Depolarization of light backscattered by randomly oriented nonspherical particles

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We derive theoretically and validate numerically general relationships for the elements of the backscattering matrix and for the linear, δ_L, and circular, δ_C, backscattering depolarization ratios for nonspherical particles in random orientation. For the practically important case of randomly oriented particles with a plane of symmetry or particles and their mirror particles occurring in equal numbers and in random orientation, $\delta_C = 2\delta_L/(1 - \delta_L)$. Extensive T-matrix computations for randomly oriented spheroids demonstrate that, although both δ_L and δ_C are indicators of particle nonsphericity, they cannot be considered a universal measure of the departure of particle shape from that of a sphere and have no simple dependence on particle size and refractive index.

It is a common practice in lidar and radar remote sensing to describe polarization characteristics of light scattered by particles in the backscattering direction in terms of linear, δ_L, and circular, δ_C, depolarization ratios.[1,2] For spherical particles both ratios are equal to zero. On the other hand, for nonspherical scatterers both δ_L and δ_C can substantially deviate from zero and thus can be considered indicators of particle nonsphericity. This explains the rapidly increasing interest in lidar and radar polarimetry as a remote sensing technique potentially capable of characterizing the shapes of scattering particles.[1–5] It also makes important and timely a theoretical and numerical study of the depolarization ratios.

The scattering of light by a particle can be described by a 4×4 real scattering matrix (see Sec. 2.5 of Ref. 6). This matrix is a function of the directions of light incidence and scattering and transforms the Stokes parameters I_0, Q_0, U_0, and V_0 of the incident light into those of the scattered light (we assume that the scattering plane acts as the plane of reference for defining the Stokes parameters of both incident and scattered light). The same description can be used for a small-volume element comprising independently scattering particles, in which case the scattering matrix is the sum of the scattering matrices of the individual particles.

For backscattering by a small-volume element comprising arbitrary particles in random orientation, the scattering matrix has the simple form (Sec. 5.32 of Ref. 6)

$$\mathbf{F} = \begin{bmatrix} a_1 & 0 & 0 & b_5 \\ 0 & a_2 & 0 & 0 \\ 0 & 0 & -a_2 & 0 \\ b_5 & 0 & 0 & a_4 \end{bmatrix}, \tag{1}$$

where $|a_2| \le a_1$, $|a_4| \le a_1$, and $|b_5| \le a_1$.[7] Also, using the backscattering theorem in Sec. 5.32 of Ref. 6, we readily verify that

$$a_4 = a_1 - 2a_2 \tag{2}$$

[cf. Eq. (33c) of Ref. 8], which implies that $a_2 \ge 0$. The backscattering matrix is further simplified for a small-volume element comprising (i) randomly oriented particles having a plane of symmetry, such as ellipsoids, and/or (ii) particles and their mirror particles in equal numbers and in random orientation. In these cases $b_5 = 0$, and we have

$$\mathbf{F} = \mathrm{diag}[a_1, a_2, -a_2, a_1 - 2a_2]. \tag{3}$$

Let us now choose a fixed plane through the direction of the incident light and use it as a reference plane for defining Stokes parameters. If the incident beam is 100% linearly polarized, parallel to this plane, its Stokes parameters can be written as $\{I_0, Q_0, U_0, V_0\} = \{1, 1, 0, 0\}$. The linear depolarization ratio, i.e., the ratio of the flux of the cross-polarized component of the backscattered light relative to that of the copolarized component, can now be written as [see Eq. (1)]

$$\delta_L = \frac{I - Q}{I + Q} = \frac{a_1 - a_2}{a_1 + a_2}, \tag{4}$$

where I and Q are the first two Stokes parameters of the backscattered light. Similarly, we can consider a fully circularly polarized incident beam with Stokes parameters $\{1, 0, 0, 1\}$ to obtain the circular backscattering depolarization ratio, δ_C, which is the ratio of the same-helicity component of the backscattered flux relative to that of the opposite-helicity component. The result for randomly oriented particles is [see Eqs. (1) and (2)]

$$\delta_C = \frac{I + V}{I - V} = \frac{a_1 + 2b_5 + a_4}{a_1 - a_4} = \frac{a_1 - a_2 + b_5}{a_2}, \tag{5}$$

where I and V are the first and fourth Stokes parameters of the backscattered light, respectively.

