125 years of radiative transfer: enduring triumphs and persisting misconceptions

Michael Mishchenko

Acknowledgments: Brian Cairns • Joop Hovenier • Michael Kahnert •
Daniel Mackowski • Larry Travis •
NASA Radiation Sciences Program (Hal Maring)

Optimists

"There have been many altered perceptions, usually direct results of changing technologies, but fewer advances in fundamental ideas"

R. M. Goody and Y. L. Yung, *Atmospheric Radiation: Theoretical Basis*
Oxford, Oxford University Press, 1989

"The subject of radiative transfer has matured to the point of being a well-developed tool."

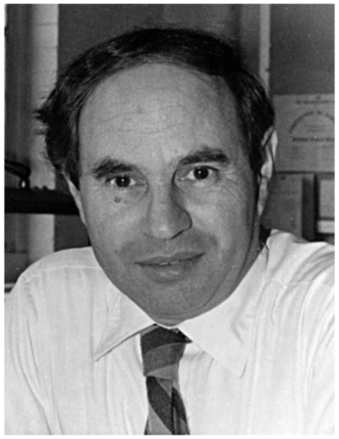
G. E. Thomas and K. Stamnes, *Radiative Transfer in the Atmosphere and Ocean*
Cambridge University Press, 1999

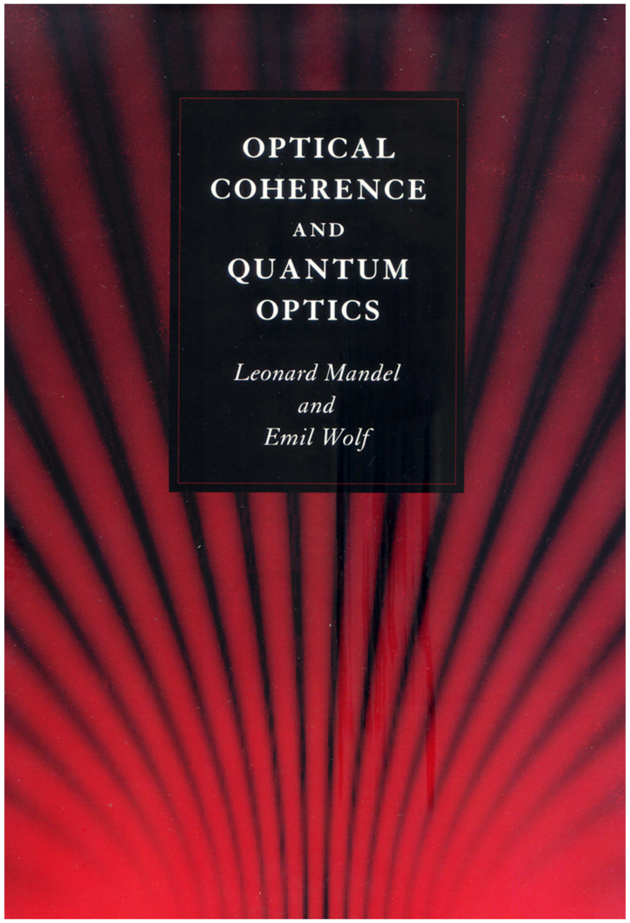
"Radiative transfer theory has reached a high point of development."

W. Zdunkovski et al., *Radiation in the Atmosphere*
Cambridge University Press, 2007

Pessimists

"In spite of the extensive use of the theory of radiative energy transfer, no satisfactory derivation of its basic equation from electromagnetic theory has been obtained up to now…" (1995)





Pessimists

Coherence and radiometry*

Emil Wolf
Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627
(Received 13 October 1977)

Recent researches have revealed that there exists an intimate connection between radiometry and the theory of partial coherence. In this paper a review is presented of some of these developments. After a brief discussion of various models for energy transport in optical fields and of some of the basic concepts of the classical theory of optical coherence, the following topics are discussed: the foundations of radiometry, the coherence properties of Lambertian sources, and the relationship between the state of coherence of a source and the directionality of the light that the source generates. Some very recent work is also described which reveals that certain sources that are spatially highly incoherent in a global sense will generate light that is just as directional as a laser beam.

J. Opt. Soc. Am. **68**, 6–17 (1978)

Optimists

"The derivation of [the radiative transfer] equation for random media with discrete particles is simple…"

A. A. Kokhanovsky, *Optics of Light Scattering Media*
Praxis, Chichester, UK, 2001

"…extending the formulation to account for … polarization… is done quite easily…"

A. A. Kokhanovsky, *Polarization Optics of Random Media*
Praxis, Chichester, UK, 2003

From phenomenology to microphysics

Phenomenological radiometery and RTT

Kepler
Bouguer
Lambert
Beer
Lommel
Khvolson
Planck
Gershun
Ambartsumian
Chandrasekhar
Rozenberg
van de Hulst
Preisendorfer
…

Microphysical RTT

Maxwell
Poynting
Heaviside
Preisendorfer
Twerski
Waterman
Gnedin
Borovoi
Barabanenkov
Wolf
Tsang
…

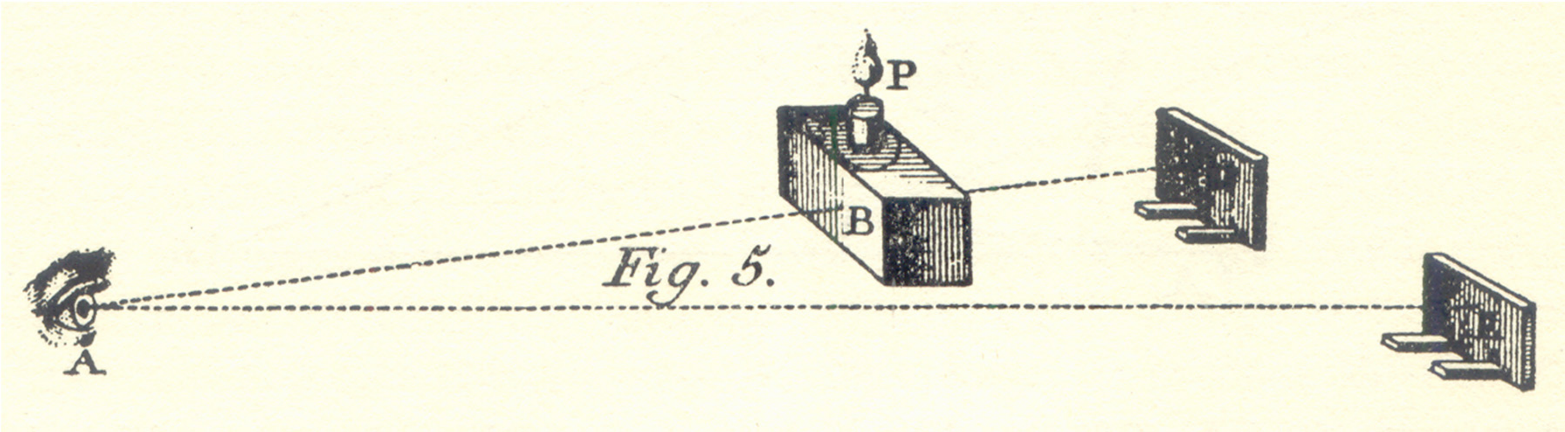
Johannes Kepler (1571–1630)

The inverse square law:

The brightness of an illuminated area diminishes as r^{-2}

Pierre Bouguer (1698–1758)

The first photometer
Eye as a null-point detector
Bouguer extinction law (not Lambert–Beer law!)



Pierre Bouguer (1698–1758)

TRAITÉ
D'OPTIQUE
SUR LA
GRADATION DE LA LUMIERE:
Ouvrage posthume de M. Bouguer, de l'Académie Royale des Sciences, &c.
ET PUBLIÉ par M. l'Abbé DE LA CAILLE, de la même Académie, &c.

Pour servir de Suite aux Mémoires de l'Académie Royale des Sciences.

A PARIS,
De l'Imprimerie de H. L. Guerin & L. F. Delatour, rue Saint Jacques, à Saint Thomas d'Aquin.
M. DCC. LX.
Avec Approbation et Privilege du Roi.

ПЬЕР БУГЕР

ОПТИЧЕСКИЙ ТРАКТАТ
О ГРАДАЦИИ СВЕТА

ПЕРЕВОД
Н. А. ТОЛСТОГО и П. П. ФЕОФИЛОВА

РЕДАКЦИЯ, СТАТЬИ И КОММЕНТАРИИ
ПРОФЕССОРА
А. А. ГЕРШУНА

ИЗДАТЕЛЬСТВО АКАДЕМИИ НАУК СССР
1950

Pierre Bouguer's
Optical Treatise on
THE GRADATION
OF LIGHT

Translated, with Introduction
and Notes by

W. E. KNOWLES MIDDLETON
National Research Council
of Canada, Ottawa

University of Toronto Press

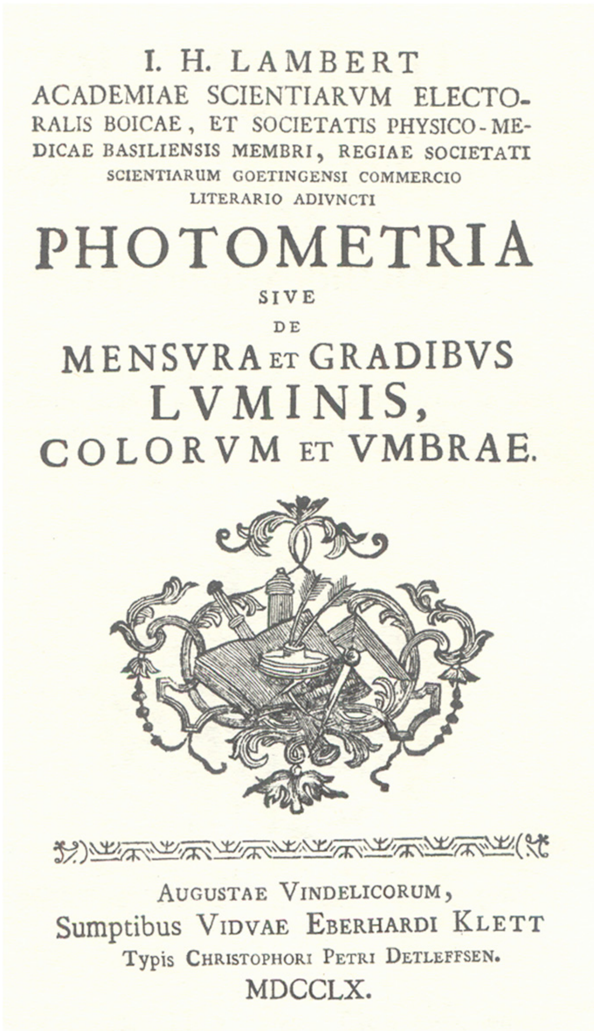
Johannes Lambert (1728–77)

Photometric concepts
Photometric quantities
Mathematical statements
System of photometric principles
Coined the term “albedo”
The cosine law
Much of engineering photometry as we know it today

And also:

Irrationality of π
Analysis of the axiom of parallel lines $\Rightarrow$ Lobachevsky–Bolyai geometry

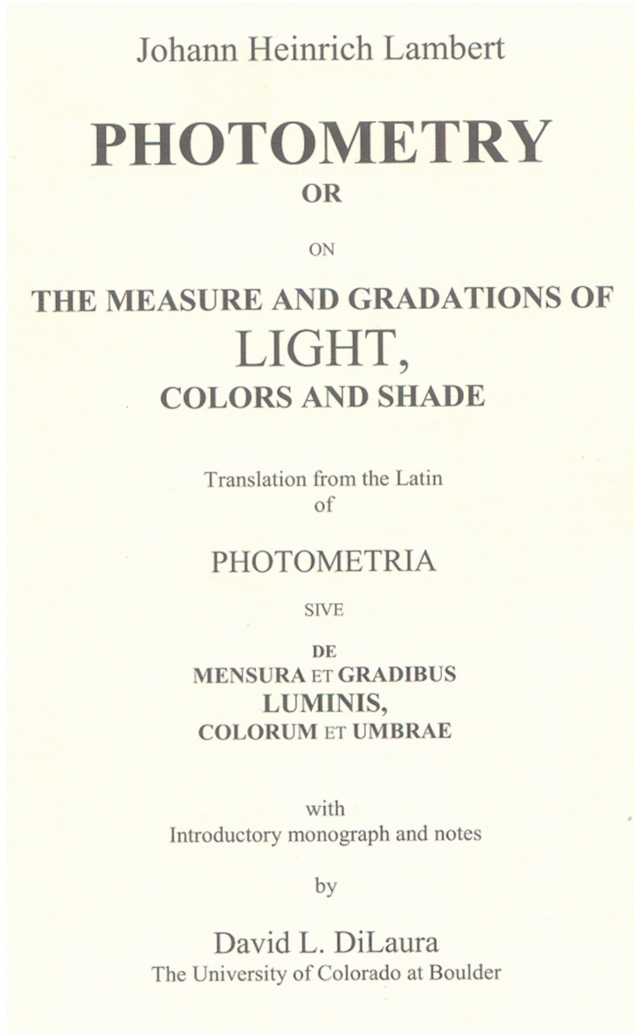
Johannes Lambert (1728–77)



I. H. LAMBERT
ACADEMIAE SCIENTIARVM ELECTORALIS BOICAE, ET SOCIETATIS PHYSICO-MEDICAE BASILIENSIS MEMBRI, REGIAE SOCIETATI SCIENTIARUM GOETINGENSI COMMERCIO LITERARIO ADIVNCTI

PHOTOMETRIA
SIVE
DE
MENSVRA ET GRADIBVS
LVMINIS,
COLORVM ET VMBRAE.

