

# Radiative transfer in plane-parallel media using spherical wavelets

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Wavelets are among the most powerful tools in signal processing owing to their multi-resolution analysis capacity and local support in both frequency and space domain [1]. In recent years, there have been a growing interest in applying wavelet-based methods to solve partial differential equations [2]. The advantages of these methods in solving partial differential equations, in particular the radiative transfer equation, include their adaptivity, sparsity [3], and, more importantly, the feasibility of obtaining data-driven solutions [4]. In this work, we will present a spherical-wavelet-based method for solving the scalar radiative transfer equation in plane-parallel media. In contrast to the conventional methods, such as the discrete-ordinates and adding/doubling methods [5], the monochromatic scalar radiance and single-scattering phase functions will be represented using spherical wavelets in a sparse and adaptive way [6]. The solution to the equation will then be obtained by combining a finite-difference solver. Owing to the capability of data processing using wavelets, this method could open more of the opportunities for cooperation between experiments and numerical simulations.

## References

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