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Approximate calculation of coherent backscattering for semi-infinite discrete random media

Victor P. Tishkovets [a,*], Michael I. Mishchenko [b]

[a] Institute of Radio Astronomy of NASU, 4 Chervonopraporna St., Kharkiv 61002, Ukraine
[b] NASA Goddard Institute for Space Studies, 2880 Broadway, New York, NY 10025, USA

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We describe an approximate method for the calculation of all characteristics of coherent backscattering for a homogeneous, semi-infinite particulate medium. The method allows one to transform a system of integral equations describing coherent backscattering exactly into a system of linear algebraic equations affording an efficient numerical solution. Comparisons of approximate theoretical results with experimental data as well as with benchmark numerical results for a medium composed of nonabsorbing Rayleigh scatterers have shown that the method can be expected to give a good accuracy.

1. Introduction

Multiple scattering of electromagnetic waves by discrete random media is the focus of interest in various science and engineering disciplines (see, e.g. [1–4] and references therein). At present, the theory of multiple scattering is adequately developed only for the case of sparse media in which scatterers are located in the far-field zones of each other [1–4]. Scattering characteristics of such discrete random media are primarily determined by the so-called ladder and cyclical diagrams [1–3]. The sum of all ladder diagrams characterizes diffuse multiple scattering and reduces to the vector radiative transfer equation. Methods for the solution of this equation in the case of isotropic and homogeneous particulate media in the form of a plane-parallel layer are now well developed, and computer codes for the calculation of the reflection matrix are available on-line (e.g. [5]).

The sum of the cyclical diagrams characterizes coherent multiple scattering which leads to the effect of coherent enhancement of backscattering (see, e.g. [2] and references therein). The calculation of characteristics of this effect constitutes a very complicated problem even in the case of a homogeneous and isotropic particulate layer. This problem is fully solved only for a semi-infinite medium composed of nonabsorbing Rayleigh scatterers provided that the incident radiation propagates normally to the boundary of medium [2,6]. A rigorous equation describing coherent backscattering for a plane-parallel medium consisting of arbitrary randomly oriented and randomly positioned scatterers has been obtained in [7,8]. A qualitative analysis and an approximate numerical solution of this equation based on retaining only several initial orders of scattering show a strong dependence of coherent backscattering on properties of the particles forming the scattering medium [8–10]. This factor is important for the interpretation of remote-sensing data obtained for various

* Corresponding author. Tel.: +380 577 061 414.
E-mail addresses: tishkovets@ri.kharkov.ua (V.P. Tishkovets), mmishchenko@giss.nasa.gov (M.I. Mishchenko).

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objects of interest. The exact numerical solution of the equation for the coherent part of the scattered radiation represents an extremely complicated problem. Therefore, numerical modeling of coherent backscattering is sometimes based on Monte-Carlo methods [11]. Unfortunately, this requires considerable expenses of computer time and becomes impracticable in the case of a medium composed of nonspherical particles.

In this paper we propose a simple approximate method for the numerical solution of the equation for the coherent part of the reflected radiation in the case of a semi-infinite medium illuminated by external radiation propagating perpendicularly to the boundary of the medium. Comparisons of approximate theoretical results with laboratory data as well as with benchmark numerical results obtained for a medium composed of nonabsorbing Rayleigh scatterers will demonstrate that the new approach is sufficiently accurate for many practical applications.

2. Basic definitions and equations

Let a homogeneous and isotropic semi-infinite particulate medium be illuminated by a plane electromagnetic wave propagating perpendicularly to the boundary of the medium. In this case the coherent part of the reflected radiation is described by the following matrix (see, in [8, Eqs. (46)–(49)]):

$$S^{(C)}_{pn\mu\nu} = \frac{2\pi\upsilon^2}{k_0^4}\sum_{qq_1LM}(-1)^L \zeta^{*(q_1\mu)(qp)}_{LM}\int_0^\infty \beta^{(z)(qn)(q_1\nu)}_{LM}\exp(-\varepsilon z)\,\mathrm{d}z, \tag{1}$$

