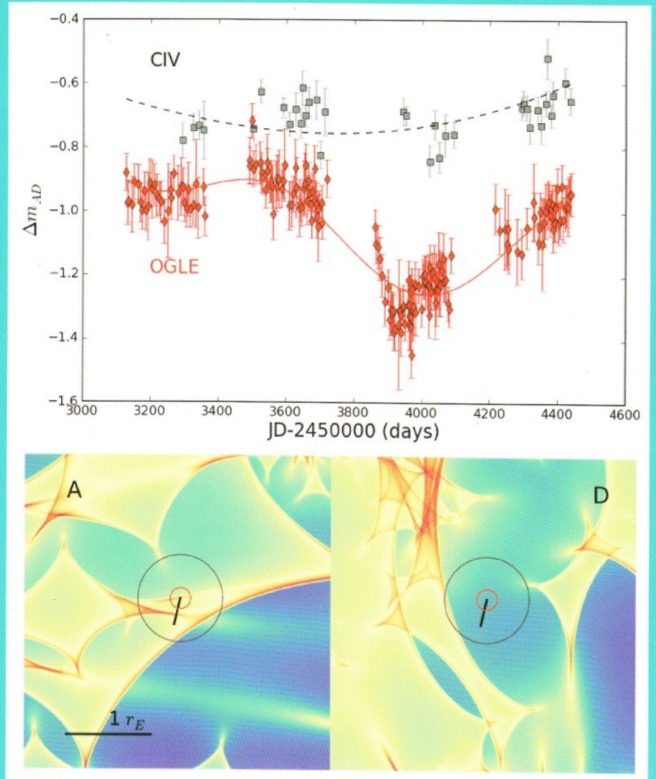
Astronomy & Astrophysics



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Contents sorted by section

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MIXING IN STARS, OCEAN AND ATMOSPHERE

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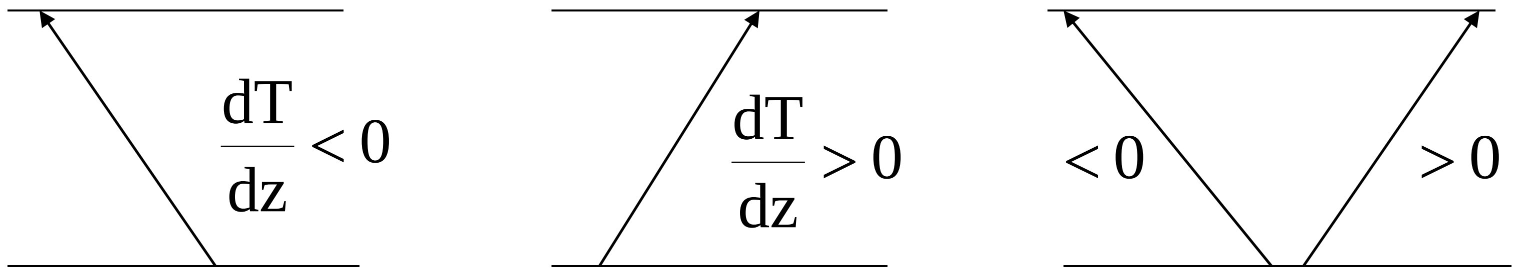
Stars **Ocean** **Atmosphere**



Unstably Stratified

Stable

U

S

U-S

Global Codes (A-OGCMs, Stellar Codes) resolve the large scales

Need: Reynolds stresses, heat fluxes, salt fluxes, etc

OCEANS

Stably stratified (cold waters at the bottom)

Eddies are in general small

Richardson Number: $Ri = N^2\Sigma^{-2}$

MIXED LAYER

- First ~ 100 meters
- Source of Mixing: Wind ~ 1TW
- Typical Values: $D=200\ cm^2s^{-1}$, $\nu=10^{-3}\ cm^2s^{-1}$, $Re=D/\nu=10^5$
- Due to Stable Stratification, Local models are OK:

