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Notes

Scattering of Gaussian beams by disordered particulate media

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ABSTRACT

A frequently observed characteristic of electromagnetic scattering by a disordered particulate medium is the absence of pronounced speckles in angular patterns of the scattered light. It is known that such diffuse speckle-free scattering patterns can be caused by averaging over randomly changing particle positions and/or over a finite spectral range. To get further insight into the possible physical causes of the absence of speckles, we use the numerically exact superposition *T*-matrix solver of the Maxwell equations and analyze the scattering of plane-wave and Gaussian beams by representative multi-sphere groups. We show that phase and amplitude variations across an incident Gaussian beam do not serve to extinguish the pronounced speckle pattern typical of plane-wave illumination of a fixed multi-particle group. Averaging over random particle positions and/or over a finite spectral range is still required to generate the classical diffuse speckle-free regime.

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1. Introduction

The two classical monographs by Viktor V. Sobolev [1,2] had served to advance various mathematical aspects of the phenomenological radiative transfer theory (RTT) intended to describe light scattering by turbid media. Since the publication of these treatises, significant progress has also been achieved in the physical foundation of the RTT [3–21]. It has been recognized in particular that a defining trait of the scattering regime described by the RTT is the absence of spike-like speckles in angular patterns of the scattered light. This recognition makes it imperative to understand what factors can serve to extinguish such sharp interference features.

The first-principles analytical derivation of the RTT from the macroscopic Maxwell equations (MMEs) given in [14,20] has demonstrated that the diffuse speckle-free regime can result from averaging the scattering patterns generated by a large multi-particle group over particle positions randomly varying in time. This derivation is based on the assumption that the incident electromagnetic field is a monochromatic plane wave and is then generalized to the case of illumination by a superposition of statistically independent quasi-monochromatic plane waves [20]. One can think of other factors potentially resulting in speckle-free scattering patterns [22,23], especially in observations of fixed particulate samples such as powder surfaces. For example, it has been shown recently on the basis of direct computer solutions of the MMEs [24] that averaging scattering patterns generated by a fixed quasi-random multi-particle configuration over a finite spectral range can be quantitatively equivalent to averaging over random particle coordinates.

The illumination scenario that has not been analyzed so far is the case of an uncollimated incident field such as a finite-width Gaussian beam characterized by strong lateral variations of both phase and amplitude. Of course, the incident Gaussian beam can formally be expanded in plane electromagnetic waves [25,26], but these waves are not statistically independent. This renders the analytical derivation of [20] inapplicable and leaves unanswered the question of whether the illumination of a fixed particulate

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sample by a finite-width laser beam can yield diffuse angular patters of the scattered light or whether ensemble and/or spectral averaging is still necessary to smooth out the speckles. The main objective of this short communication is to answer this question based on direct computer simulations of electromagnetic scattering using the superposition *T*-matrix method (STMM) [27–29].

2. Modeling methodology

Our analysis is based on the comparison of far-field scattering patterns generated by two types of particulate object. The first one (hereinafter Object 1) is a fixed configuration of *N* identical spherical particles quasi-randomly populating an imaginary spherical volume *V* having a radius *R* (Fig. 1). The particle coordinates are assigned by running a random-number generator and making sure that the particle volumes do not overlap.

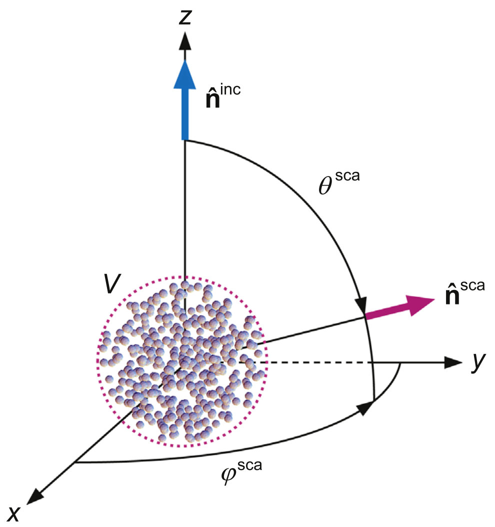

Fig. 1. Scattering geometry.