where the matrix $S^{(C)}_{pn\mu\nu}$ is defined per unit area of the boundary of the medium, $p, n, \mu, \nu, q, q_1 = \pm 1$, $k_0 = 2\pi/\lambda$, is the wave number, λ is the wavelength of the incident radiation, υ is the particle number density, the asterisk denotes complex conjugation,

$$\varepsilon = \mathrm{Im}(\eta)\left(1 - \frac{1}{\cos\vartheta}\right) + \mathrm{i}(1+\cos\vartheta)\left(\frac{\mathrm{Re}(\eta)-1}{\cos\vartheta} + 1\right), \tag{2}$$

η is the complex effective refractive index of the medium, and ϑ is the scattering angle (Fig. 1). Eq. (1) assumes the use of the circular-polarization representation of the Stokes column vector.

The coefficients $\beta^{(z)(qn)(q_1\nu)}_{LM}$ are determined from the following system of equations:

$$\begin{aligned}\beta^{(z)(pn)(\mu\nu)}_{LM} = {} & \exp(-\varepsilon^* z)B^{(z)(pn)(\mu\nu)}_{LM} + \frac{2\pi\upsilon}{k_0^3}\sum_{qq_1lm} i^{M-m}\chi_l^{(pq)(\mu q_1)}\int \beta^{(y)(qn)(q_1\nu)}_{lm} \\ & \times \exp(-\tau\rho)\, d^L_{MN_0}(\omega)\, d^l_{mN_0}(\omega)\, J_{m-M}(\rho\sin\vartheta\sin\omega)\sin\omega\,\mathrm{d}\omega\,\mathrm{d}\rho.\end{aligned} \tag{3}$$

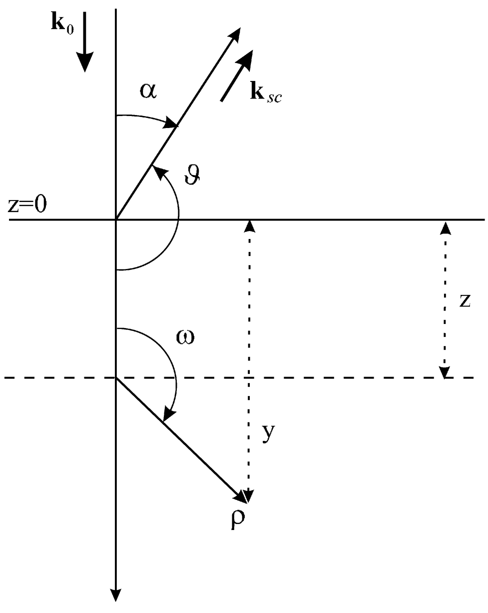

Fig. 1. Geometry of scattering by a semi-infinite medium. The incident light propagates normally to the boundary of the medium ($z = 0$). The directions of incidence and scattering are indicated by the vectors $\mathbf{k}_0$ and $\mathbf{k}_{sc}$, respectively.

Here, $\tau = 2\,\mathrm{Im}(\eta), N_0 = \mu - p, y = z - \rho\cos\omega, d^L_{MN}(\omega)$ is the Wigner d function [12], and $J_m(x)$ is the Bessel function. The angle ω (Fig. 1) is measured from the direction given by $-\mathbf{k}_0$. Furthermore,

$$B^{(z)(pn)(\mu\nu)}_{LM} = \sum_{lm} \zeta^{(pn)(\mu\nu)}_{lm} i^{M-m} \int d^L_{MN_0}(\omega) d^l_{mN_0}(\omega) \exp(-\tau_1\rho) J_{m-M}(\rho\sin\vartheta\sin\omega)\sin\omega\,\mathrm{d}\omega\,\mathrm{d}\rho, \tag{4}$$

where $\tau_1 = 2\,\mathrm{Im}(\eta) - \varepsilon^*\cos\omega$. The coefficients $\chi^{(pn)(\mu\nu)}_l$ and $\zeta^{(pn)(\mu\nu)}_{lm}$ are determined by the amplitude scattering matrix of the particles forming the medium. In particular, for spherical particles they are given by [7,8]

