+\omega')\langle\mathbf{E}^{\rm te}(\mathbf{r},\omega)\otimes[\mathbf{E}^{\rm te}(\mathbf{r}',\omega)]^*\rangle_\xi \tag{123}$$

and

$$\langle\mathbf{E}^{\rm tf}(\mathbf{r},\omega)\otimes\mathbf{E}^{\rm tf}(\mathbf{r}',\omega')\rangle_\xi=\delta(\omega+\omega')\langle\mathbf{E}^{\rm tf}(\mathbf{r},\omega)\otimes[\mathbf{E}^{\rm tf}(\mathbf{r}',\omega)]^*\rangle_\xi. \tag{124}$$

As a consequence, $\langle\mathbf{E}(\mathbf{r},t)\otimes\mathbf{E}(\mathbf{r}',t)\rangle_\xi$ is independent of t and is given by

$$\langle\mathbf{E}(\mathbf{r},t)\otimes\mathbf{E}(\mathbf{r}',t)\rangle_\xi=\langle\mathbf{E}^{\rm te}(\mathbf{r},t)\otimes\mathbf{E}^{\rm te}(\mathbf{r}',t)\rangle_\xi+\langle\mathbf{E}^{\rm tf}(\mathbf{r},t)\otimes\mathbf{E}^{\rm tf}(\mathbf{r}',t)\rangle_\xi, \tag{125}$$

where

$$\langle\mathbf{E}^{\rm te}(\mathbf{r},t)\otimes\mathbf{E}^{\rm te}(\mathbf{r}',t)\rangle_\xi=2{\rm Re}\int_0^\infty{\rm d}\omega\langle\mathbf{E}^{\rm te}(\mathbf{r},\omega)\otimes[\mathbf{E}^{\rm te}(\mathbf{r}',\omega)]^*\rangle_\xi, \tag{126}$$

$$\langle\mathbf{E}^{\rm tf}(\mathbf{r},t)\otimes\mathbf{E}^{\rm tf}(\mathbf{r}',t)\rangle_\xi=2{\rm Re}\int_0^\infty{\rm d}\omega\langle\mathbf{E}^{\rm tf}(\mathbf{r},\omega)\otimes[\mathbf{E}^{\rm tf}(\mathbf{r}',\omega)]^*\rangle_\xi, \tag{127}$$

and "Re" stands for "the real part of". In particular, the ensemble-averaged total Poynting vector is time-independent and given by

$$\langle\mathbf{S}(\mathbf{r},t)\rangle_\xi=\langle\mathbf{S}^{\rm te}(\mathbf{r},t)\rangle_\xi+\langle\mathbf{S}^{\rm tf}(\mathbf{r},t)\rangle_\xi, \tag{128}$$

where

$$\langle\mathbf{S}^{\rm te}(\mathbf{r},t)\rangle_\xi=2{\rm Re}\int_0^\infty{\rm d}\omega\langle\mathbf{E}^{\rm te}(\mathbf{r},\omega)\times[\mathbf{H}^{\rm te}(\mathbf{r},\omega)]^*\rangle_\xi, \tag{129}$$

$$\langle\mathbf{S}^{\rm tf}(\mathbf{r},t)\rangle_\xi=2{\rm Re}\int_0^\infty{\rm d}\omega\langle\mathbf{E}^{\rm tf}(\mathbf{r},\omega)\times[\mathbf{H}^{\rm tf}(\mathbf{r},\omega)]^*\rangle_\xi. \tag{130}$$

These results will be instrumental in the forthcoming discussion of relevant optical observables and, fundamentally, will enable separate computation of the corresponding elastic-scattering and thermal-emission second moments of the total electromagnetic field.

13. Conclusions

This paper has summarized a fairly straightforward generalization of the standard quasi-polychromatic elastic-scattering formalism for a finite (multiparticle) object [9,12,17,58] intended to allow for thermal self-emission. So far we have focused only on fundamental equations describing the total electromagnetic field and its second moments. We have recapitulated the main axioms of FED, formulated the general scattering–emission problem for a fixed object, and derived the SEVIE, the Lippmann–Schwinger equation for the dyadic transition operator, the generalized scattering–emission version of the Foldy equations, and the far-field limit of the total field. We have shown that in the framework of FED, the computation of the total field is completely separated into the independent computations of the elastically scattered and emitted fields. The same is true of the calculation of quadratic/bilinear combinations in the total electromagnetic field. Subsequent publications will describe the application of this formalism to the computation of optical observables entering the generalized far-field, single-scattering, and radiative-transfer approximations along the lines of Refs. [12,17].

Finally we note that although the incident field is usually assumed to be a plane polychromatic beam, the results of Sections 7–10 are valid as long as the incident field satisfies Eq. (56). Also, the formulas of Section 9 were derived under the assumption that the entire object is a collection of distinct particles. It is clear however that these formulas remain valid for any finite object occupying a volume $V_{\rm INT}$ provided that $V_{\rm INT}$ is subdivided arbitrarily into $N>1$ non-overlapping parts V_i.

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