AUGUSTAE VINDELICORUM,
Sumptibus VIDVAE EBERHARDI KLETT
Typis CHRISTOPHORI PETRI DETLEFFSEN.
MDCCLX.



Johann Heinrich Lambert

PHOTOMETRY
OR
ON
THE MEASURE AND GRADATIONS OF
LIGHT,
COLORS AND SHADE

Translation from the Latin
of

PHOTOMETRIA
SIVE
DE
MENSURA ET GRADIBUS
LUMINIS,
COLORUM ET UMBRAE

with
Introductory monograph and notes

by

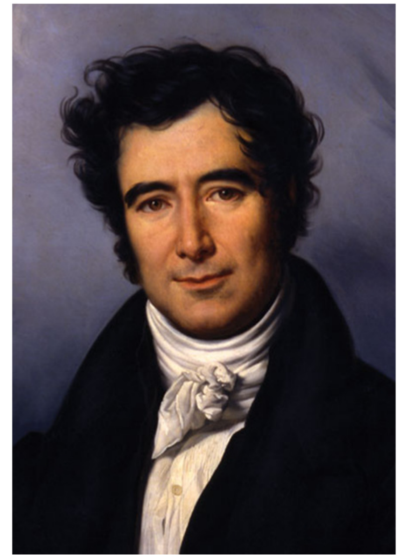
David L. DiLaura
The University of Colorado at Boulder

German translation: 1892
French translation: 1997

François Arago (1786–1853)

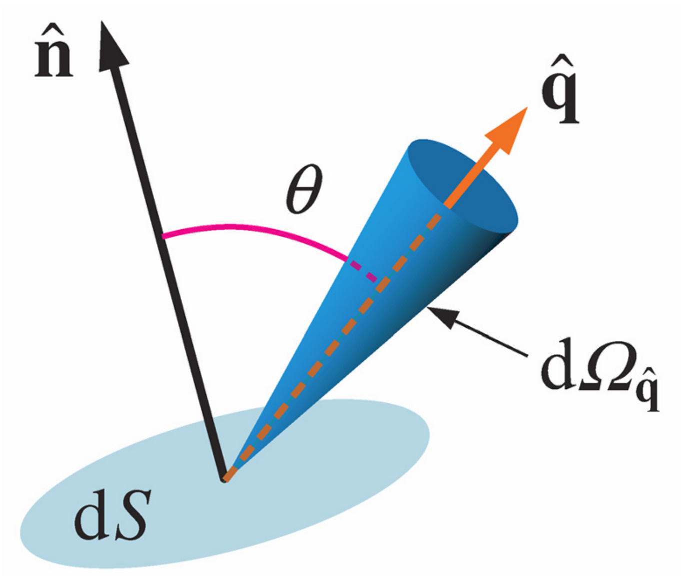
Criticized photometry for ignoring polarization of light

Photometric measurement must be part of polarimetric measurement



Paradigm shift: specific intensity

From real surface elements to imaginary ones



Eugen von Lommel (1837–99)

XI. ***Die Photometrie der diffusen Zurückwerfung; von E. Lommel.***

(Aus den Sitzungsber. d. math. phys. Classe d. K. Acad. zu München, mitgetheilt vom Hrn. Verf.)

In einer früheren Abhandlung „über Fluorescenz"[1]) habe ich in einem: „Ueber die Grundsätze der Photometrie", überschriebenen Abschnitt gezeigt, dass in der theoretischen Photometrie nicht, wie bis dahin üblich war, die Flächenelemente einer leuchtenden Oberfläche, sondern die Volumenelemente des leuchtenden Körpers als lichtstrahlend zu betrachten seien. Demgemäss wurden der theoretischen Behandlung photometrischer Probleme die folgenden drei Sätze zu Grunde gelegt:

I. Die von einem Volumenelement nach einem anderen strahlende Lichtmenge ist dem Quadrate ihrer Entfernung umgekehrt proportional.

II. Die von einem Volumenelement ausstrahlende und auf ein Flächenelement fallende Lichtmenge ist dem Cosinus des Incidenzwinkels proportional.

III. Das von einem Volumenelement ausstrahlende Licht wird auf seinem Wege innerhalb des strahlenden Körpers nach Maassgabe des Absorptionsgesetzes geschwächt.

1) Lommel, Wied. Ann. **10**. p. 449 u. 631. 1880.

Sitzber. Acad. Wissensch. München **17**, 95–124 (1887)

8 Web of Knowledge citations

Eugen von Lommel (1837–99)

Eugen von Lommel.

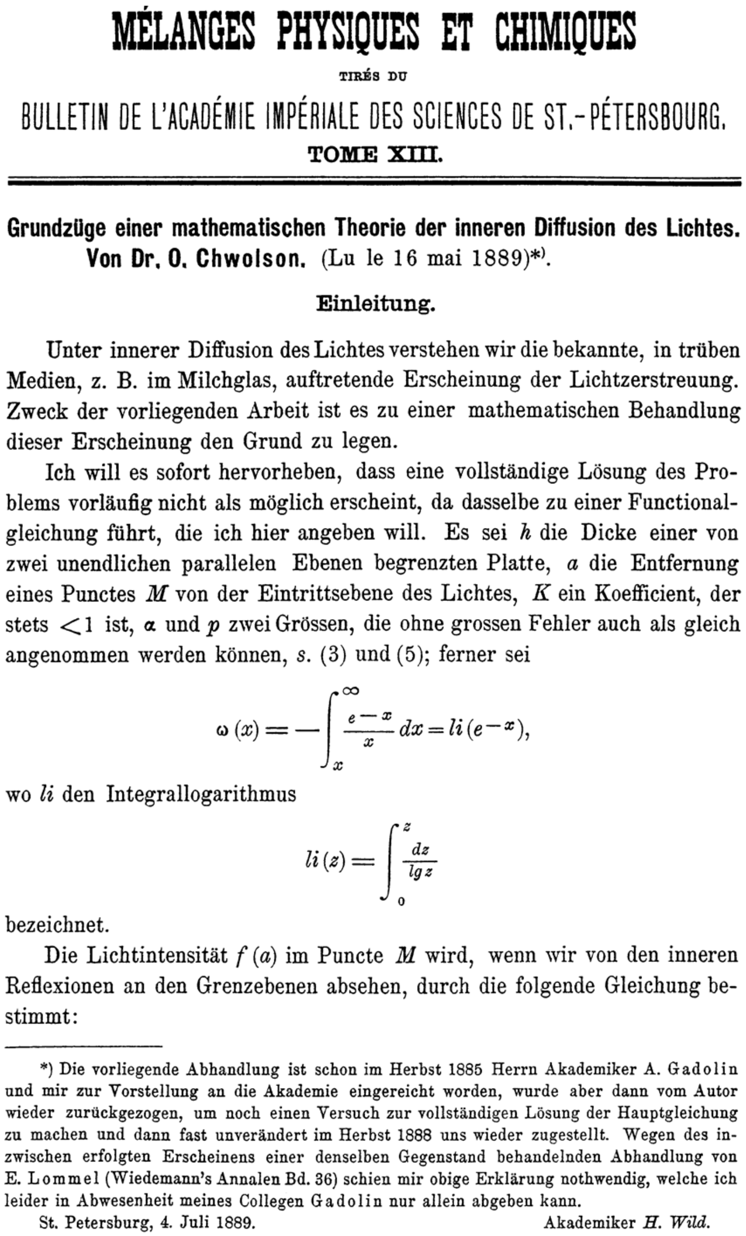
Von **Ludwig Boltzmann** in Wien.

Ein arbeitsreiches Leben, in der Jugend nicht ohne Entbehrungen und Mühsale, später aber reich an Erfolgen und köstlichen Früchten, durchschnitt der Tod, als am 19. Juni 1899 Professor Dr. Eugen von Lommel starb. Obwohl der Verfasser dieser Zeilen mit ihm vier Jahre lang als engster Fachcollege zusammenwirkte und in vertrautester freundschaftlicher Beziehung stand, so sprach er mit ihm doch fast ausschließlich über Wissenschaft und Berufsthätigkeit. Die nachfolgenden biographischen Notizen und Charakterschilderungen verdankt er der Güte der Witwe des Verewigten, teilweise entnimmt er sie auch einem Aufsatze Professor Günther's. So mußte also Lommel sterben, bis der Verfasser in die Eigentümlichkeiten seines Gemütes und Herzens nähern Einblick gewann.

Also:
Lommel differential equation
Lommel function
Lommel–Weber function
Lommel polynomial

Jahresbericht der Deutschen Mathematiker-Vereinigung (Leipzig, Teubner, 1900)

Orest Khvolson (1852–1934)



MÉLANGES PHYSIQUES ET CHIMIQUES

TIRÉS DU

BULLETIN DE L'ACADÉMIE IMPÉRIALE DES SCIENCES DE ST.-PÉTERSBOURG.

TOME XIII.

Grundzüge einer mathematischen Theorie der inneren Diffusion des Lichtes.
Von Dr. O. Chwolson. (Lu le 16 mai 1889)*).

Einleitung.

Unter innerer Diffusion des Lichtes verstehen wir die bekannte, in trüben Medien, z. B. im Milchglas, auftretende Erscheinung der Lichtzerstreuung. Zweck der vorliegenden Arbeit ist es zu einer mathematischen Behandlung dieser Erscheinung den Grund zu legen.

Ich will es sofort hervorheben, dass eine vollständige Lösung des Problems vorläufig nicht als möglich erscheint, da dasselbe zu einer Functionalgleichung führt, die ich hier angeben will. Es sei h die Dicke einer von zwei unendlichen parallelen Ebenen begrenzten Platte, a die Entfernung eines Punctes M von der Eintrittsebene des Lichtes, K ein Koefficient, der stets <1 ist, α und p zwei Grössen, die ohne grossen Fehler auch als gleich angenommen werden können, s. (3) und (5); ferner sei

$$\omega(x) = -\int_x^\infty \frac{e^{-x}}{x}\,dx = li(e^{-x}),$$

wo li den Integrallogarithmus

$$li(z) = \int_0^z \frac{dz}{lg\,z}$$

bezeichnet.

Die Lichtintensität $f(a)$ im Puncte M wird, wenn wir von den inneren Reflexionen an den Grenzebenen absehen, durch die folgende Gleichung bestimmt:

*) Die vorliegende Abhandlung ist schon im Herbst 1885 Herrn Akademiker A. Gadolin und mir zur Vorstellung an die Akademie eingereicht worden, wurde aber dann vom Autor wieder zurückgezogen, um noch einen Versuch zur vollständigen Lösung der Hauptgleichung zu machen und dann fast unverändert im Herbst 1888 uns wieder zugestellt. Wegen des inzwischen erfolgten Erscheinens einer denselben Gegenstand behandelnden Abhandlung von E. Lommel (Wiedemann's Annalen Bd. 36) schien mir obige Erklärung nothwendig, welche ich leider in Abwesenheit meines Collegen Gadolin nur allein abgeben kann.

St. Petersburg, 4. Juli 1889. Akademiker *H. Wild.*

Bull. l'Acad. Impériale Sci. St. Pétersbourg **33**, 221–256 (1889)

2 Web of Knowledge citations

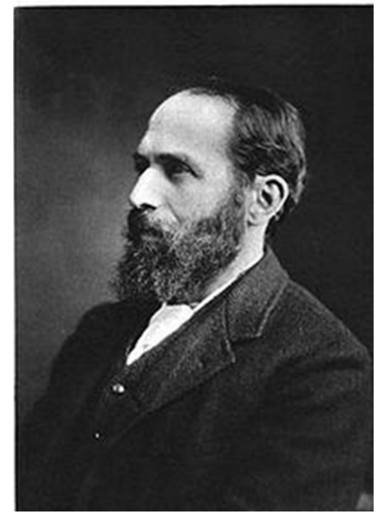
Sir Arthur Schuster (1851–1934)

RADIATION THROUGH A FOGGY ATMOSPHERE

By ARTHUR SCHUSTER

1. In discussing the transmission of light through a mass of gas, it is usual to consider only the effects of emission and absorption, and to neglect all effects of scattering. But when the absorbing mass holds fine particles of matter in suspension, the scattered light materially affects the character of the transmitted radiation. I propose to discuss the conditions under which "bright line" spectra or "dark line" spectra may be obtained from a radiating mass of gas, taking account of scattering. I call an atmosphere "foggy" when scattering takes place to an appreciable extent. The applications of the results of this investigation are, however, much wider than the title chosen would seem to imply, for there is some scattering even from the molecules of a homogeneous substance, and to that extent all bodies fall within the definition and may be called "foggy."