$$\chi^{(pn)(\mu\nu)}_{L_1} = \sum_{Ll} \frac{(2L+1)(2l+1)}{4} \langle a^{(pn)}_L a^{*(\mu\nu)}_l \rangle C^{L_1M_0}_{L-nl\nu} C^{L_1N_0}_{L-pl\mu},$$

$$\zeta^{(pn)(\mu\nu)}_{L_1M_1} = \sum_{Llm} \frac{(2L+1)(2l+1)}{4} \langle a^{(pn)}_L a^{*(\mu\nu)}_l \rangle (-1)^{l+m} C^{L_1M_1}_{L-nl-m} C^{L_1N_0}_{L-pl\mu} d^l_{m\nu}(\vartheta). \tag{5}$$

Here, $a^{(pn)}_L = a_L + pnb_L$, a_L, b_L are the standard Lorenz–Mie coefficients [2], the angular brackets denote averaging over particle properties, $M_0 = \nu - n$, and the C's are Clebsch–Gordan coefficients [12].

The integrals in Eqs. (3) and (4) are defined by (see Fig. 1)

$$\int = \int_0^{\pi/2} \sin\omega\,\mathrm{d}\omega \int_0^{z/\cos\omega} \mathrm{d}\rho + \int_{\pi/2}^{\pi} \sin\omega\,\mathrm{d}\omega \int_0^{\infty} \mathrm{d}\rho. \tag{6}$$

The equations given above describe coherent backscattering for a semi-infinite, homogeneous and isotropic medium composed of arbitrary scatterers. The meaning of these equations is as follows. The coefficients of Eq. (4) characterize the interference of double scattered waves. They determine the solution of the system of Eq. (3) which yields the coefficients $\beta^{(z)(pn)(\mu\nu)}_{LM}$ as functions of z. If one keeps in Eq. (3) only the first term, the matrix of Eq. (1) will describe coherent backscattering in the double-scattering approximation [8,9]. The dependence of characteristics of coherent backscattering on the properties of the medium in the framework of this approximation is discussed in [8–10]. Unfortunately, the numerical solution of the system of Eq. (3) for all scattering orders represents a very complicated problem. In this paper we consider an approximate solution of this system.

Let us denote

$$\gamma^{(pn)(\mu\nu)}_{LM} = 2\,\mathrm{Re}(\varepsilon) \int_0^{\infty} \beta^{(z)(pn)(\mu\nu)}_{LM} \exp(-\varepsilon z)\,\mathrm{d}z. \tag{7}$$

Then the matrix equation (1) becomes

$$S^{(C)}_{pn\mu\nu} = \frac{\pi\upsilon^2}{k_0^4\,\mathrm{Re}(\varepsilon)} \sum_{qq_1LM} (-1)^L \zeta^{*(q_1\mu)(qp)}_{LM} \gamma^{(z)(qn)(q_1\nu)}_{LM}. \tag{8}$$

To derive an equation for the determination of the coefficients in Eq. (7), we multiply the coefficients of Eq. (3) by $2\,\mathrm{Re}(\varepsilon)\exp(-\varepsilon z)$ and integrate over z. The integration of the first term on the right-hand side of Eq. (3) yields the following result (see [8–10]):

$$\begin{aligned} F^{(pn)(\mu\nu)}_{LM} &= 2\,\mathrm{Re}(\varepsilon) \int_0^{\infty} B^{(z)(pn)(\mu\nu)}_{LM} \exp(-2\,\mathrm{Re}(\varepsilon) z)\,\mathrm{d}z \\ &= \sum_{lm} \zeta^{(pn)(\mu\nu)}_{lm} \int_0^{\pi} d^L_{MN_0}(\omega) d^l_{mN_0}(\omega) I_{|m-M|}(c,f)\sin\omega\,\mathrm{d}\omega. \end{aligned} \tag{9}$$