The depolarization ratios have several interesting properties. First, in view of Eqs. (4) and (5), δ_L and

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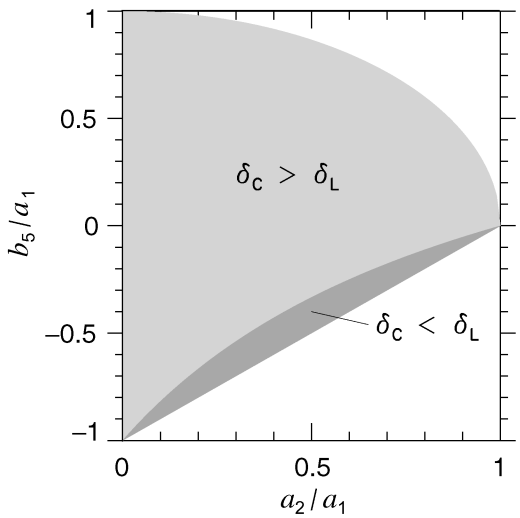

Fig. 1. Domain (shaded area) of possible combinations of b_5/a_1 and a_2/a_1 for backscattering by randomly oriented particles [see inequality (7)]. In the darker region $\delta_C < \delta_L$, whereas in the lighter region $\delta_C > \delta_L$.

δ_C are both nonnegative, since the absolute values of Q and V can never exceed the value of I. This is a result of the general property[6] of Stokes parameters, $I^2 \geq Q^2 + U^2 + V^2$. Second, since $0 \leq a_2 \leq a_1$, we have $0 \leq \delta_L \leq 1$. Third, for (isotropic) spheres we have $b_5 = 0$ and $a_1 = a_2 = -a_4$ so that both δ_L and δ_C vanish. In other words, if the incident light is fully linearly polarized the backscattered signal is fully linearly polarized in the same plane, whereas if the incident light is fully circularly polarized the backscattered light is also fully circularly polarized, albeit in the opposite sense. This is, generally, not the case for nonspherical particles. Therefore δ_L and δ_C can be used as indicators of nonsphericity of particles. Fourth, since $a_2 \geq 0$, Eq. (5) entails

$$a_1 - a_2 + b_5 \geq 0\,, \tag{6}$$

because δ_C is nonnegative. Combining inequality (6) and the general inequality $b_5^2 \leq a_1^2 - a_2^2$ [see Eq. (231) of Ref. 7], we have the inequality

$$(a_2/a_1 - 1) \leq b_5/a_1 \leq [1 - (a_2/a_1)^2]^{1/2}. \tag{7}$$

This inequality is demonstrated in Fig. 1, which also shows the domains in which $\delta_C > \delta_L$ and $\delta_C < \delta_L$. To establish the boundary between these two domains, we first note that $\delta_C > \delta_L$ if $(a_1 - a_2 + b_5)/a_2 > (a_1 - a_2)/(a_1 + a_2)$ [cf. Eqs. (4) and (5)]. Multiplying both sides of the latter inequality by the nonnegative denominators gives $\delta_C > \delta_L$ if

$$b_5/a_1 > (a_2/a_1 - 1)(a_2/a_1 + 1)^{-1} = -\delta_L\,. \tag{8}$$

As shown in Fig. 1, this is the case for most of the possible combinations of b_5/a_1 and a_2/a_1. Finally, if $b_5 = 0$, i.e., if the backscattering matrix is given by Eq. (3), we have from Eq. (5) $\delta_C = (a_1 - a_2)/a_2$, which shows that δ_C is a monotonically increasing function of δ_L, namely,