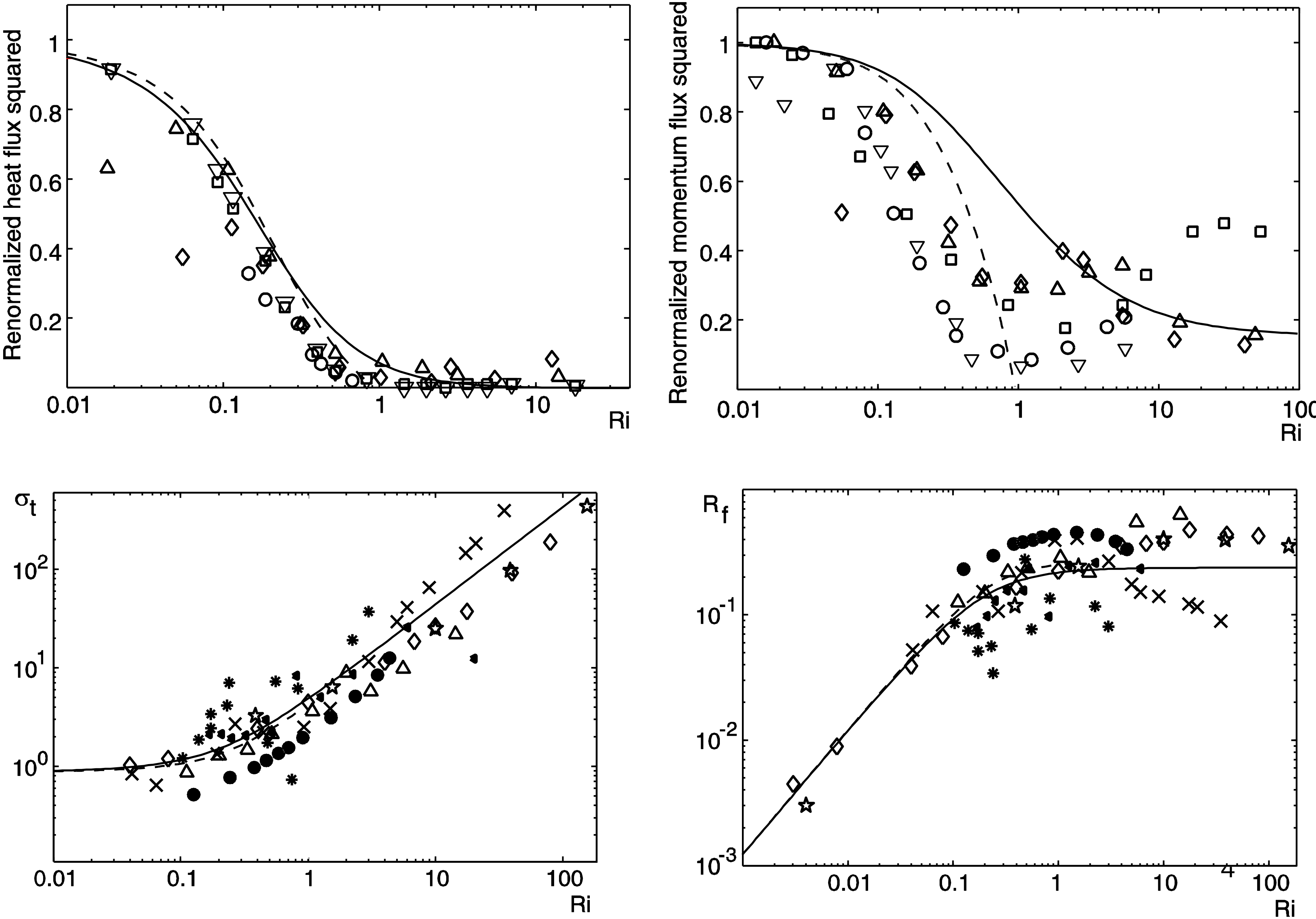
$$\overline{uw} = -K_m \frac{\partial U}{\partial z}, \quad \overline{w\theta} = -K_h \frac{\partial T}{\partial z}, \quad \overline{w\sigma} = -K_s \frac{\partial S}{\partial z}$$