According to the investigations of Lord Rayleigh, the greater part of the light we receive from the sky is due to light scattered by the molecules of the air. This involves a diminution in the intensity of the direct rays amounting in our atmosphere to roughly 5 per cent. The effective thickness of stellar atmospheres may be great compared with that of the shell of air which surrounds our globe, and hence the effects of scattering may be of primary importance in interpreting the nature of stellar atmospheres.



Astrophys. J. **21**, 1–22 (1905)

NASA Goddard Institute for Space Studies

Max Planck (1858–1947)

“The state of the radiation at a given instant and at a given point of the medium cannot be represented… by a single vector (that is, a single directed quantity). All heat rays which at a given instant pass through the same point of the medium are perfectly independent of one another, and in order to specify completely the state of the radiation the intensity of radiation must be known in all the directions, infinite in number, which pass through the point in question.”

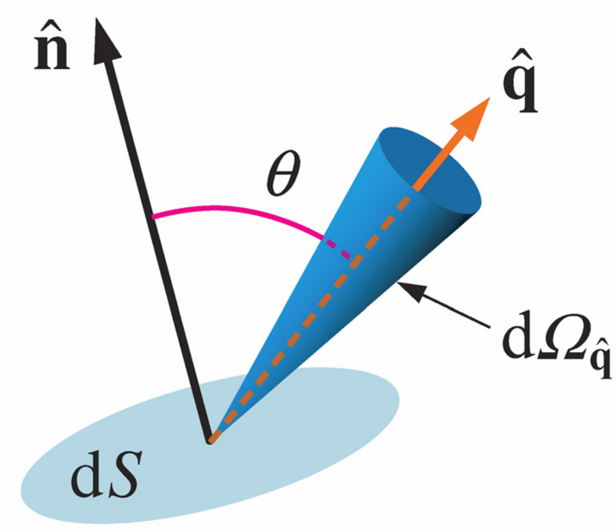
M. Planck, *Theorie der Wärmestrahlung*, 1906

Specific intensity

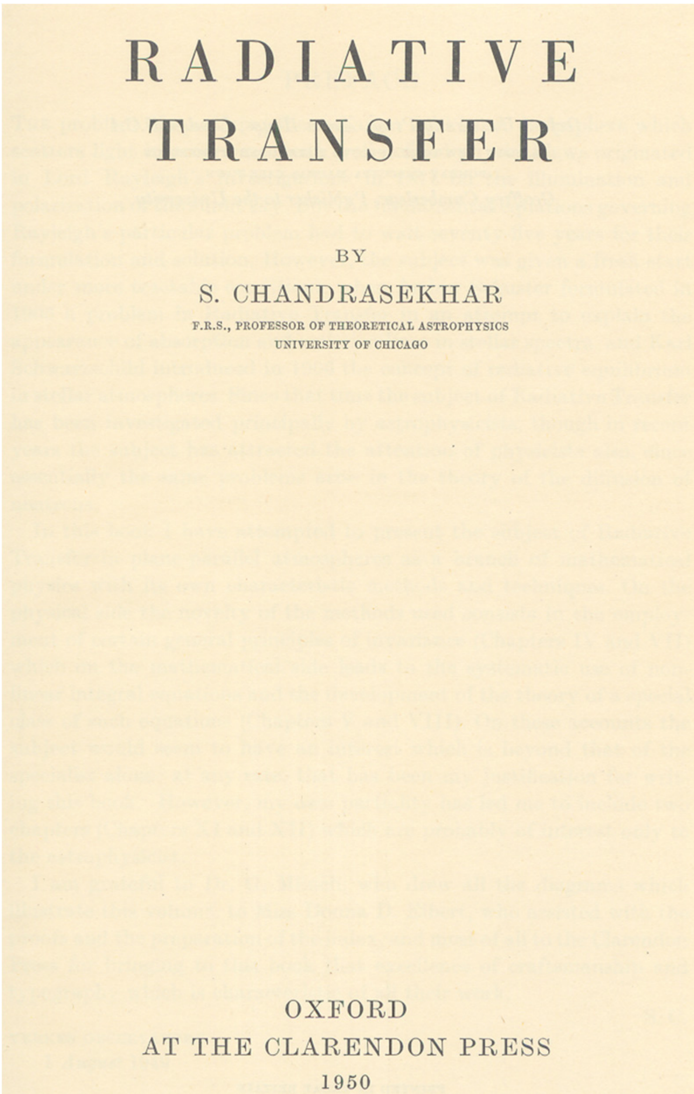
The amount of radiant energy dE which is transported across an element of area dS and in directions confined to an element of solid angle $d\Omega$, during a time dt is described in terms of the specific intensity I by

$$dE = I \cos\theta \, dS \, d\Omega \, dt,$$

where θ is the angle which the direction considered makes with the outward normal to dS.



Subrahmanyan Chandrasekhar (1910–95)



RADIATIVE TRANSFER

BY

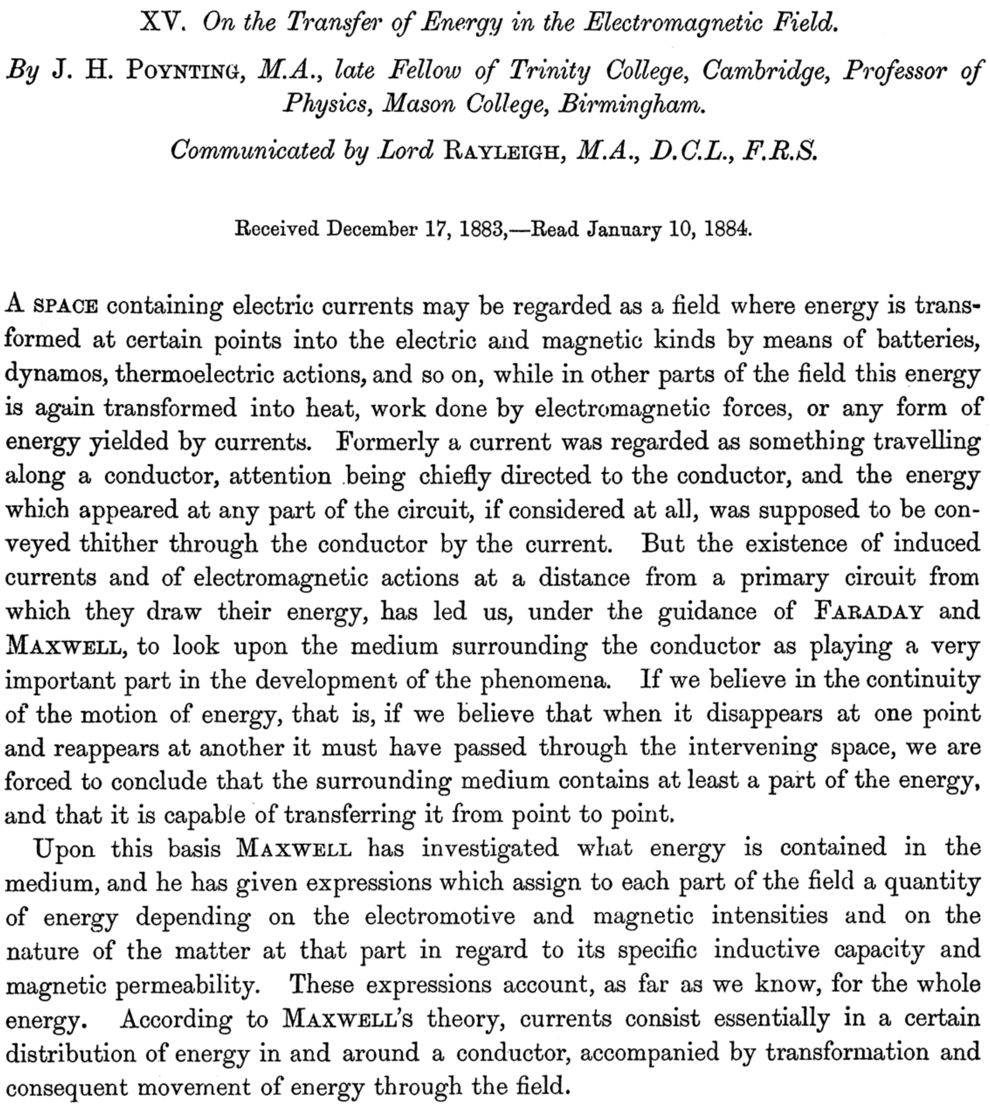
S. CHANDRASEKHAR

F.R.S., PROFESSOR OF THEORETICAL ASTROPHYSICS
UNIVERSITY OF CHICAGO

OXFORD
AT THE CLARENDON PRESS
1950

$$dE = I \cos\theta \, dS \, d\Omega \, dt$$

John Henry Poynting (1852–1914)


XV. *On the Transfer of Energy in the Electromagnetic Field.*

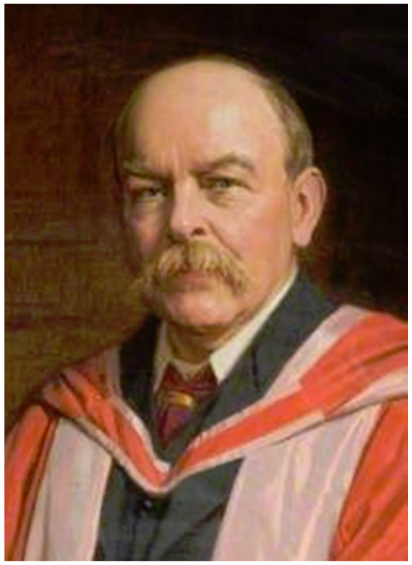
By J. H. Poynting, *M.A., late Fellow of Trinity College, Cambridge, Professor of Physics, Mason College, Birmingham.*

Communicated by Lord Rayleigh, *M.A., D.C.L., F.R.S.*

Received December 17, 1883,—Read January 10, 1884.

A space containing electric currents may be regarded as a field where energy is transformed at certain points into the electric and magnetic kinds by means of batteries, dynamos, thermoelectric actions, and so on, while in other parts of the field this energy is again transformed into heat, work done by electromagnetic forces, or any form of energy yielded by currents. Formerly a current was regarded as something travelling along a conductor, attention being chiefly directed to the conductor, and the energy which appeared at any part of the circuit, if considered at all, was supposed to be conveyed thither through the conductor by the current. But the existence of induced currents and of electromagnetic actions at a distance from a primary circuit from which they draw their energy, has led us, under the guidance of Faraday and Maxwell, to look upon the medium surrounding the conductor as playing a very important part in the development of the phenomena. If we believe in the continuity of the motion of energy, that is, if we believe that when it disappears at one point and reappears at another it must have passed through the intervening space, we are forced to conclude that the surrounding medium contains at least a part of the energy, and that it is capable of transferring it from point to point.

Upon this basis Maxwell has investigated what energy is contained in the medium, and he has given expressions which assign to each part of the field a quantity of energy depending on the electromotive and magnetic intensities and on the nature of the matter at that part in regard to its specific inductive capacity and magnetic permeability. These expressions account, as far as we know, for the whole energy. According to Maxwell's theory, currents consist essentially in a certain distribution of energy in and around a conductor, accompanied by transformation and consequent movement of energy through the field.



The Poynting vector

"The Poynting vector $\boldsymbol{S}(\mathbf{r}, t)$ defined by

$$\boldsymbol{S}(\mathbf{r}, t) = \boldsymbol{E}(\mathbf{r}, t) \times \boldsymbol{H}(\mathbf{r}, t)$$

is the intensity of energy flow at a point $\mathbf{r}$ in the field; i.e., gives the energy per second crossing a unit area whose normal is oriented in the direction of the vector $\boldsymbol{E}(\mathbf{r}, t) \times \boldsymbol{H}(\mathbf{r}, t)$."