Here,

$$I_m(c,f) = i^{-m} \frac{c^m}{\sqrt{c^2+f^2}\left(f + \sqrt{c^2+f^2}\right)^m},$$

$$c = \sin\vartheta\sin\omega,$$

$$f = 2\,\mathrm{Im}(\eta) + |\cos\omega|\mathrm{Im}(\eta)\left(1 - \frac{1}{\cos\vartheta}\right) + \mathrm{i}\,\cos\omega(1+\cos\vartheta)\left(\frac{\mathrm{Re}(\eta)-1}{\cos\vartheta} + 1\right). \tag{10}$$

The integration of the second term on the right-hand side of Eq. (3) corresponding to the radiation coming to the point z "from above" (the first term in Eq. (6); see Fig. 1), is not very complicated and yields

$$2\,\mathrm{Re}(\varepsilon) i^{M-m} \int_0^{\infty} \exp(-\varepsilon z)\,\mathrm{d}z \int_0^{z/\cos\omega} \beta^{(y)(qn)(q_1\nu)}_{lm} \exp(-\tau\rho) J_{m-M}(c\rho)\,\mathrm{d}\rho = \gamma^{(qn)(q_1\nu)}_{lm} I_{|m-M|}(c,f). \tag{11}$$

The main complexity lies in integrating the term corresponding to the radiation coming "from below" (the second term in Eq. (6)). We will calculate it approximately, assuming that

$$\beta^{(y)(qn)(q_1\nu)}_{lm} = \beta^{(z)(qn)(q_1\nu)}_{lm} \exp(-\sigma\tau\rho|\cos\omega|). \tag{12}$$

In other words, we assume that the coefficient $\beta_{lm}^{(y)(qn)(q_1\nu)}$ at a point y can be approximately represented as the value of this coefficient at the point z multiplied by $\exp(-\sigma\tau\rho|\cos\omega|)$, where σ is a parameter whose value can be determined from a certain condition. In the theory of radiation transport in a semi-infinite atmosphere, a representation similar to Eq. (12) is applied in order to describe the radiation field at large optical depths [13]. An equation for the calculation of σ and an expression for σ as a function of particle characteristics can be found in [13]. However, this equation is valid only for very large optical depths and cannot be directly used to determine σ in our case since the characteristics of radiation reflected by the medium are controlled primarily by the upper layers of the medium. Therefore, to determine σ we will invoke a condition which will be considered latter (see Eq. (18) and the explanation below).

The integration of the last term, upon taking full account of Eq. (12), gives the following expression:

$$2\,\mathrm{Re}(\varepsilon)i^{M-m}\int_0^\infty \exp(-\varepsilon z)\,\mathrm{d}z\int_0^\infty \beta_{lm}^{(y)(qp)(q_1\nu)}\exp(-\tau\rho)J_{m-M}(c\rho)\,\mathrm{d}\rho=\gamma_{lm}^{(qn)(q_1\nu)}I_{|m-M|}(c,g), \tag{13}$$

where

$$g=2\,\mathrm{Im}(\eta)(1+\sigma|\cos\omega|). \tag{14}$$

Thus, the approach considered leads to the following system of linear algebraic equations for the coefficients of Eq. (7):

$$\gamma_{LM}^{(pn)(\mu\nu)}=F_{LM}^{(pn)(\mu\nu)}+\frac{2\pi\upsilon}{k_0^3}\sum_{qq_q lm}\chi_l^{(pq)(\mu q_1)}\gamma_{lm}^{(qn)(q_1\nu)}G_{LMlm}, \tag{15}$$

where

$$\begin{aligned}G_{LMlm}&=\int_0^{\pi/2}I_{|M_2|}(c,f)d_{MN_0}^L(\omega)d_{mN_0}^l(\omega)\sin\omega\,\mathrm{d}\omega+\int_{\pi/2}^{\pi}I_{|M_2|}(c,g)d_{MN_0}^L(\omega)d_{mN_0}^l(\omega)\sin\omega\,\mathrm{d}\omega\\&=\sum_{L_2=|L-l|}^{L+l}C_{LMl-m}^{L_2M_2}C_{LN_0l-N_0}^{L_20}(-1)^m\int_0^{\pi/2}(I_{|M_2|}(c,f)+(-1)^{L_2+M_2}I_{|M_2|}(c,g))d_{M_20}^{L_2}(\omega)\sin\omega\,\mathrm{d}\omega\end{aligned} \tag{16}$$

and $M_2=M-m$.