Critical issue: **Ri(cr)**

Mellor-Yamada Ri(cr)=0.2; Empirical studies Ri(cr)=O(1)

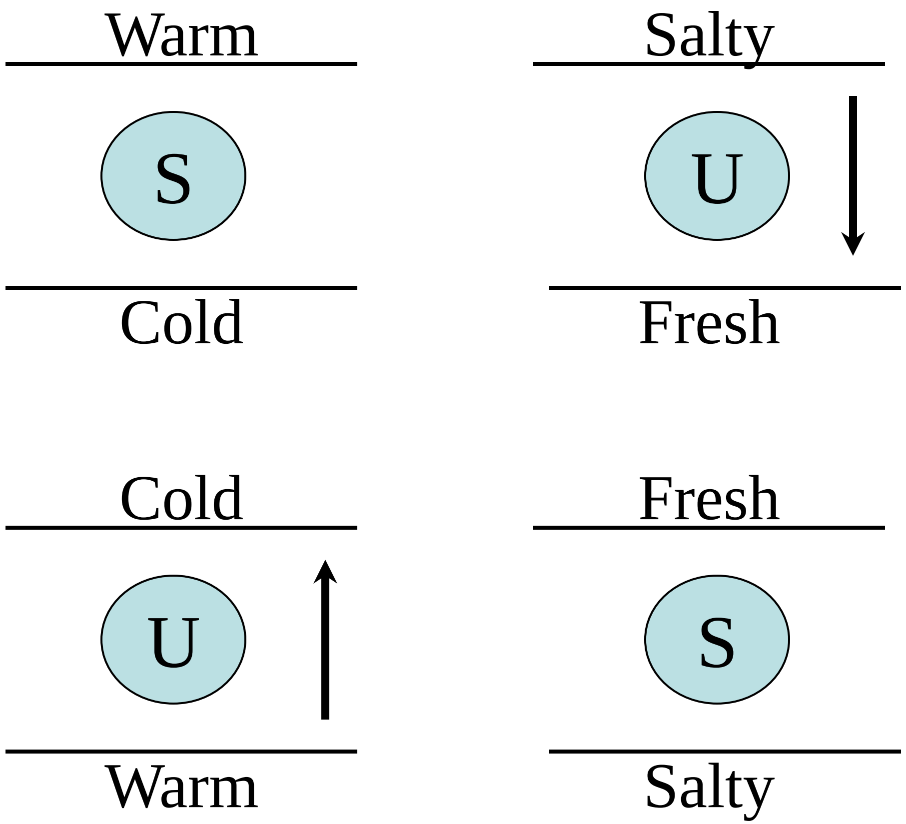
- RSM Ri(cr)= 1

No Ri(cr)



Renormalized heat flux squared
1
0.8
0.6
0.4
0.2
0.01
0.1
1
10
Ri
Renormalized momentum flux squared
1
0.8
0.6
0.4
0.2
0.01
0.1
1
10
100
Ri
σt
10^2
10^1
10^0
0.01
0.1
1
10
100
Ri
Rf
10^-1
10^-2
10^-3
0.01
0.1
1
10
100
Ri

Double Diffusion



SF (Med.)

DC (water/ice)

$$R_\rho = \frac{\alpha_S \partial S/\partial z}{\alpha_T \partial T/\partial z}, \quad \gamma = \frac{\alpha_T \overline{w\theta}}{\alpha_s \overline{ws}}$$