J. A. Stratton, *Electromagnetic Theory*, 1941

Max Planck (1858–1947)


VORLESUNGEN

ÜBER DIE

THEORIE DER WÄRMESTRAHLUNG

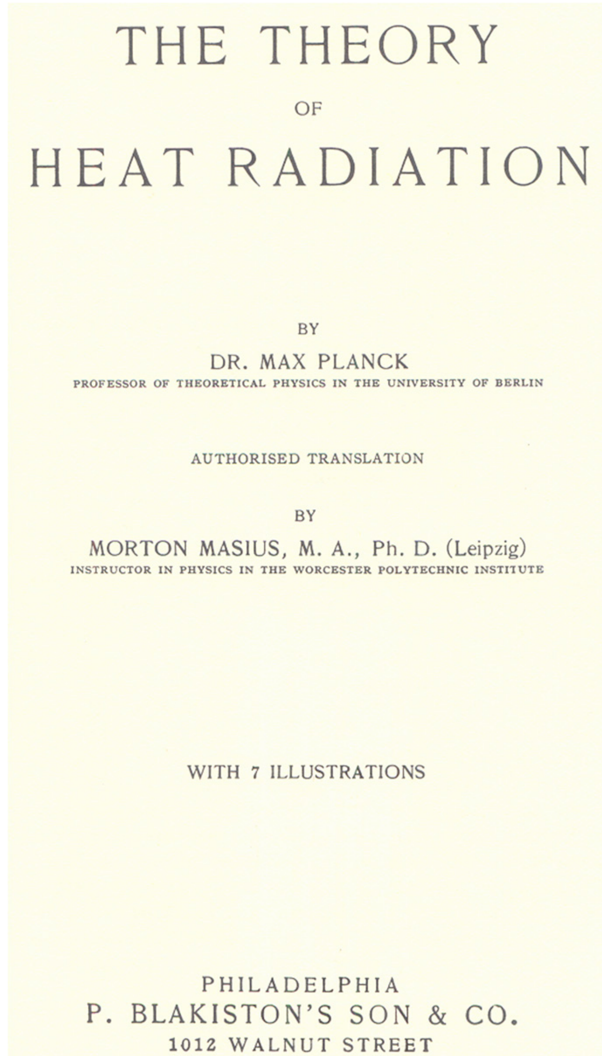
VON

DR. MAX PLANCK,

PROFESSOR DER THEORETISCHEN PHYSIK
AN DER UNIVERSITÄT BERLIN

MIT 6 ABBILDUNGEN

LEIPZIG, 1906
VERLAG VON JOHANN AMBROSIUS BARTH


THE THEORY

OF

HEAT RADIATION

BY

DR. MAX PLANCK
PROFESSOR OF THEORETICAL PHYSICS IN THE UNIVERSITY OF BERLIN

AUTHORISED TRANSLATION

BY

MORTON MASIUS, M. A., Ph. D. (Leipzig)
INSTRUCTOR IN PHYSICS IN THE WORCESTER POLYTECHNIC INSTITUTE

WITH 7 ILLUSTRATIONS

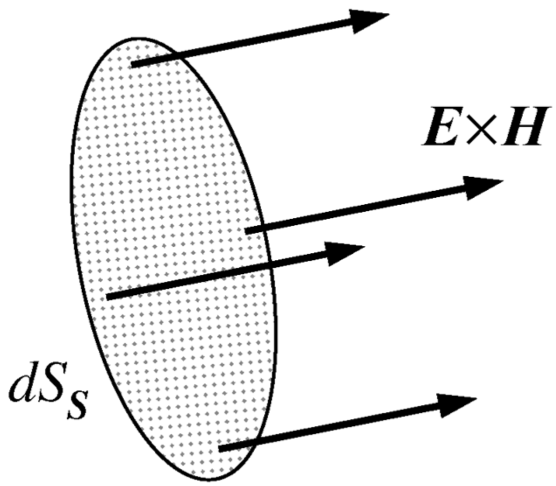
PHILADELPHIA
P. BLAKISTON'S SON & CO.
1012 WALNUT STREET

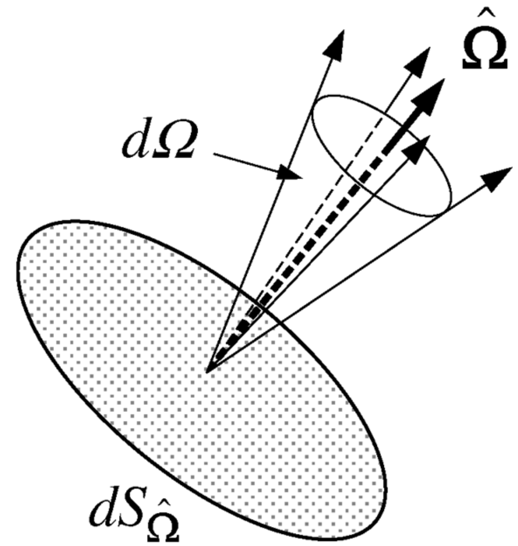
"rays…

at a given instant pass through the same point of the medium…

in all the directions, infinite in number…"

Poynting vector vs. specific intensity

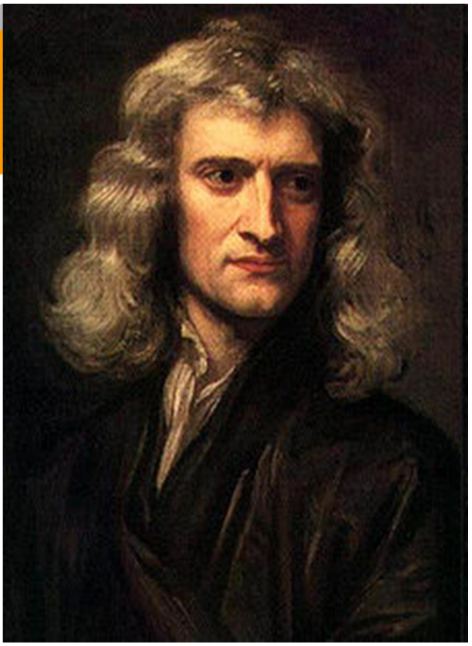




Dimension of the Poynting vector: $\mathrm{Wm^{-2}s^{-1}}$

Dimension of the specific intensity: $\mathrm{Wm^{-2}s^{-1}sr^{-1}}$

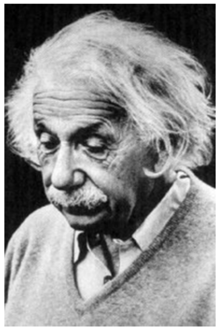
Sir Isaac Newton (1642–1727)



Light is composed of indivisible particles or *corpuscles*, which propagate along straight lines in a homogeneous medium and are refracted by accelerating into a denser medium.

“The energy of a light ray spreading out from a point source is not continuously distributed over an increasing space but consists of a finite number of energy quanta which are localized at points in space, which move without dividing, and which can only be produced and absorbed as complete units.”

A. Einstein, 1905



Photonic confusion

A “simple” way to introduce the RTE:

1. Use the concept of photons:

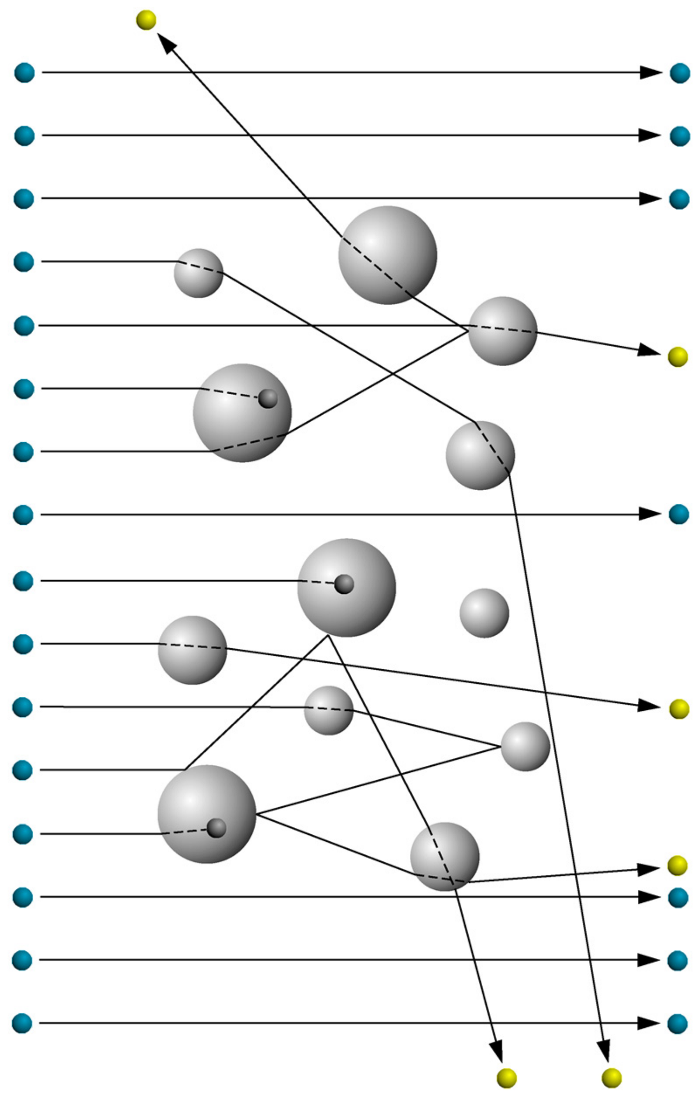
“Light is a shower of particles” or “a staccato of little discrete packets of energy called photons” (G. W. Petty, *A First Course in Atmospheric Radiation*, 2006)

Photons are “blobs of energy without phases” (C. F. Bohren and E. E. Clothiaux, *Fundamentals of Atmospheric Radiation*, 2006).

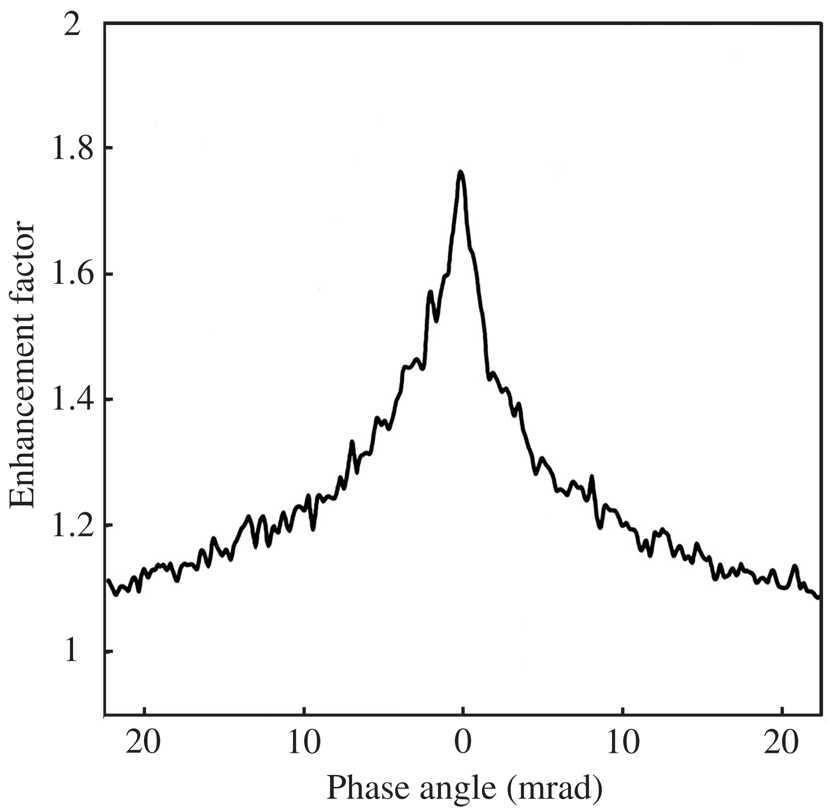
Photonic confusion

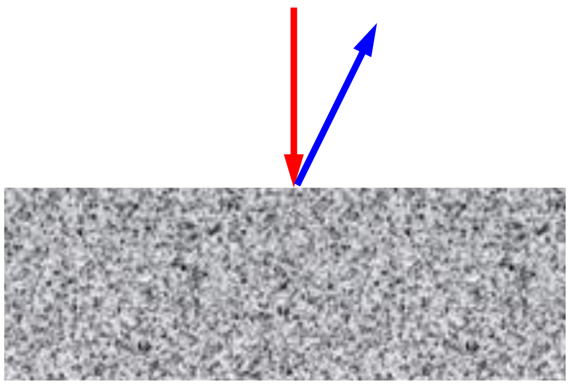
2. Describe the radiation field in terms of the so-called "photon gas"

3. Postulate that the photon gas satisfies the Boltzmann kinetic equation (e.g., Pomraning, *The Equations of Radiation Hydrodynamics*, 1973)



Coherent backscattering (weak localization)





van Albada *et al*. 1985

Photonic confusion and QED (1927–32)

1. To explain the photoelectric effect one does not need photons. Use the Planck's semi-classical approach: quantize the matter and leave the electromagnetic field classical.

2. Real QED photons are not localized particles of light (Heisenberg, Jordan, Dirac):

(i) There is no photon wave function in the configuration space.

(ii) A photon is a quantum of a normal mode of the electromagnetic field.

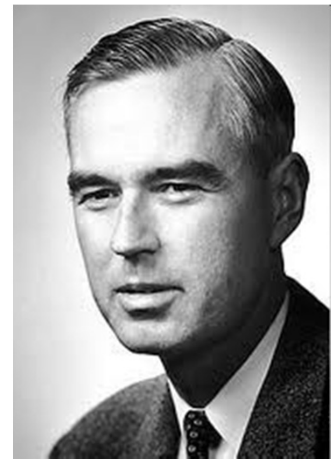
(iii) Each photons occupies the entire quantization domain.