The second type of object (hereinafter Object 2) is a spherical volume *V* of discrete random medium whose far-field scattering properties are modeled by summing the results obtained by averaging over the equiprobable orientation distribution of Object 1 and of its mirror counterpart. It has been shown previously [20,30] that this type of averaging yields numerical results that are quantitatively equivalent to those obtained by ensemble averaging over random particle positions throughout *V*. Specifically, it has been demonstrated that different statistical realizations of Object 2 yield virtually indistinguishable far-zone scattering characteristics.

The transformation of the time-averaged Stokes column vector of the incident Gaussian beam ("inc") into the time-averaged Stokes column vector of the scattered spherical wavefront ("sca") in the far zone of a fixed particulate object is described by the phase matrix **Z**:

$$\mathbf{I}^{\rm sca}(\rho\hat{\mathbf{n}}^{\rm sca}) = \frac{1}{\rho^2}\mathbf{Z}(\hat{\mathbf{n}}^{\rm sca}, \hat{\mathbf{n}}^{\rm inc})\mathbf{I}^{\rm inc}, \tag{1}$$

where $\hat{\mathbf{n}}^{\rm inc} = \{\theta^{\rm inc}, \varphi^{\rm inc}\}$ is the unit vector in the incidence direction, $\hat{\mathbf{n}}^{\rm sca} = \{\theta^{\rm sca}, \varphi^{\rm sca}\}$ is that in the scattering direction, ρ is the distance from the center of the object to the far-zone observation point, and $\{\theta, \varphi\}$ are the zenith and azimuth angles in the fixed (laboratory) spherical coordinate system centered at the object (Fig. 1).

In what follows, we assume that $\theta^{\rm inc} = 0$. The shape of the Gaussian beam is specified by the non-negative beam