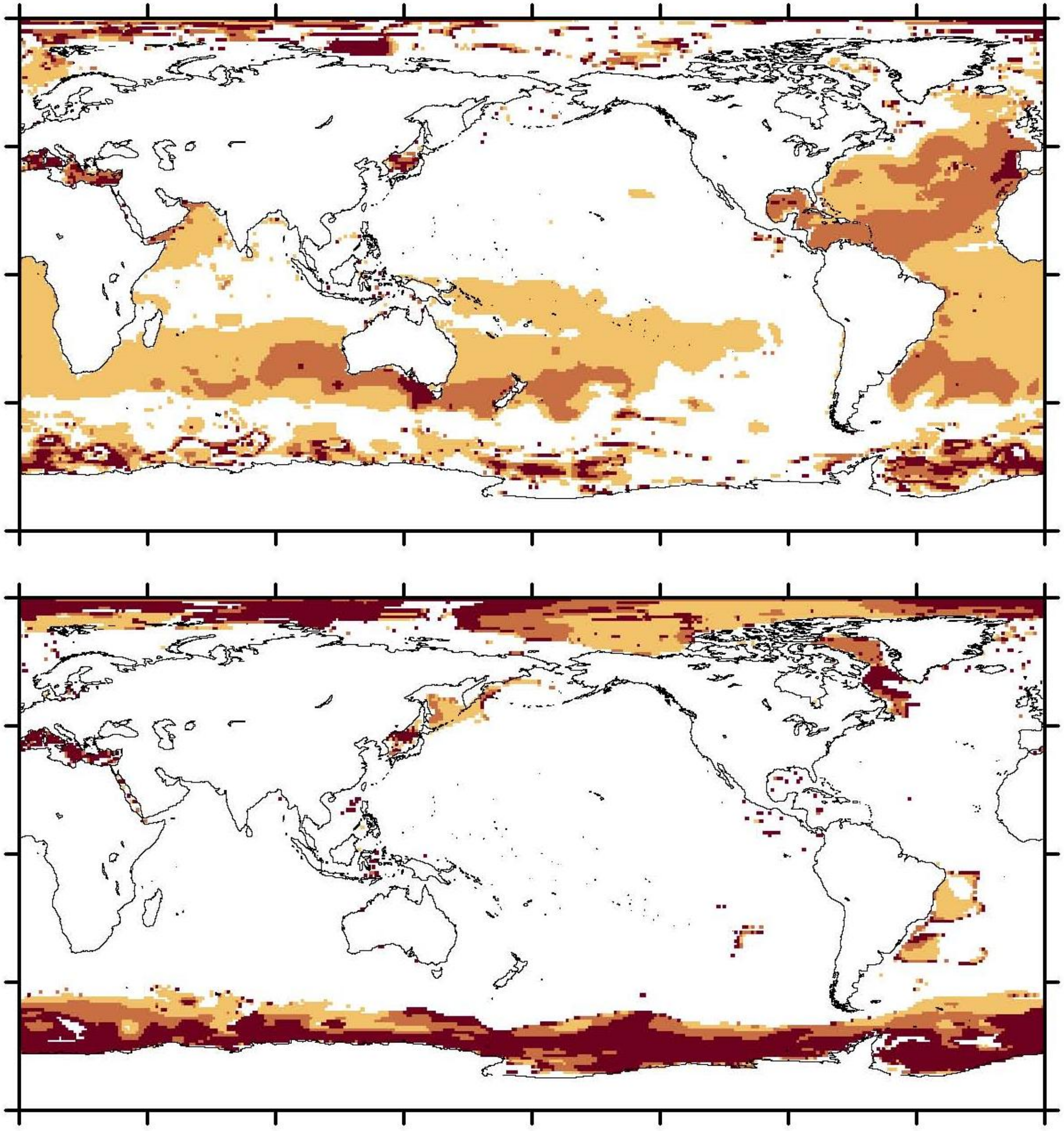
SF: $R_\rho < 1; \quad \gamma < 1$

DC: $R_\rho > 1; \quad \gamma > 1$

Double Diffusion SF, DC

SF

DC



DOUBLE DIFFUSION

- ✓ Weak turbulence: $K(h) \neq K(s)$
- ✓ NATRE: $K(s)=1.38K(h)$

SALT FINGERS

- ✓ Salty-warm /cold-fresh waters (Mediterranean Sea)
- ✓ **Star analogy: helium flask off center**
- ✓ **Higher μ, hot/colder, low μ gas**