Photonic confusion and QED

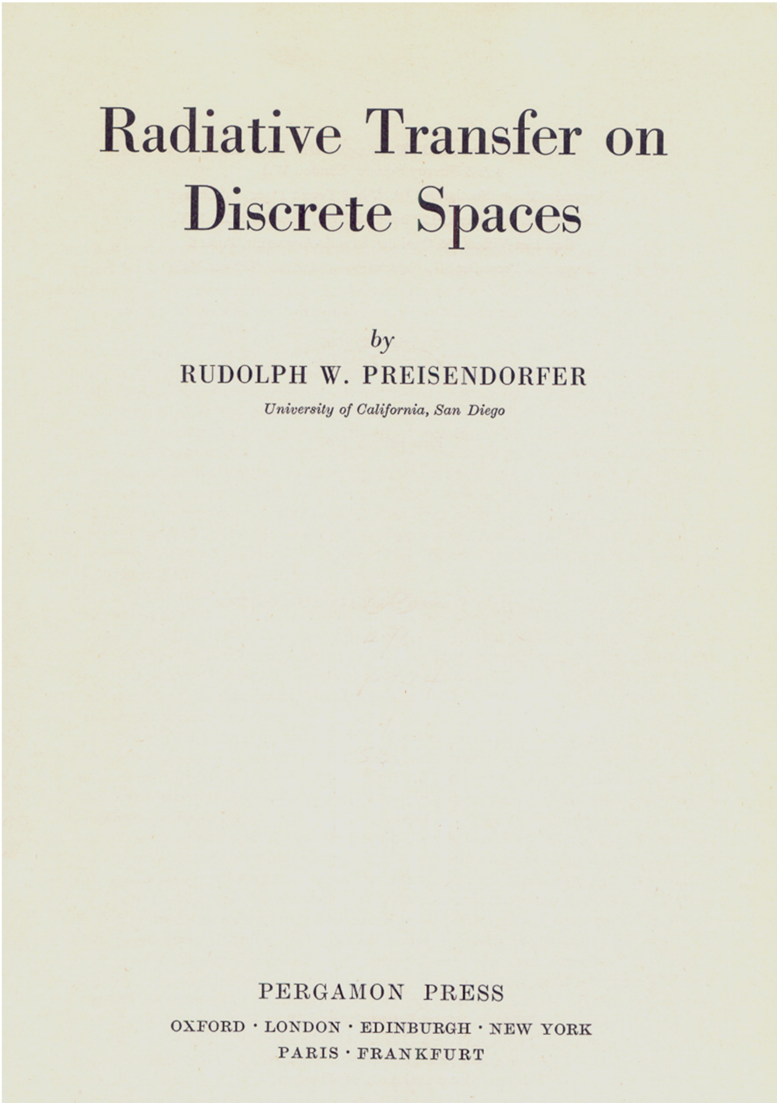
3. Willis Lamb Jr.:

(i) “There is no such thing as a photon. Only a comedy of errors and historical accidents led to its popularity…” (“Antiphoton”, 1995).

(ii) Stay away from photons if you don’t know what they are.



Overarching dilemma (1965)


Radiative Transfer on Discrete Spaces

by

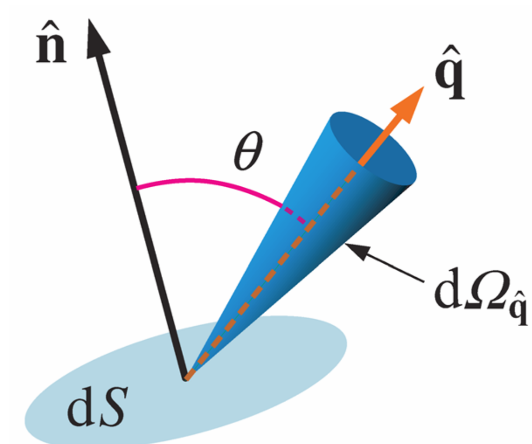
RUDOLPH W. PREISENDORFER

University of California, San Diego

PERGAMON PRESS

OXFORD · LONDON · EDINBURGH · NEW YORK

PARIS · FRANKFURT



Overarching dilemma (1965)

Mainland of physics

Tiny island of radiative transfer

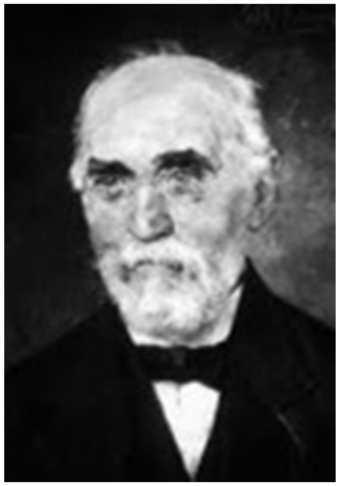
Overarching dilemma (1965)

Mainland of physics

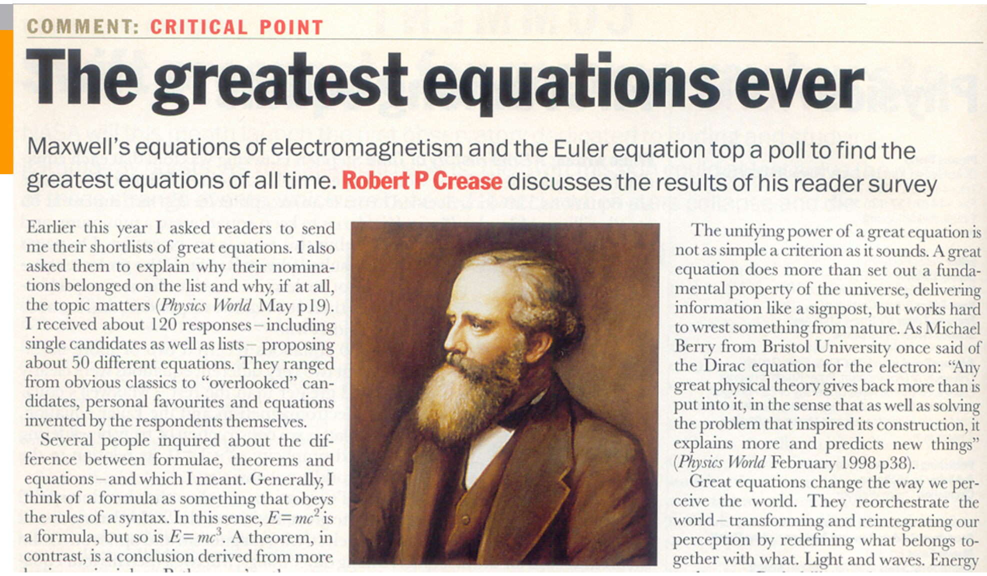
Tiny island of radiative transfer

Relativity theory

The development of the relativity theory
by Henri Poincaré and Hendrik Lorentz reasserted the
fundamental character of Maxwell's
electromagnetics.



Maxwell’s theory


COMMENT: CRITICAL POINT

The greatest equations ever

Maxwell’s equations of electromagnetism and the Euler equation top a poll to find the greatest equations of all time. **Robert P Crease** discusses the results of his reader survey

Earlier this year I asked readers to send me their shortlists of great equations. I also asked them to explain why their nominations belonged on the list and why, if at all, the topic matters (*Physics World* May p19). I received about 120 responses – including single candidates as well as lists – proposing about 50 different equations. They ranged from obvious classics to “overlooked” candidates, personal favourites and equations invented by the respondents themselves.

Several people inquired about the difference between formulae, theorems and equations – and which I meant. Generally, I think of a formula as something that obeys the rules of a syntax. In this sense, $E=mc^2$ is a formula, but so is $E=mc^3$. A theorem, in contrast, is a conclusion derived from more

The unifying power of a great equation is not as simple a criterion as it sounds. A great equation does more than set out a fundamental property of the universe, delivering information like a signpost, but works hard to wrest something from nature. As Michael Berry from Bristol University once said of the Dirac equation for the electron: “Any great physical theory gives back more than is put into it, in the sense that as well as solving the problem that inspired its construction, it explains more and predicts new things” (*Physics World* February 1998 p38).

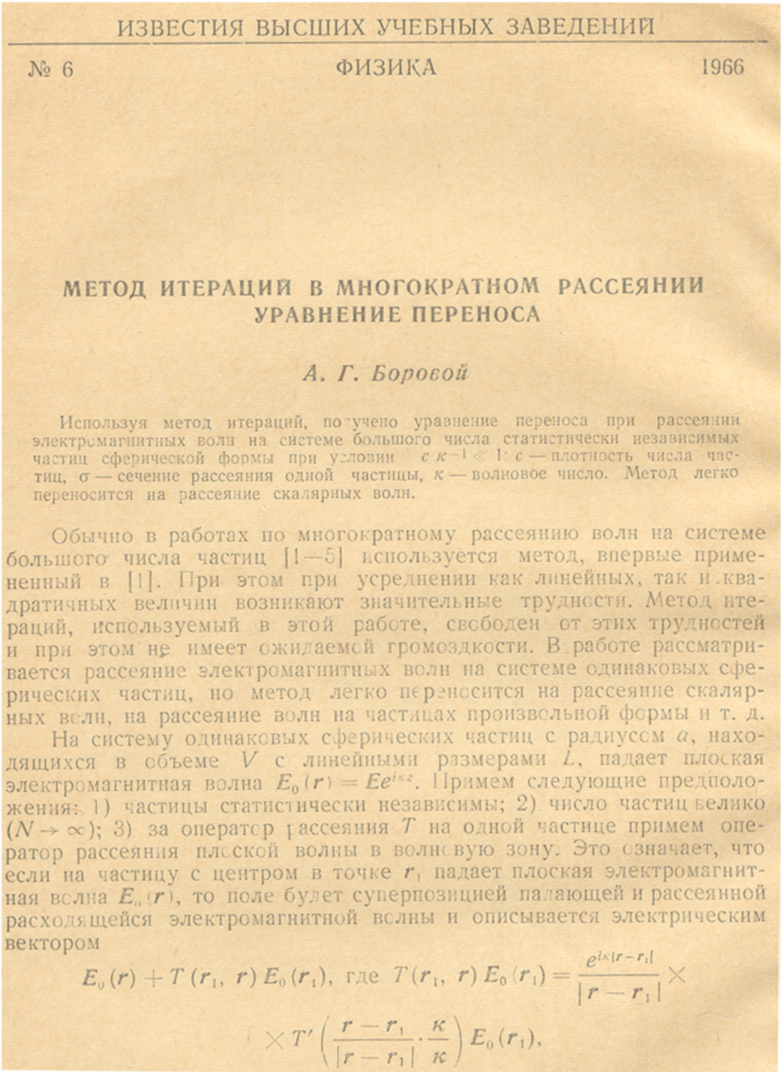
Great equations change the way we perceive the world. They reorchestrate the world – transforming and reintegrating our perception by redefining what belongs together with what. Light and waves. Energy

Maxwell’s equations of electromagnetism topped a poll to find the greatest equations of all time:

R. P. Crease, The greatest equations ever, *Phys. World*, Oct. 2004

The Poincaré–Planck–Lewis–*et al*. $E = mc^2$ and the Hilbert–Einstein equations of gravity were not even close!

First step (Anatoli Borovoi, 1966)



ИЗВЕСТИЯ ВЫСШИХ УЧЕБНЫХ ЗАВЕДЕНИЙ

№ 6 ФИЗИКА 1966

МЕТОД ИТЕРАЦИЙ В МНОГОКРАТНОМ РАССЕЯНИИ УРАВНЕНИЕ ПЕРЕНОСА

А. Г. Боровой

Используя метод итераций, получено уравнение переноса при рассеянии электромагнитных волн на системе большого числа статистически независимых частиц сферической формы при условии $c\kappa^{-1} \ll 1$; c — плотность числа частиц, σ — сечение рассеяния одной частицы, κ — волновое число. Метод легко переносится на рассеяние скалярных волн.

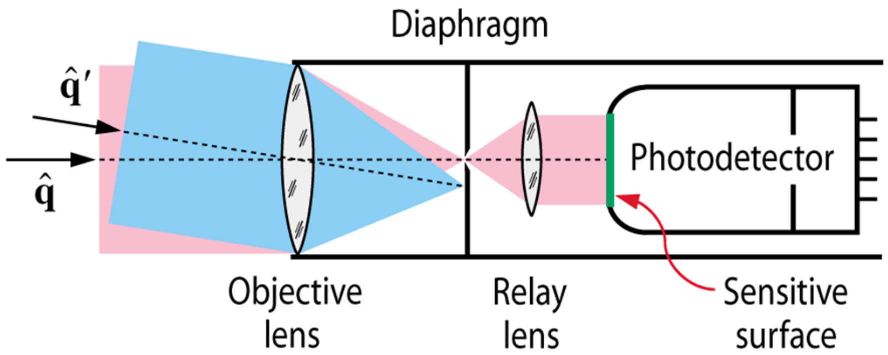
Обычно в работах по многократному рассеянию волн на системе большого числа частиц [1—5] используется метод, впервые примененный в [1]. При этом при усреднении как линейных, так и квадратичных величин возникают значительные трудности. Метод итераций, используемый в этой работе, свободен от этих трудностей и при этом не имеет ожидаемой громоздкости. В работе рассматривается рассеяние электромагнитных волн на системе одинаковых сферических частиц, но метод легко переносится на рассеяние скалярных волн, на рассеяние волн на частицах произвольной формы и т. д.