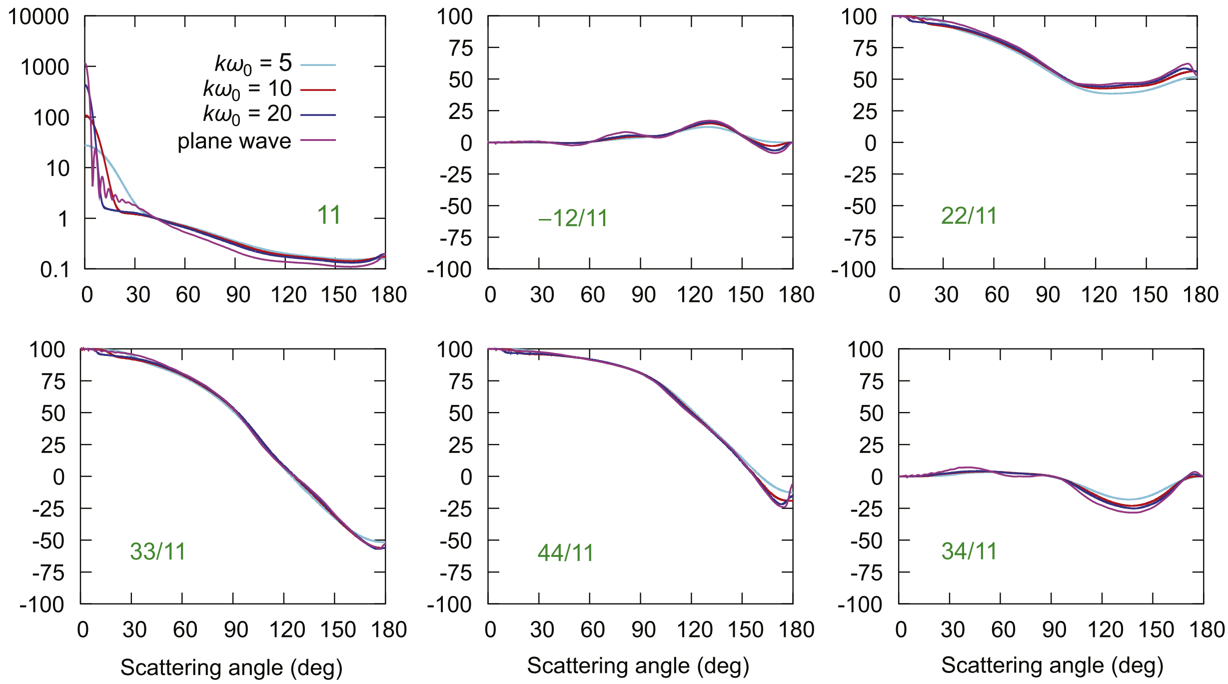

Fig. 2. Elements of the matrix $\tilde{\mathbf{F}}^{(2)}(\Theta)$. "11" denotes the phase function $\tilde{F}_{11}^{(2)}(\Theta)$, while "*ij*/11" denotes $\tilde{F}_{ij}^{(2)}(\Theta)/\tilde{F}_{11}^{(2)}(\Theta)$.

width parameter ω_0, i.e., the distance from the z-axis in the $z=0$ plane required for the electric-field amplitude to decrease by a factor of $1/e$ [29].

In the case of Object 1, the normalized scattering matrix is defined as

$$\tilde{\mathbf{F}}^{(1)}(\Theta) = C^{(1)}\mathbf{Z}(\theta^{\text{sca}} = \Theta,\ \varphi^{\text{sca}} = 0;\ \theta^{\text{inc}} = 0,\ \varphi^{\text{inc}} = 0), \tag{2}$$

where $C^{(1)}$ is a normalization constant. In the case of Object 2, the normalized scattering matrix is calculated according to

$$\tilde{\mathbf{F}}^{(2)}(\Theta) = C^{(2)}\langle \mathbf{Z}(\theta^{\text{sca}} = \Theta,\ \varphi^{\text{sca}};\ \theta^{\text{inc}} = 0,\ \varphi^{\text{inc}} = \varphi^{\text{sca}};\ \Psi)\rangle_\Psi, \tag{3}$$

where $\langle\cdots\rangle_\Psi$ denotes ensemble averaging inasmuch as it is modeled by averaging over the uniform orientation distribution of Object 1 and its mirror counterpart with respect to the laboratory coordinate system. As a consequence of taking the average over the uniform orientation distribution, $\tilde{\mathbf{F}}^{(2)}(\Theta)$ is independent of φ^{sca}. The normalization constants in Eqs. (2) and (3) are chosen such that the (1,1) element of each scattering matrix (i.e., the phase function) satisfies the standard normalization condition

$$\frac{1}{2}\int_0^\pi \mathrm{d}\Theta \tilde{F}_{11}(\Theta)\sin\Theta = 1. \tag{4}$$

The normalized scattering matrices $\tilde{\mathbf{F}}^{(1)}(\Theta)$ and $\tilde{\mathbf{F}}^{(2)}(\Theta)$ are computed using the numerically exact STMM solver of the MMEs described in [27–29]. Note that this technique affords an efficient quasi-analytical orientation-averaging procedure that is highly accurate and, unlike the numerical integration approach, yields $\tilde{\mathbf{F}}^{(2)}(\Theta)$ results completely devoid of residual "noise" (cf. [31]). In all computations, R is fixed at a value implying the volume size parameter $kR=50$, where k is the wave number in the host medium, while r is fixed at a value implying the particle size parameter $kr=4$. The refractive index of the particles is fixed at $m=1.32$ and their number is fixed at $N=200$. Figs. 2 and 3 summarize the results of computations for the following values of the dimensionless beam-width parameter: $k\omega_0 = 5$, 10, 20, and ∞ (plane wave).