The solution of the system of Eq. (15) can be obtained, for example, by the method of iterations. The substitution of this solution into Eq. (8) gives the circular-polarization matrix $S_{pn\nu\mu}^{(C)}$. However, in practice the linear-polarization basis is often used. The linear-polarization reflection matrix for the interference component $\mathbf{R}^{(C)}$ has the form (see, for example [9])

$$\begin{aligned}&R_{11}^{(C)}=U\sum_{pn}S_{pnpn}^{(C)},\quad R_{21}^{(C)}=-U\sum_{pn}S_{pn-pn}^{(C)},\quad R_{22}^{(C)}=U\sum_{pn}S_{pn-p-n}^{(C)},\\&R_{33}^{(C)}=U\sum_{pn}S_{pn-p-n}^{(C)}i^{p-n},\quad R_{44}^{(C)}=U\sum_{pn}S_{pnpn}^{(C)}i^{p-n},\quad R_{43}^{(C)}=-iU\sum_{pn}S_{pnp-n}^{(C)}i^{p-n},\end{aligned} \tag{17}$$

where $U=-\pi/2k_0^2\cos\vartheta$. It is assumed that the incident flux per unit area perpendicular to the incident beam is proportional to π, and the Stokes parameters are defined as in [2].

In the case of exact backscattering ($\vartheta=180^\circ$), the matrix $\mathbf{R}^{(C)}$ is related to the incoherent reflection matrix $\mathbf{R}^{(L)}$ via simple formulas [2]. In particular, these formulas yield

$$2R_{11}^{(L)}=R_{11}^{(C)}+R_{22}^{(C)}-R_{33}^{(C)}+R_{44}^{(C)}+R_{11}^1+R_{22}^1-R_{33}^1+R_{44}^1, \tag{18}$$

where $\mathbf{R}^1$ is the single-scattering reflection matrix. We will use the latter formula to determine σ in Eq. (12). Specifically, having solved the radiative transfer equation for given parameters of the medium, we determine the element $R_{11}^{(L)}$. (In the case of a plane-parallel layer, computer codes for the numerical solution of the radiative transfer equation are available on-line [5].) Then, when solving the system (15) with different values of the parameter σ, we select the value such that the equality (18) is satisfied.

Thus, in the framework of the above approach the contribution of the double-scattering component is calculated exactly, while the contribution of the higher orders of scattering is calculated approximately. Below we will evaluate the accuracy of this approximation using exact numerical results computed for a medium composed of nonabsorbing Rayleigh scatterers [2,6] as well as experimental data published in [14].

3. Analysis of accuracy

We compare first the results of calculations for a semi-infinite medium composed of nonabsorbing Rayleigh scatterers. Figs. 2a and b show the dependence of the enhancement factor

$$\tilde{\zeta}_I(\tilde{q})=\frac{R_{11}^{(L)}(0)+R_{11}^{(C)}(\tilde{q})}{R_{11}^{(L)}(0)} \tag{19}$$