На систему одинаковых сферических частиц с радиусом a, находящихся в объеме V с линейными размерами L, падает плоская электромагнитная волна $E_0(r) = Ee^{ikz}$. Примем следующие предположения: 1) частицы статистически независимы; 2) число частиц велико ($N \to \infty$); 3) за оператор рассеяния T на одной частице примем оператор рассеяния плоской волны в волновую зону. Это означает, что если на частицу с центром в точке r_1 падает плоская электромагнитная волна $E_0(r)$, то поле будет суперпозицией падающей и рассеянной расходящейся электромагнитной волны и описывается электрическим вектором

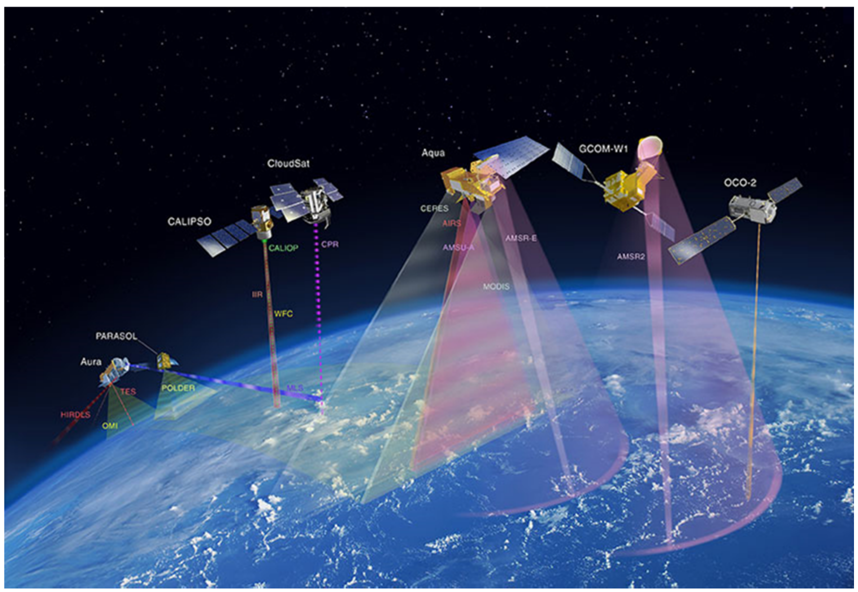
$$E_0(r) + T(r_1, r)E_0(r_1), \text{ где } T(r_1, r)E_0(r_1) = \frac{e^{i\kappa|r-r_1|}}{|r-r_1|} \times$$

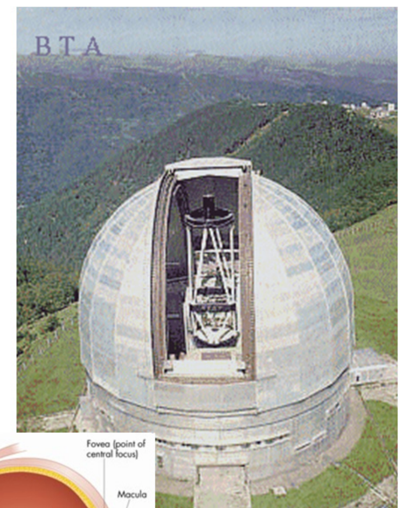
$$\times T'\left(\frac{r-r_1}{|r-r_1|} \cdot \frac{\kappa}{\kappa}\right) E_0(r_1),$$

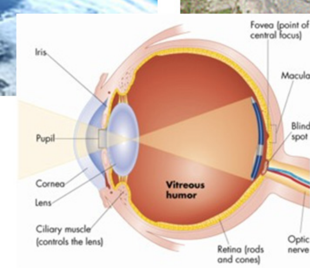
Well-collimated radiometers



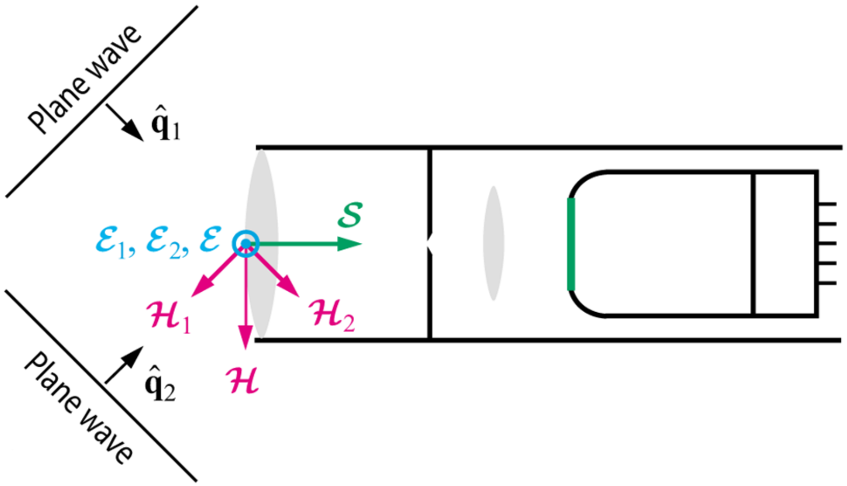
A well-collimated radiometer is not an electromagnetic flux-meter. It operates in the wave domain rather than in the energy (or Poynting-vector) domain.

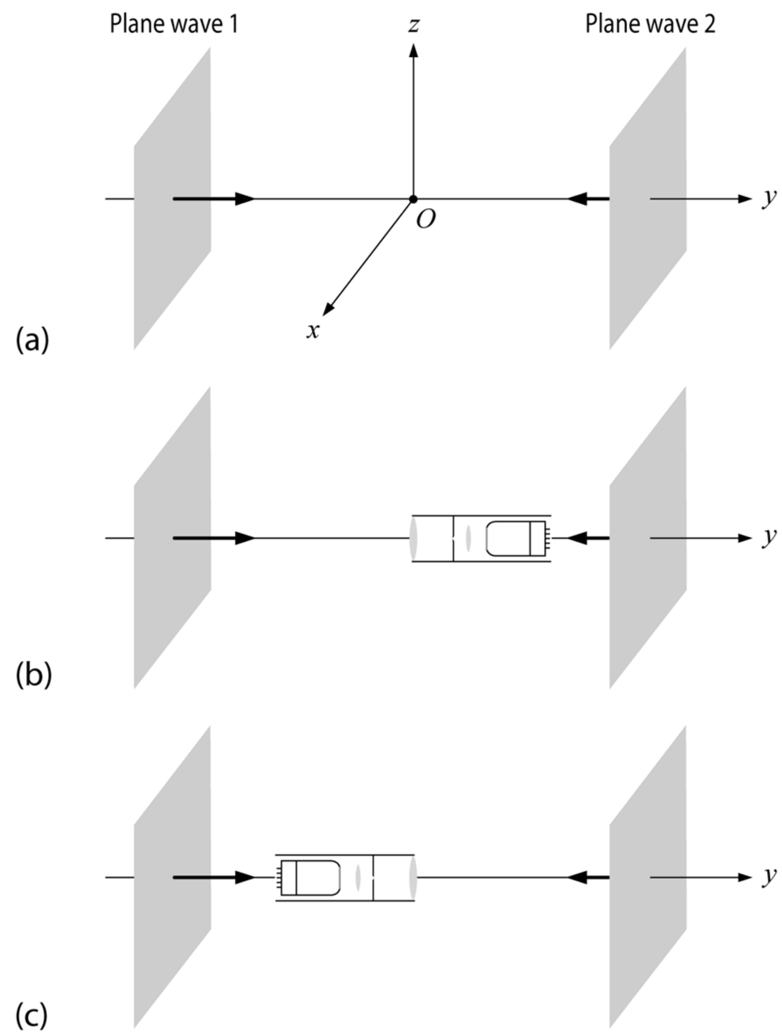






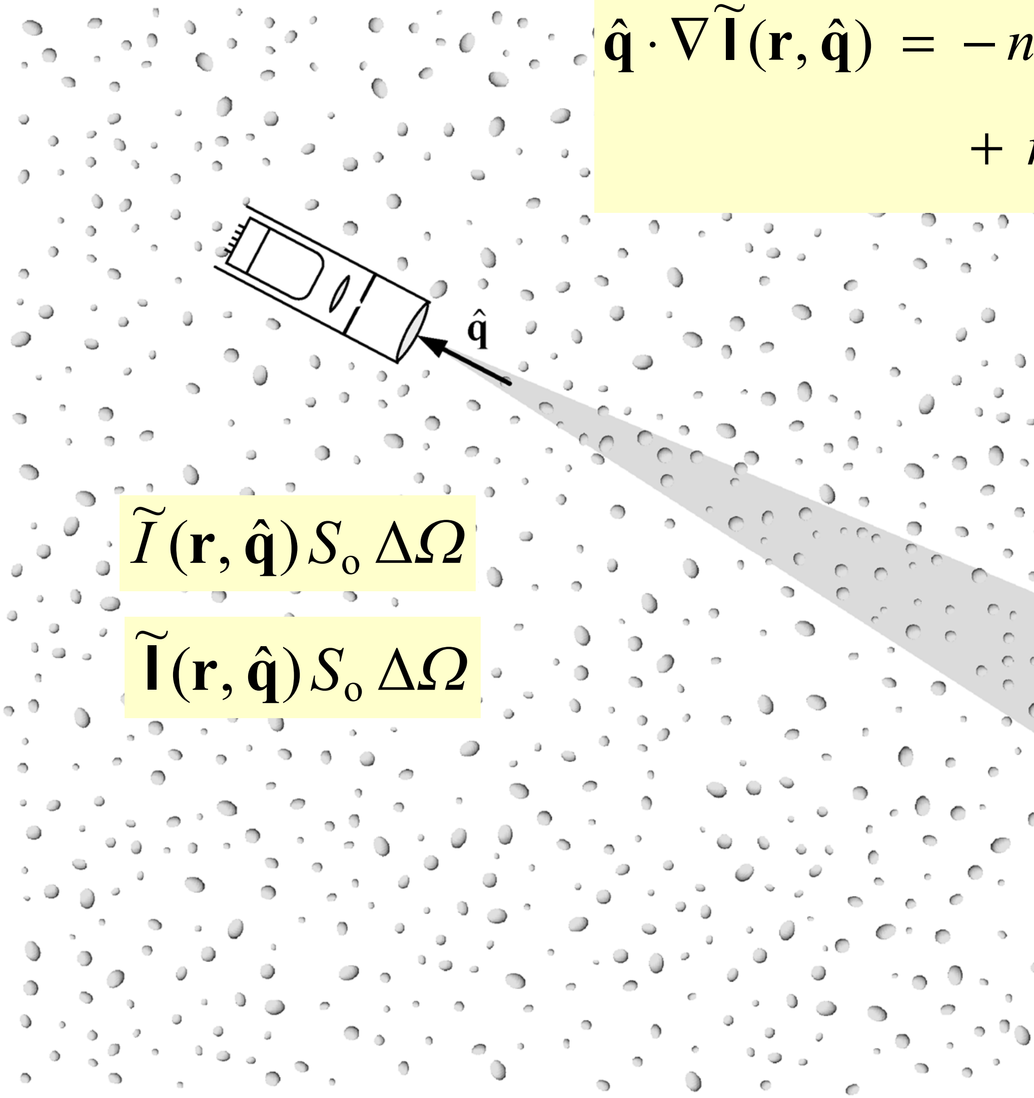
Well-collimated radiometers





Reading of a well-collimated radiometer

$$\hat{\mathbf{q}} \cdot \nabla \tilde{\mathbf{I}}(\mathbf{r}, \hat{\mathbf{q}}) = -n_0 \langle \mathbf{K}(\hat{\mathbf{q}}) \rangle_\xi \tilde{\mathbf{I}}(\mathbf{r}, \hat{\mathbf{q}}) + n_0 \int_{4\pi} \mathrm{d}\hat{\mathbf{q}}' \langle \mathbf{Z}(\hat{\mathbf{q}}, \hat{\mathbf{q}}') \rangle_\xi \tilde{\mathbf{I}}(\mathbf{r}, \hat{\mathbf{q}}')$$



$$\tilde{I}(\mathbf{r}, \hat{\mathbf{q}}) S_\mathrm{o} \Delta\Omega$$

$$\tilde{\mathbf{I}}(\mathbf{r}, \hat{\mathbf{q}}) S_\mathrm{o} \Delta\Omega$$

$$\tilde{\mathbf{I}}(\mathbf{r}, \hat{\mathbf{q}}) = \begin{bmatrix} \tilde{I}(\mathbf{r}, \hat{\mathbf{q}}) \\ \tilde{Q}(\mathbf{r}, \hat{\mathbf{q}}) \\ \tilde{U}(\mathbf{r}, \hat{\mathbf{q}}) \\ \tilde{V}(\mathbf{r}, \hat{\mathbf{q}}) \end{bmatrix}$$

- Far-field approximation
- Ergodicity
- The Twersky approximation
- The ladder approximation