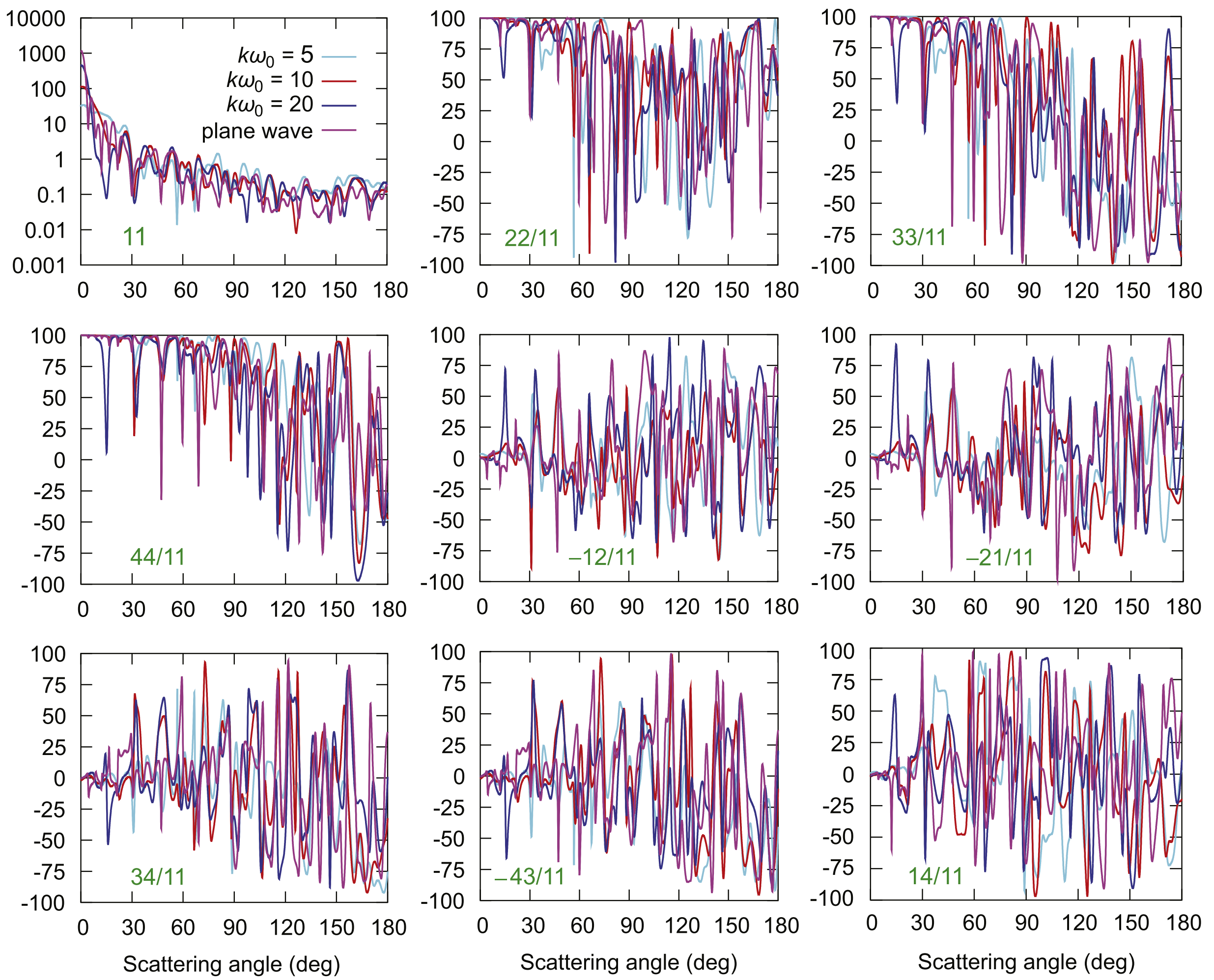

Fig. 3. Elements of the matrix $\tilde{\mathbf{F}}^{(1)}(\Theta)$. "11" denotes the phase function $\tilde{F}_{11}^{(1)}(\Theta)$, while "$ij$/11" denotes $\tilde{F}_{ij}^{(1)}(\Theta)/\tilde{F}_{11}^{(1)}(\Theta)$.