DIFFUSIVE CONVECTION

- ✓ Cold-fresh/warm-salty waters (Read Sea)
- ✓ **Star analogy: Semi-convection SN 1987A-Red vs. Blue**

$$Ri, \quad R_\rho = \frac{\beta \partial S/\partial z}{\alpha \partial T/\partial z}$$

Lab. Data

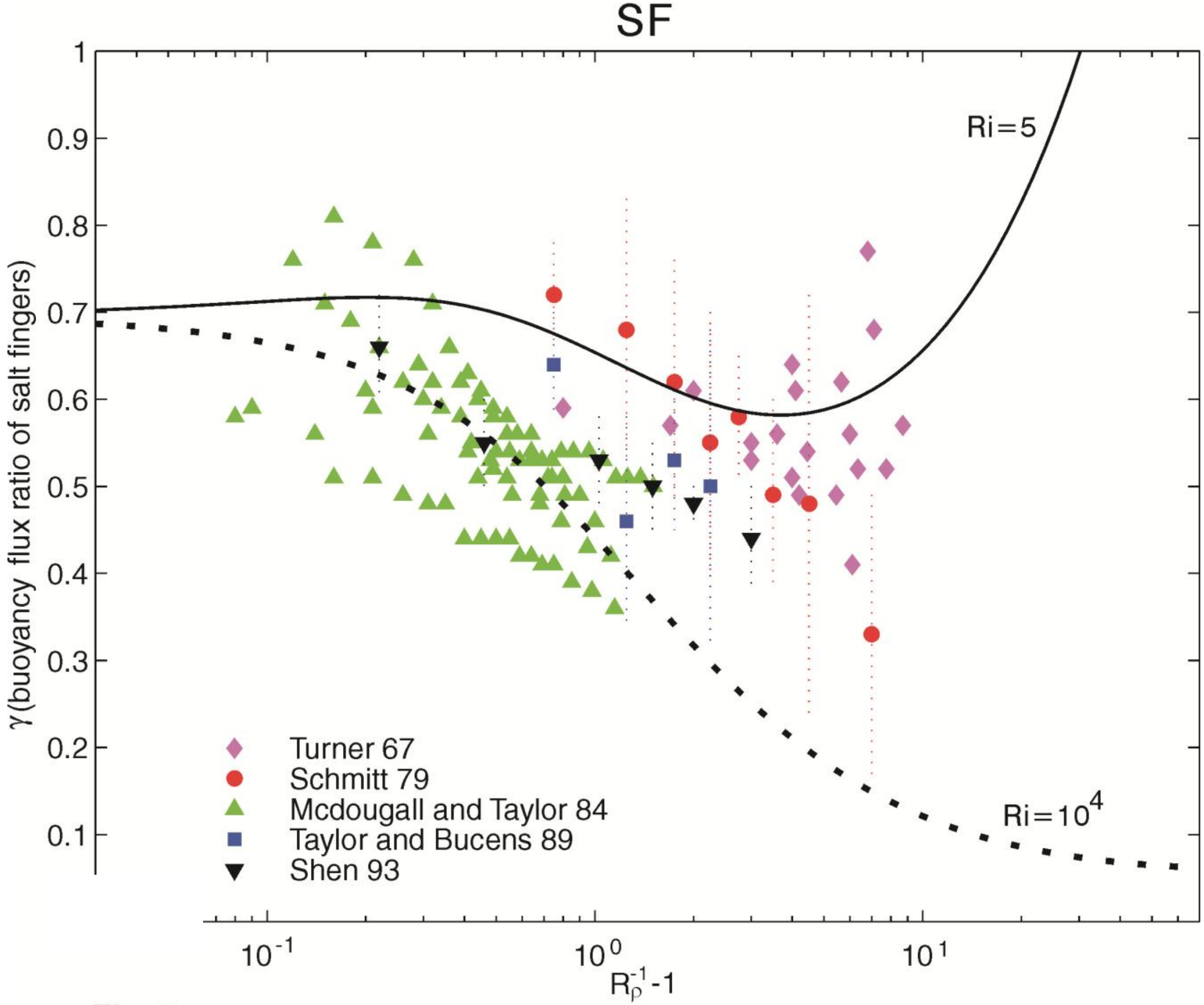
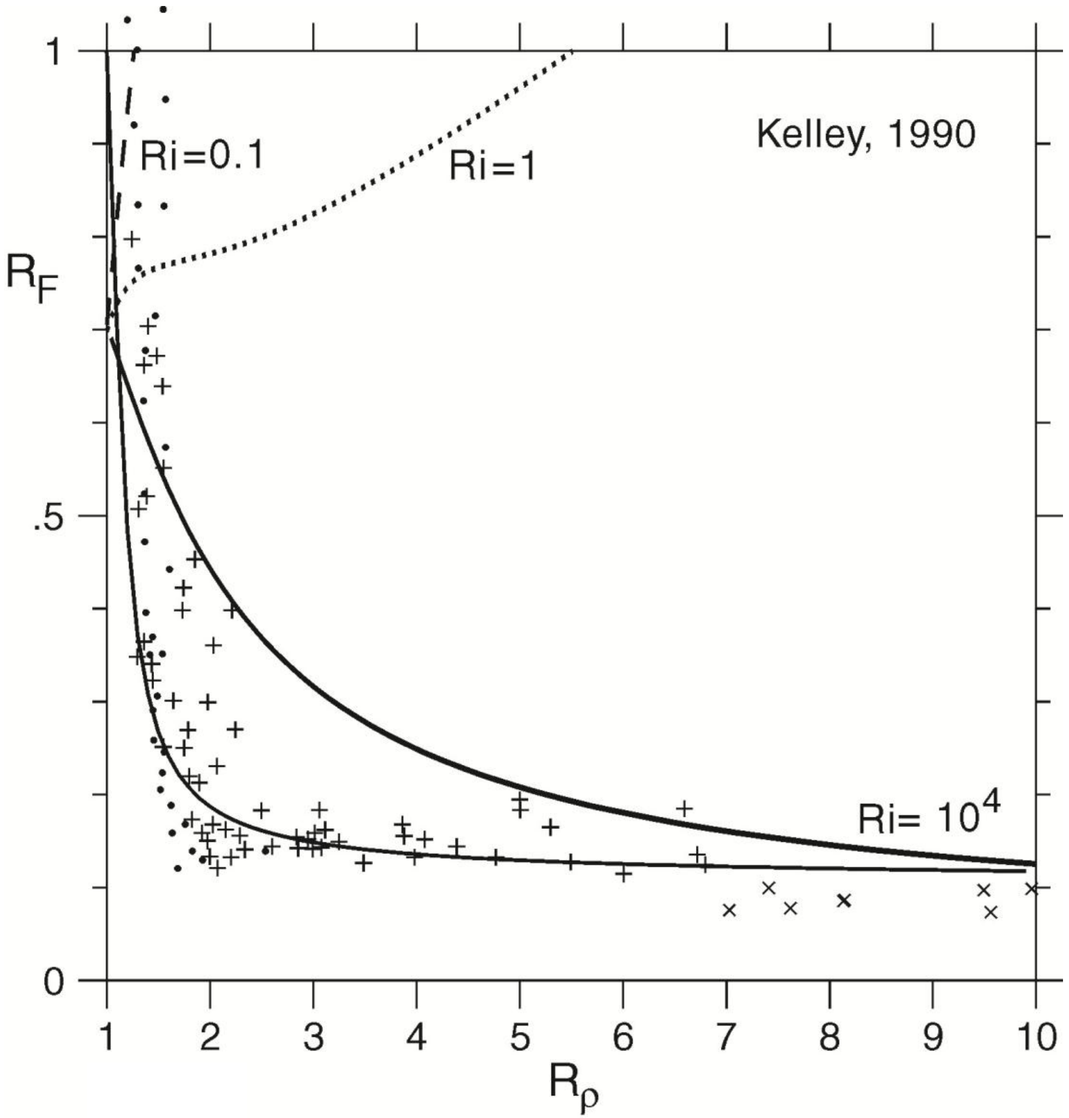