$$\delta_C = 2\delta_L/(1 - \delta_L)\,. \tag{9}$$

This relation is shown in Fig. 2. It is readily verified that in this case

$$0 \leq 2\delta_L \leq \delta_C \leq \infty\,. \tag{10}$$

Importantly, Eq. (9) implies that δ_C increases with changing particle shape, refractive index, or size if δ_L increases and decreases if δ_L decreases. Therefore positions of local maxima and minima for δ_C coincide with those for δ_L, as was observed in Refs. 9 and 10.

We have validated inequalities (9) and (10) numerically by computing δ_L and δ_C for randomly oriented spheroids, Chebyshev particles, and bispheres, using the T-matrix approach as described in Ref. 11. In all the cases considered, we found that relations (9) and (10) were accurate within the specified accuracy of T-matrix computations.

To investigate the possible influence of particle asphericity on backscattering depolarization and thus on the potential remote sensing content of the linear and circular depolarization ratios, we have performed extensive computations of δ_L and δ_C for monodispersions of randomly oriented oblate and prolate spheroids (Plate I). The refractive index $1.5 + 0.005i$ is typical of mineral tropospheric aerosols and polar stratospheric cloud particles in the visible, while the refractive index $1.78 + 0.005i$ is characteristic of water ice at millimeter and centimeter wavelengths. Because spheroids are particles with a plane of symmetry, Eq. (9) applies and can be used to compute δ_C. We see that the left and right panels of Plate I are dramatically different, thus demonstrating the strong dependence of δ_L (and thus of δ_C) on refractive index. Also, as Plate I shows, depolarization ratios are, in general, strongly dependent on the particle size parameter and aspect ratio. Importantly, depolarization ratios do not systematically increase with increasing aspect ratio; thus both δ_L and δ_C cannot be considered a universal measure of the departure of particle shape from that of a sphere (see also Refs. 9 and 12). Indeed, for prolate spheroids with index of refraction $1.5 + 0.005i$ maximal depolarization is observed for aspect ratios as small as 1.05–1.1. Similarly, depolarization does not exhibit a systematic dependence on size parameter so that, depending on particle shape and refractive index, maximal values of depolarization ratios can be reached at either small or large size parameters. As a result, large (size parameter greater than 20) and strongly aspherical particles can produce nearly zero depolarization ratios. Inter-

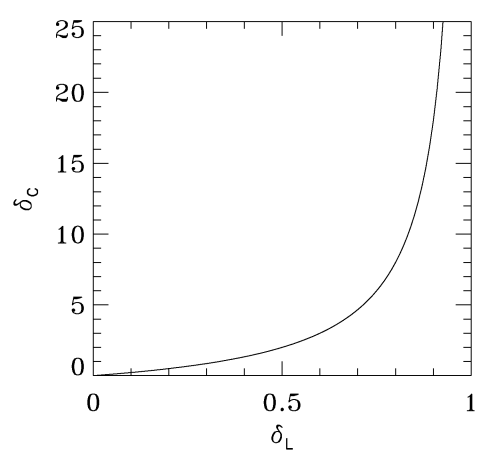

Fig. 2. δ_C versus δ_L for randomly oriented particles with the backscattering matrix given by Eq. (3).