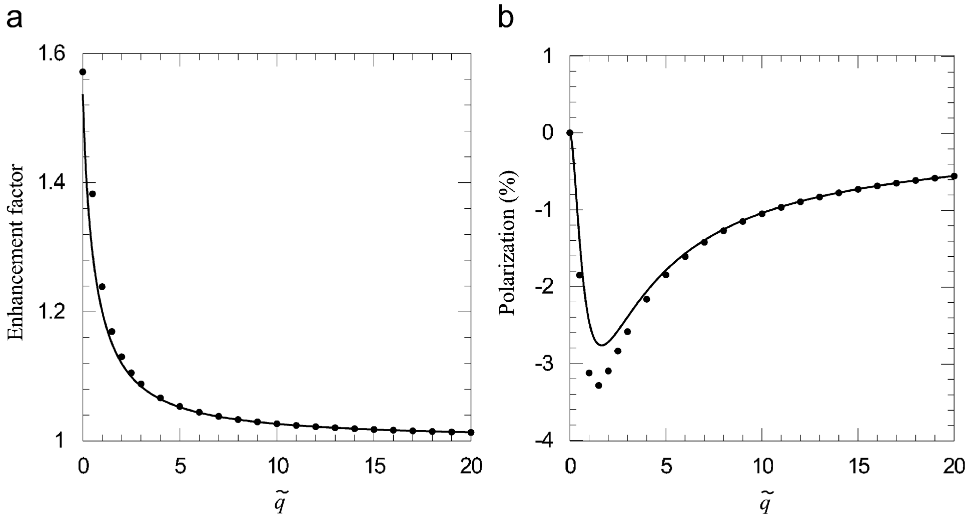

Fig. 2. Enhancement factor $\tilde{\zeta}_I(\tilde{q})$ and degree of linear polarization $P(\tilde{q})$ as functions of the parameter $\tilde{q}$ for a semi-infinite medium composed of nonabsorbing Rayleigh particles with $\tilde{x} = 0.01$ and $\tilde{m} = 1.5 + \mathrm{i}0$. The solid curves depict the exact results of [2], whereas the dots show the result of using approximate formulas.

and the degree of linear polarization

$$P(\tilde{q}) = \frac{R_{21}^{(C)}(\tilde{q})}{R_{11}^{(L)}(0) + R_{11}^{(C)}(\tilde{q})} \tag{20}$$

on the parameter $\tilde{q} = k_0 l_{\mathrm{ext}} \alpha$, where α is the phase angle (Fig. 1) and

$$l_{\mathrm{ext}} = \frac{1}{2k_0 \,\mathrm{Im}(\eta)}. \tag{21}$$

We use the following expression for the effective refractive index of the medium [15]:

$$\eta = 1 + \mathrm{i}\frac{\upsilon}{2k_0} S(0). \tag{22}$$

The forward-scattering amplitude is

$$S(0) = \frac{2\pi}{k_0^2} \sum_l (2l+1)(a_l + b_l) \tag{23}$$

and $\mathrm{Re}(S(0)) = C_{\mathrm{ext}}$, where C_{ext} is the extinction cross-section per particle. The microphysical parameters of the particles are as follows: $\tilde{x} = 0.01$ and $\tilde{m} = 1.5 + \mathrm{i}0$, where $\tilde{x}$ is the size parameter and $\tilde{m}$ is the refractive index. In Fig. 2, the solid curves depict the numerically exact results [2,6], while the dots show the result of using the approximate equations derived above along with the value $\sigma = -0.21$ determined from equality (18). This particular value implies that $(R_{11}^{(C)} + R_{22}^{(C)} - R_{33}^{(C)} + R_{44}^{(C)} + R_{11}^{1} + R_{22}^{1} - R_{33}^{1} + R_{44}^{1})/2 = 1.149$, whereas the solution of the radiative transfer equation yields $R_{11}^{(L)} = 1.147$.

It should be noted that a negative value of σ results in an increase in the coefficients of Eq. (12) with increasing depth z. This behavior can be explained by the increasing radiation density in a nonabsorbing (or slightly absorbing) medium with increasing optical depth [16].

