

# Approximate $T$ -matrix and optical properties of spheroidal particles to third order in size parameter

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Nanoparticles are widely used for example in chemistry, medicine, cancer treatment, surface enhanced spectroscopy, partly due to their strong electric field enhancements. Shapes such as nano-rods, disks and deformed spheres are commonly approximated in theory by spheroids to obtain analytic results. The most significant property of small illuminated particles is the induced dipole moment, for which there is a simple expression for spheroids in the electrostatic limit – this involves the “static depolarization factors”. There have been attempts to extend this using “dynamic depolarization factors” [1,2], but these extensions are not consistent with the second order series expansion in size-parameter.

In electromagnetic scattering, the so-called  $T$ -matrix encompasses the optical response of a scatterer for any incident excitation and is most commonly defined using the basis of multipolar fields [3], generalizing the concept of polarizability of a scatterer. Following our recent paper [4], we calculate the series expansion of the  $T$ -matrix for a spheroidal particle in the small-size/long-wavelength limit, up to third lowest order with respect to  $X$ .  $\mathbf{T}$  is calculated from the extended boundary condition method with a linear system involving two infinite matrices  $\mathbf{P}$  and  $\mathbf{Q}$ , whose matrix elements are integrals on the particle surface. The limiting form of the  $P$ - and  $Q$ -matrices for spheroids [5,6], ensures that this Taylor expansion can be obtained by considering only multipoles of order 3 or less (i.e., dipoles, quadrupoles and octupoles). The lowest order is  $O(X^3)$  and equivalent to the quasi-static or Rayleigh approximation. Expressions to order  $O(X^5)$  are obtained by Taylor expansion of the integrals in  $\mathbf{P}$  and  $\mathbf{Q}$  followed by matrix inversion. We then apply a radiative correction, which makes the resulting expressions valid up to order  $O(X^6)$ . Orientation-averaged extinction, scattering, and absorption cross-sections are then derived.

These results provide a simple alternative to the exact  $T$ -matrix method for spheroidal particles smaller than the wavelength, in a size range much larger than for the Rayleigh approximation. The approach also provides details of quadrupole and octopole interactions and their spectral peaks are identified.

## References

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