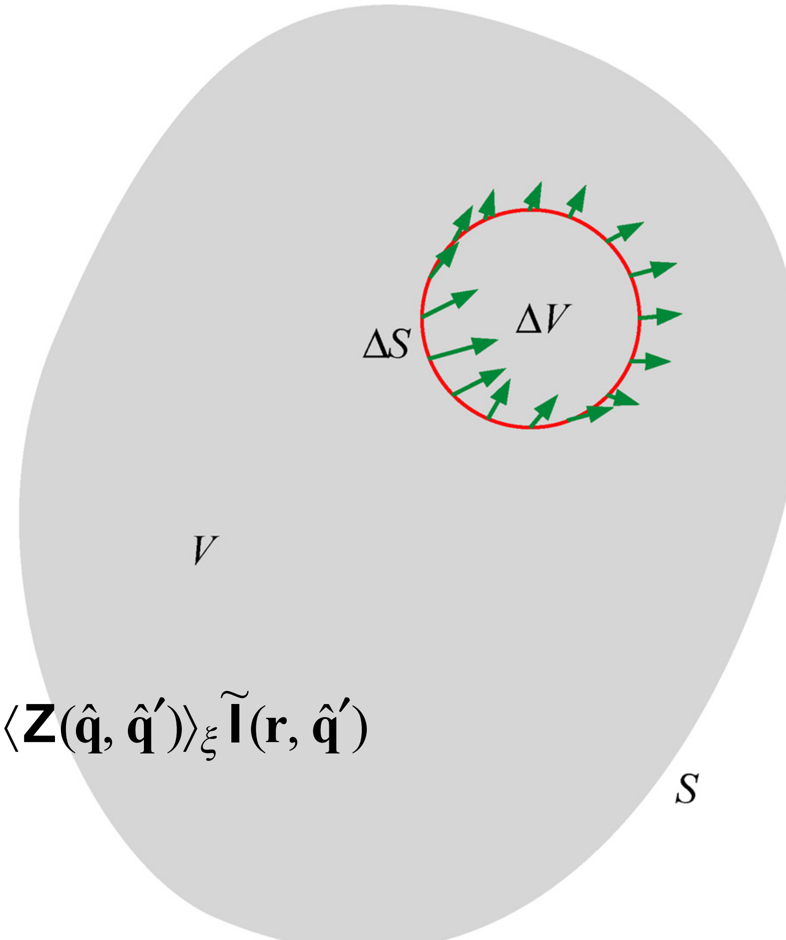
Radiation-budget problem

$$W_{\Delta S} = -\mathrm{Re}\int_{\Delta S} \mathrm{d}S\,\langle \mathbf{S}(\mathbf{r})\rangle \cdot \hat{\mathbf{n}}(\mathbf{r})$$

$$\langle \mathbf{S}(\mathbf{r})\rangle = \int_{4\pi} \mathrm{d}\hat{\mathbf{q}}\,\hat{\mathbf{q}}\,\tilde{I}(\mathbf{r},\hat{\mathbf{q}})$$

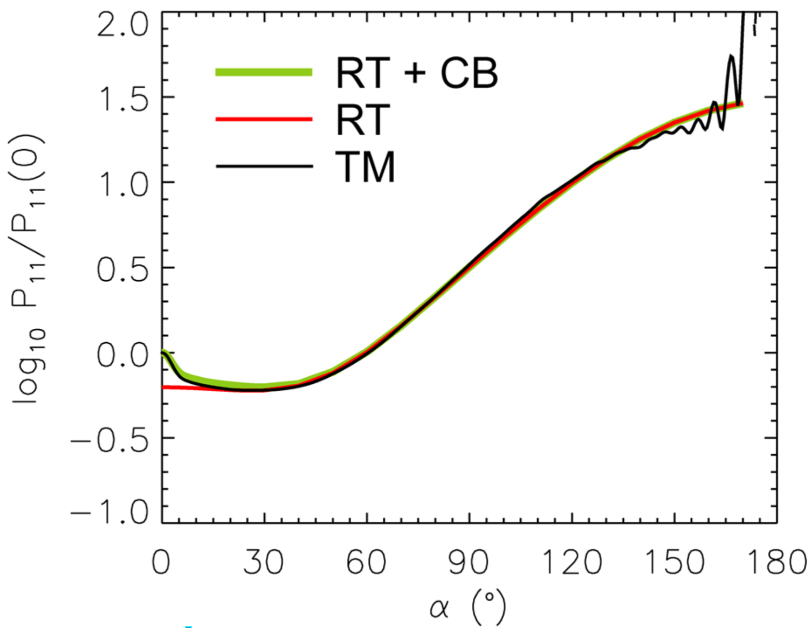
$$\tilde{\mathbf{I}}(\mathbf{r},\hat{\mathbf{q}}) = \begin{bmatrix} \tilde{I}(\mathbf{r},\hat{\mathbf{q}}) \\ \tilde{Q}(\mathbf{r},\hat{\mathbf{q}}) \\ \tilde{U}(\mathbf{r},\hat{\mathbf{q}}) \\ \tilde{V}(\mathbf{r},\hat{\mathbf{q}}) \end{bmatrix}$$

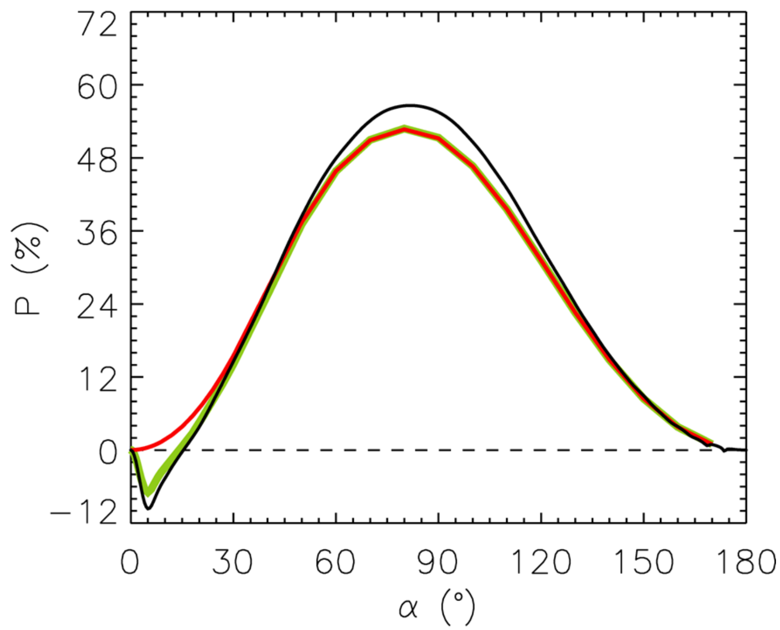
$$\hat{\mathbf{q}} \cdot \nabla \tilde{\mathbf{I}}(\mathbf{r},\hat{\mathbf{q}}) = -n_0 \langle \mathbf{K}(\hat{\mathbf{q}})\rangle_{\xi} \tilde{\mathbf{I}}(\mathbf{r},\hat{\mathbf{q}}) + n_0 \int_{4\pi} \mathrm{d}\hat{\mathbf{q}}' \langle \mathbf{Z}(\hat{\mathbf{q}},\hat{\mathbf{q}}')\rangle_{\xi} \tilde{\mathbf{I}}(\mathbf{r},\hat{\mathbf{q}}')$$

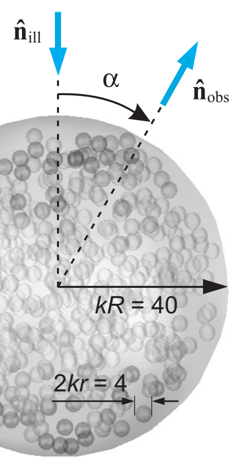


- Far-field approximation
- Ergodicity
- The Twersky approximation
- The ladder approximation

T-matrix vs Monte Carlo







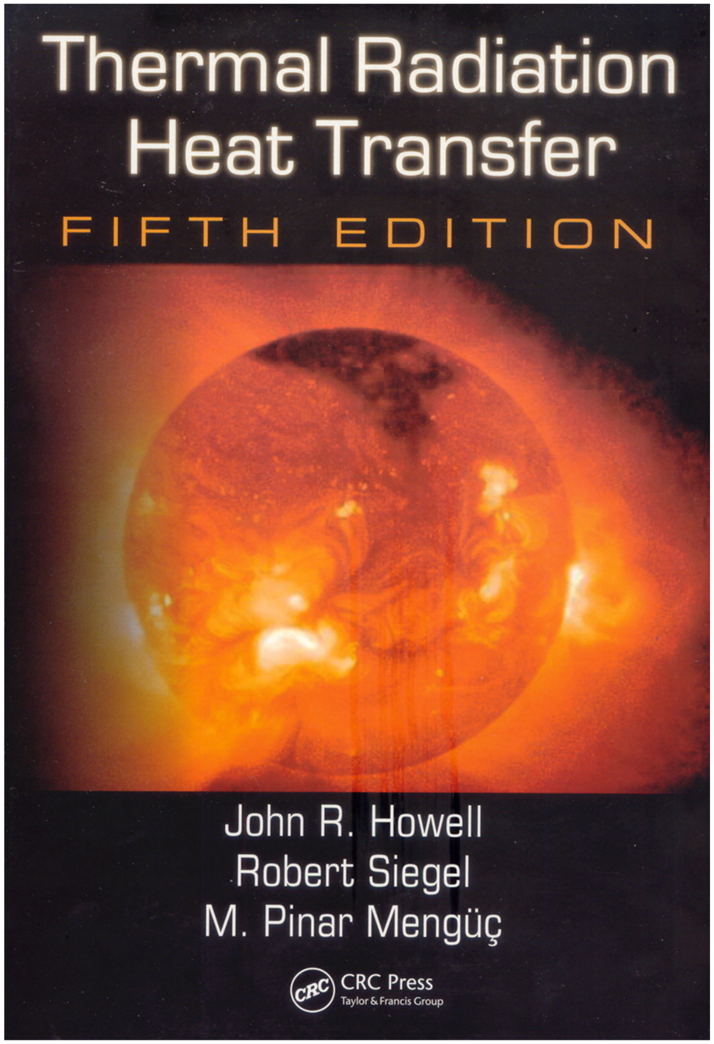
Muinonen *et al*. 2012

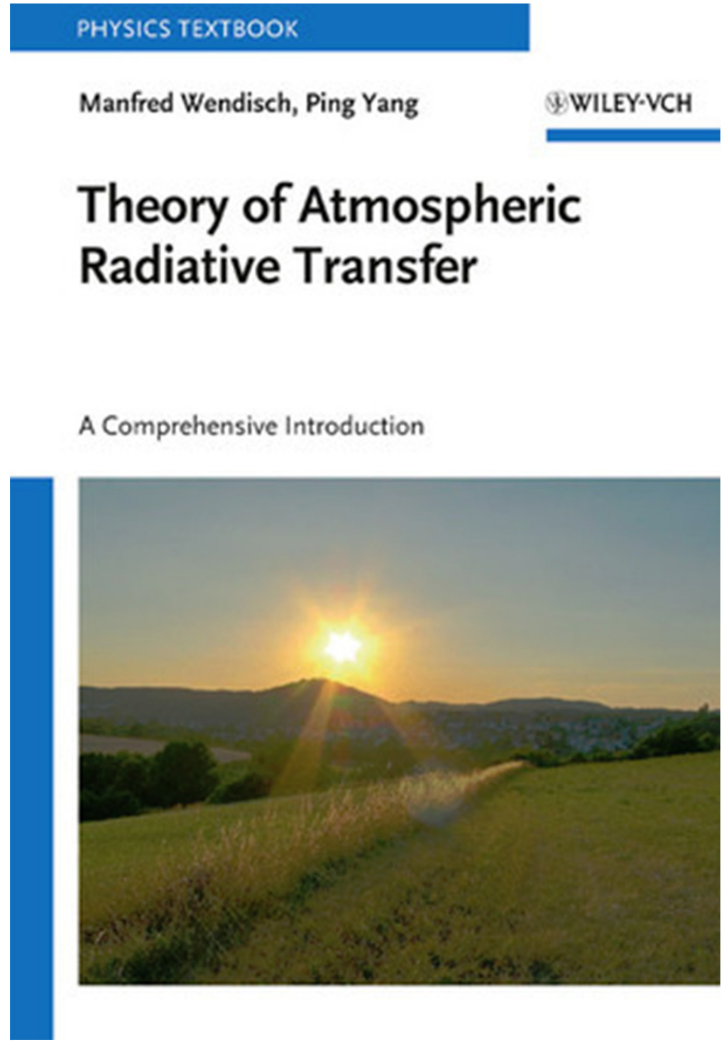
Paradigm shift

1. EM energy transport is always monodirectional. There is no such thing as “specific intensity”. The formal solution of the purely mathematical transport equation does not describe instantaneous multidirectional EM energy flow.
2. WCRs do not measure multidirectional EM energy flow either.
3. Directional radiometry ≠ measurement of multidirectional EM energy transfer and its theoretical modeling.
4. However, the particular measurement afforded by a WCR can sometimes be modeled by solving the transport equation.
4. Directional radiometry = measurement with a WCR and its theoretical modeling.
5. RT and CB are twins.
6. The majority of published RT results are safe but for the wrong reason.