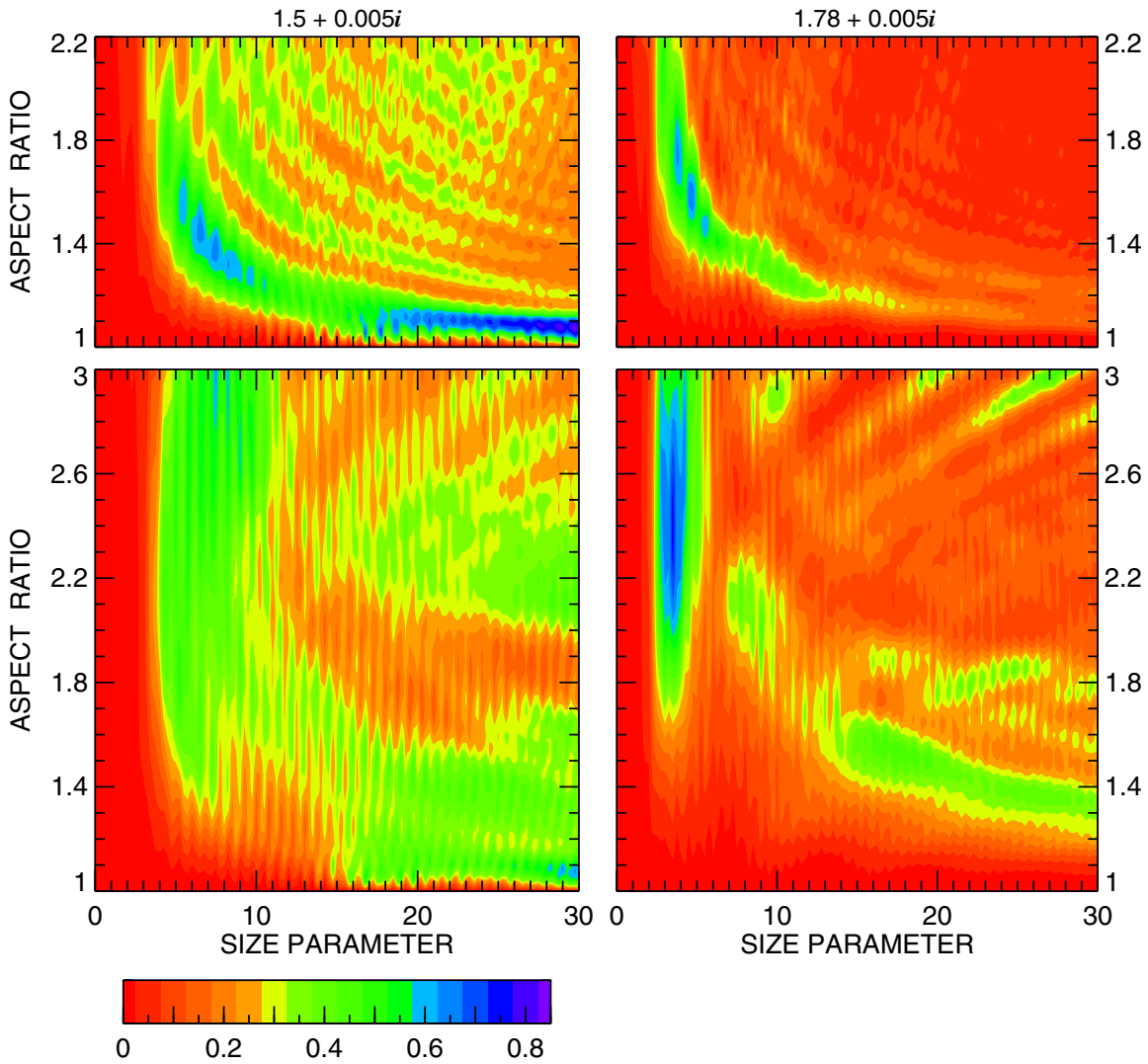

Plate I. Color diagram of δ_L as a function of the aspect ratio (ratio of the larger to the smaller spheroidal axes) and the equal-surface-area-sphere size parameter for prolate (upper panels) and oblate (lower panels) spheroids with indices of refraction 1.5+0.005*i* (left panels) and 1.78+0.005*i* (right panels).

estingly, for the refractive index $1.78 + 0.005i$ maximal depolarization is found at size parameters smaller than 6, i.e., for particles smaller than a wavelength.

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