3. Discussion and conclusions

The uniform orientation distribution of Object 1 and its mirror counterpart causes the dimensionless scattering matrix $\tilde{\mathbf{F}}^{(2)}(\Theta)$ to have the following well-known symmetric structure [20,32]:

$$\tilde{\mathbf{F}}^{(2)}(\Theta) = \begin{bmatrix} \tilde{F}_{11}^{(2)}(\Theta) & \tilde{F}_{12}^{(2)}(\Theta) & 0 & 0 \\ \tilde{F}_{12}^{(2)}(\Theta) & \tilde{F}_{22}^{(2)}(\Theta) & 0 & 0 \\ 0 & 0 & \tilde{F}_{33}^{(2)}(\Theta) & \tilde{F}_{34}^{(2)}(\Theta) \\ 0 & 0 & -\tilde{F}_{34}^{(2)}(\Theta) & \tilde{F}_{44}^{(2)}(\Theta) \end{bmatrix}, \tag{5}$$

with

$$\tilde{F}_{12}^{(2)}(0) = \tilde{F}_{34}^{(2)}(0) = \tilde{F}_{12}^{(2)}(\pi) = \tilde{F}_{34}^{(2)}(\pi) = 0. \tag{6}$$

Fig. 2 demonstrates that the elements of the matrix (5) are smooth functions of the scattering angle for all four values of the dimensionless width parameter $k\omega_0$. We can thus conclude that averaging over the ensemble always results in optical traits typical of a spherical volume of discrete random medium irrespective of the lateral width of the incident beam.

Similarly, it is quite obvious from Fig. 3 that there is no radical qualitative difference between the cases of illumination of a fixed quasi-random multi-sphere object by a plane electromagnetic wave or a Gaussian beam of finite lateral width. Irrespective of $k\omega_0$, the angular profiles of all scattering-matrix elements are burdened by pronounced speckles in the form of sharp large-amplitude maxima and minima. The average amplitude of these spike-like features does not seem to decrease with a decrease of $k\omega_0$ from infinity all the way down to 5. Furthermore, the matrix $\tilde{\mathbf{F}}^{(1)}(\Theta)$ obviously lacks the symmetric block-diagonal structure of the matrix $\tilde{\mathbf{F}}^{(2)}(\Theta)$ described by Eqs. (5) and (6). We can thus conclude that, by itself, illuminating a fixed particulate sample by an uncollimated yet fully coherent beam cannot serve to eradicate the speckles and yield scattering patterns typical of the diffuse scattering regime.

The latter conclusion should not be considered totally unexpected and is in fact consistent with the qualitative explanation of speckles given in [24]. Indeed, since the phase of the Gaussian beam is not constant across the beam, the phase difference between the two multi-particle paths shown in Fig. 6 of [24] depends on $k\omega_0$ as well as on the specific coordinates of particles 1 and 1′. Nonetheless, in the case of a fixed multi-particle group this phase difference remains constant in time rather than randomly changes, thereby causing a static contribution to the speckle pattern.

The main implications of this result are as follows. In the case of true discrete random media such as ergodic low-density suspensions of particles in gases and liquids, the nondetection of speckles in the scattered light is most typically explained by random movements of particles during the time required to take an optical measurement. In the case of a fixed particulate sample such as a powder surface, the absence of speckles can be caused by using a polychromatic rather than monochromatic source of illumination. If the source of illumination is a coherent laser beam then the absence of speckles in the angular distribution of light scattered by a fixed particulate sample cannot be explained by the finite width of the beam and can only be caused by using a detector of light with very poor angular resolution.

Acknowledgments

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