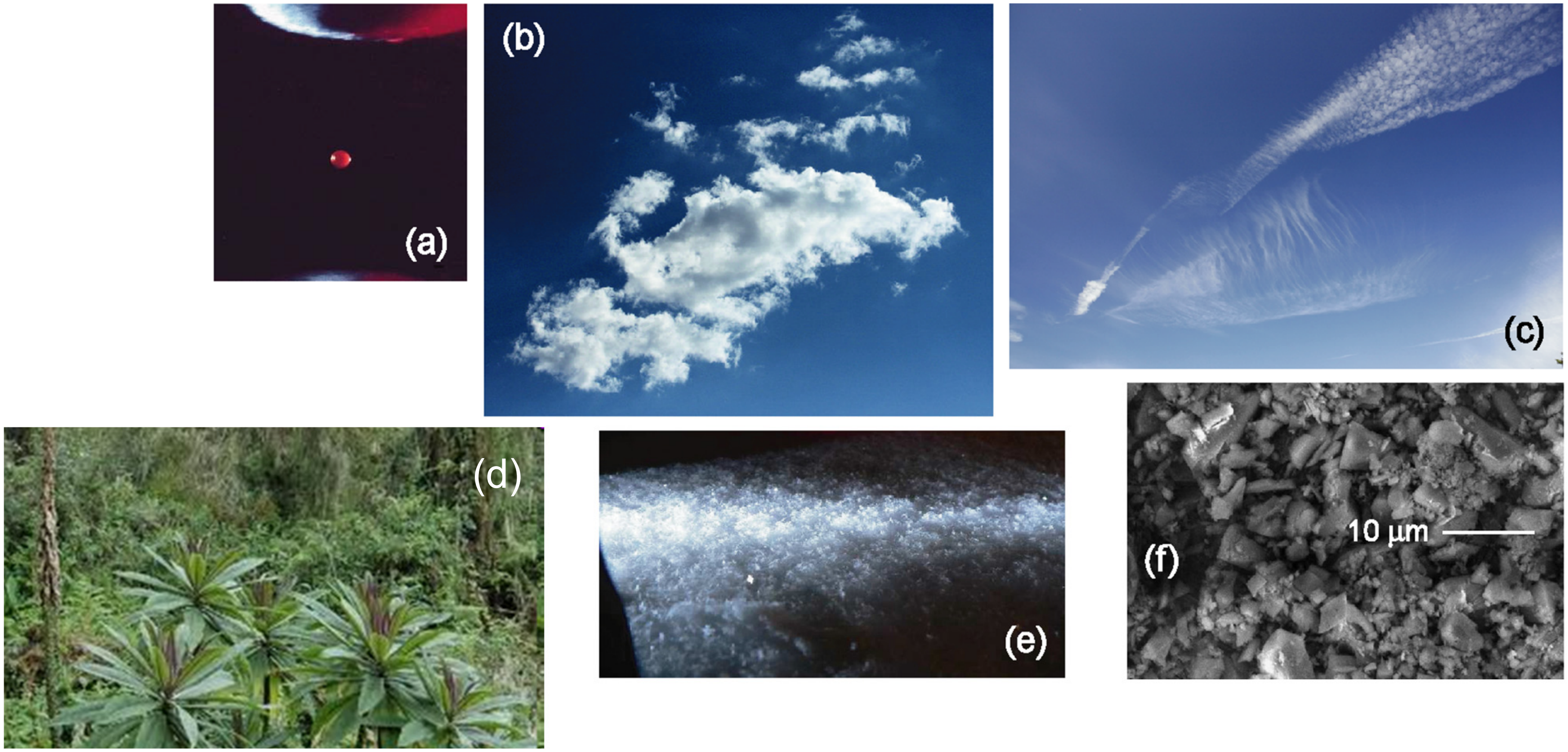
Preisendorfer's bridge

Further information





Single- and multiple-scattering objects



Further information

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Review

Directional radiometry and radiative transfer: A new paradigm

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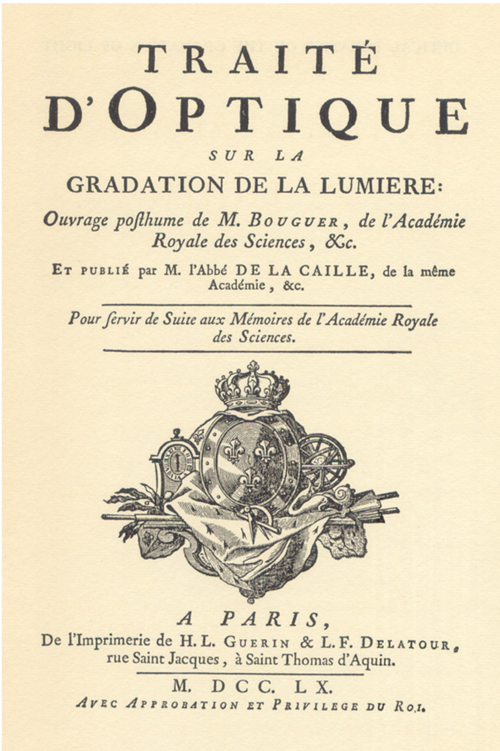
Back-up slides

Further information


TRAITÉ
D'OPTIQUE
SUR LA
GRADATION DE LA LUMIERE:
Ouvrage posthume de M. Bouguer, de l'Académie Royale des Sciences, &c.
Et publié par M. l'Abbé DE LA CAILLE, de la même Académie, &c.
Pour servir de Suite aux Mémoires de l'Académie Royale des Sciences.
A PARIS,
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M. DCC. LX.
Avec Approbation et Privilege du Roi.

Photometrie der diffusen Zurückwerfung. 473

XI. ***Die Photometrie der diffusen Zurückwerfung; von E. Lommel.***

(Aus den Sitzungsber. d. math. phys. Classe d. K. Acad. zu München, mitgetheilt vom Hrn. Verf.)

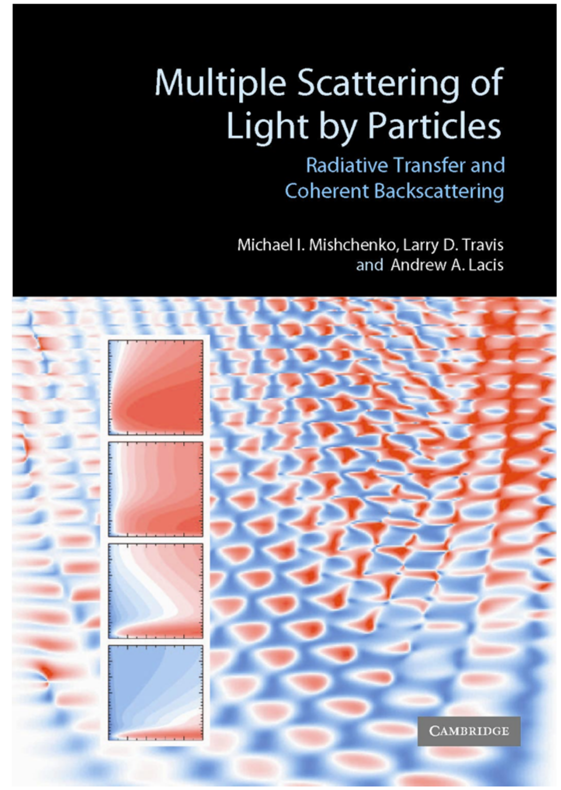
In einer früheren Abhandlung „über Fluorescenz"[1]) habe ich in einem: „Ueber die Grundsätze der Photometrie", überschriebenen Abschnitt gezeigt, dass in der theoretischen Photometrie nicht, wie bis dahin üblich war, die Flächenelemente einer leuchtenden Oberfläche, sondern die Volumenelemente des leuchtenden Körpers als lichtstrahlend zu betrachten seien. Demgemäss wurden der theoretischen Behandlung photometrischer Probleme die folgenden drei Sätze zu Grunde gelegt:

I. Die von einem Volumenelement nach einem anderen strahlende Lichtmenge ist dem Quadrate ihrer Entfernung umgekehrt proportional.

II. Die von einem Volumenelement ausstrahlende und auf ein Flächenelement fallende Lichtmenge ist dem Cosinus des Incidenzwinkels proportional.

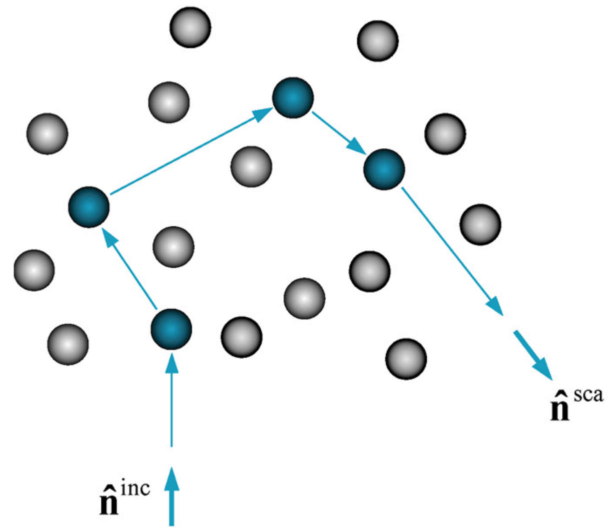
III. Das von einem Volumenelement ausstrahlende Licht wird auf seinem Wege innerhalb des strahlenden Körpers nach Maassgabe des Absorptionsgesetzes geschwächt.

1) Lommel, Wied. Ann. 10. p. 449 u. 631. 1880.



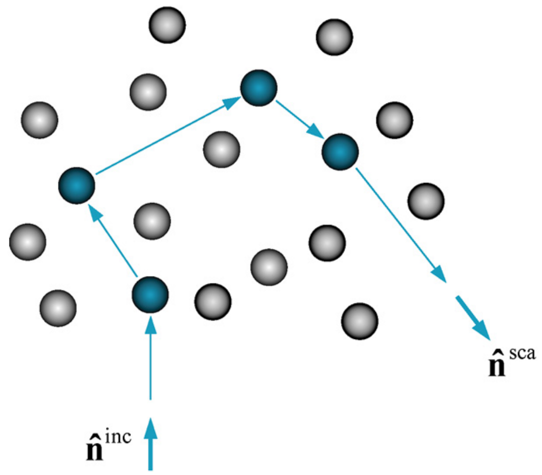
Foldy–Lax equations

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}^{\mathrm{inc}}(\mathbf{r}) + \sum_{i=1}^{N} \mathbf{E}_i(\mathbf{r})$$



Radiative transfer equation

1. Each particle is located in the far-field zones of all other particles; the observation point is also located in the far-field zones of all the particles forming the scattering medium

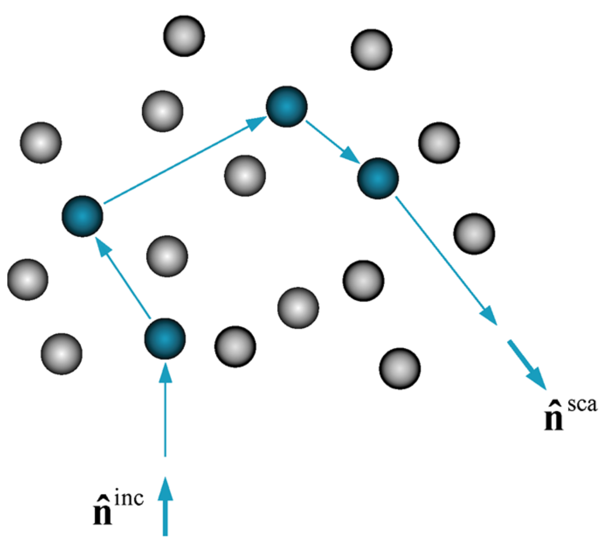


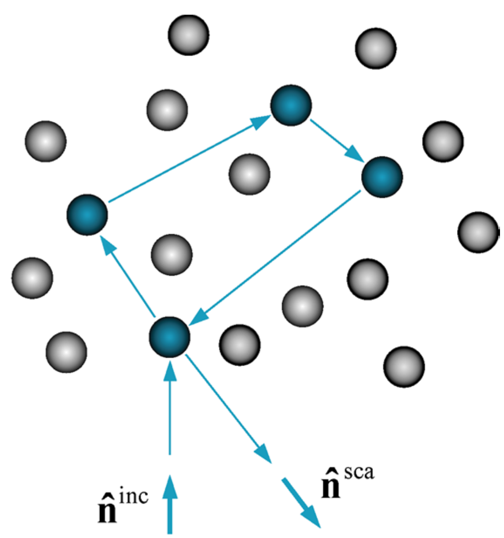
2. (a) Ergodicity:
   (b) The position and state of each particle are statistically independent of each other and of those of all other particles
   (c) The spatial distribution of the particles throughout the medium is random and statistically uniform

Twersky approximation

3. All multi-particle sequences going through a particle two and more times are neglected (the Twersky approximation). This is justified when the total number of particles in the scattering medium is very large.

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}^{\text{inc}}(\mathbf{r}) + \sum_{i=1}^{N} \mathbf{E}_i(\mathbf{r})$$





Ladder approximation

$$\mathbf{E}_i(\mathbf{r}) \bullet [\mathbf{E}_j(\mathbf{r})]^*$$