Fig. 2 demonstrates a high accuracy of the approximation as applied to the medium composed of nonabsorbing Rayleigh scatterers. We will now analyze the adequacy of the approximation as applied to a medium composed of wavelength-sized scatterers. In Fig. 3 the experimental data (dots) show the measured co- and cross-polarized components of light scattered by monodisperse spherical polystyrene particles suspended in water [14], i.e., $R_{VV} = (R_{11} + R_{22} + 2R_{12})/2$ and $R_{VH} = (R_{11} - R_{22})/2$, respectively. Here, $\mathbf{R} = \mathbf{R}^{(L)} + \mathbf{R}^{(C)}$ is the reflection matrix. The diameter of the particles is 460 nm, the wavelength of the incident light is 515 nm, and the volume concentration of the particles is $\xi = \upsilon V_0 = 0.1$, where V_0 is the volume of a particle. The solid curves depict the results of theoretical computations. The refractive index of the polystyrene particles is taken to be $\tilde{m} = 1.59 + \mathrm{i}0$, while the value of the parameter σ determined from Eq. (18) is -0.0796. This value corresponds to $(R_{11}^{(C)} + R_{22}^{(C)} - R_{33}^{(C)} + R_{44}^{(C)} + R_{11}^{1} + R_{22}^{1} - R_{33}^{1} + R_{44}^{1})/2 = 1.173$, whereas the respective solution of the radiative transfer equation gives $R_{11}^{(L)} = 1.176$. The calculated co- and cross-polarized components are normalized to the experimental data at $\alpha = 0.6^{\circ}$. The effective refractive index of the medium is calculated using Eq. (23).

As seen from Fig. 3, the approximation provides a good accuracy also for the case of a medium composed of wavelength-sized particles. The relative accuracy of the calculations is better than 10%, which should be acceptable in many applications.

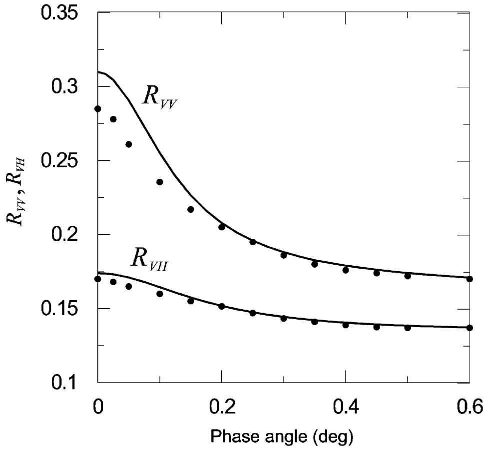

Fig. 3. Comparison of experimental (dots; [14]) and theoretical (solid curves) results for a semi-infinite medium composed of monodisperse spherical polystyrene particles suspended in water.

4. Conclusion

The exact numerical solution of the equation for the coherent part of the scattered radiation constitutes an extremely complicated problem. Therefore the effect of coherent backscattering enhancement as a function of particle microphysical parameters has not been studied in requisite detail. We have proposed a simple approximate method for the calculation of characteristics of coherent backscattering for a semi-infinite medium composed of arbitrary particles. The method is based on the transformation of an exact system of integral equations into a system of linear algebraic equations which can be solved readily. Comparisons of approximate theoretical results with benchmark numerical data computed for a medium composed of nonabsorbing Rayleigh scatterers as well as with experimental data obtained for a medium composed of wavelength-sized particles have shown that our approximation can be expected to give a good accuracy. Unfortunately, we have no reliable experimental or theoretical results which would allow us to test the proposed technique for larger particles and other values of the parameters characterizing the scattering medium. Note, however, that the comparison of the experimental and theoretical results was made without invoking any free model parameters: only the actual, independently measured physical parameters (particle size, refractive index, and concentration as well as the wavelength of light) were used in the calculation.

The approximation can be easily extended to the case of oblique illumination by using the results of [8]. For media consisting of nonspherical particles, a similar approach for the calculation of the coefficients $\zeta_{lm}^{(pn)(\mu\nu)}$ in Eq. (5) can be used (see in [8, Eq. (52)]).

It should finally be noted that such a simple theoretical technique can be developed only for a semi-infinite medium. In the case of a medium of finite optical depth the analytical integration in Eqs. (9), (11) and (13) results in very complex formulae and does not yield a system of algebraic equations similar to that in Eq. (15).

Acknowledgments

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