Fig. 2

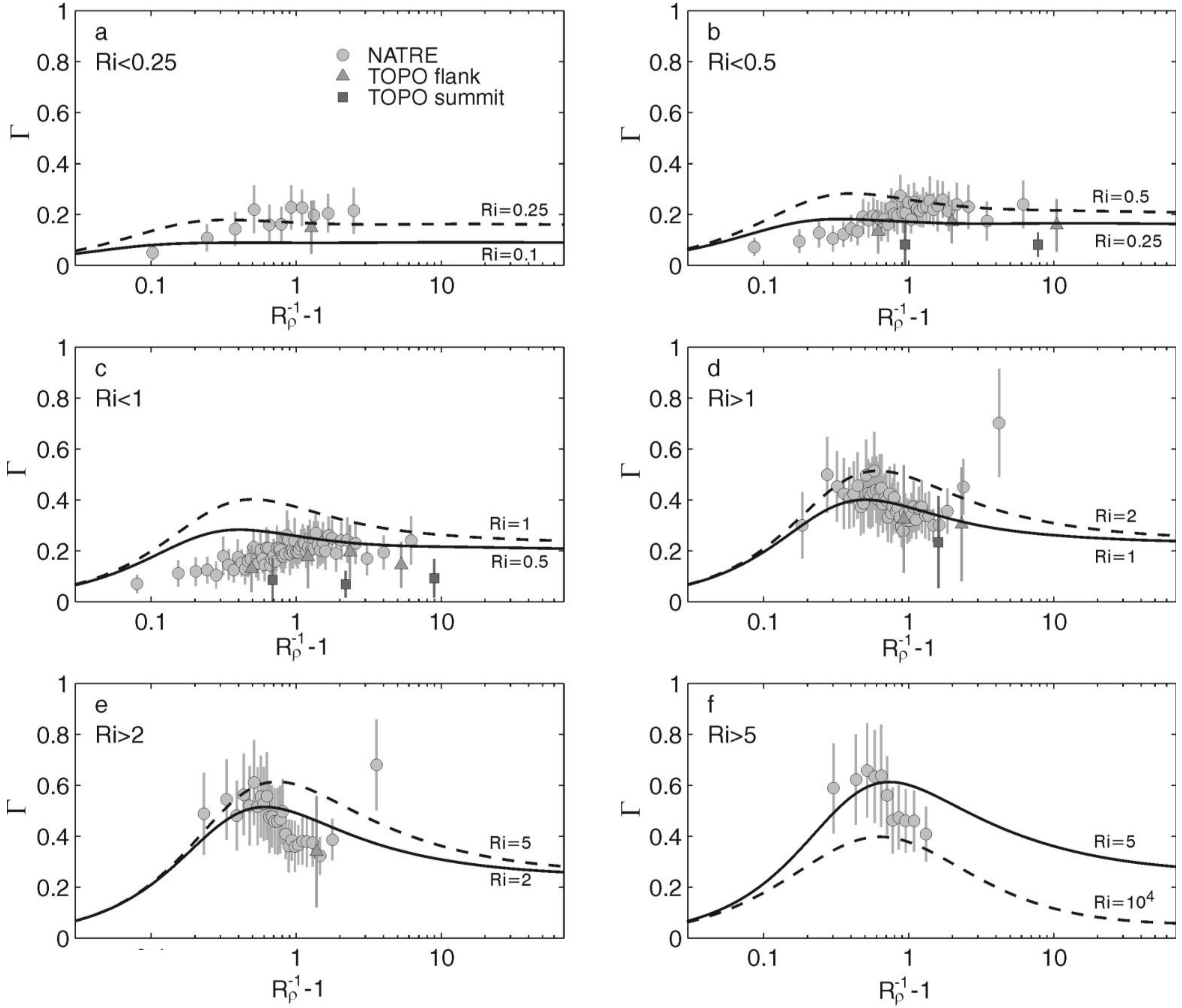
Lab. Data

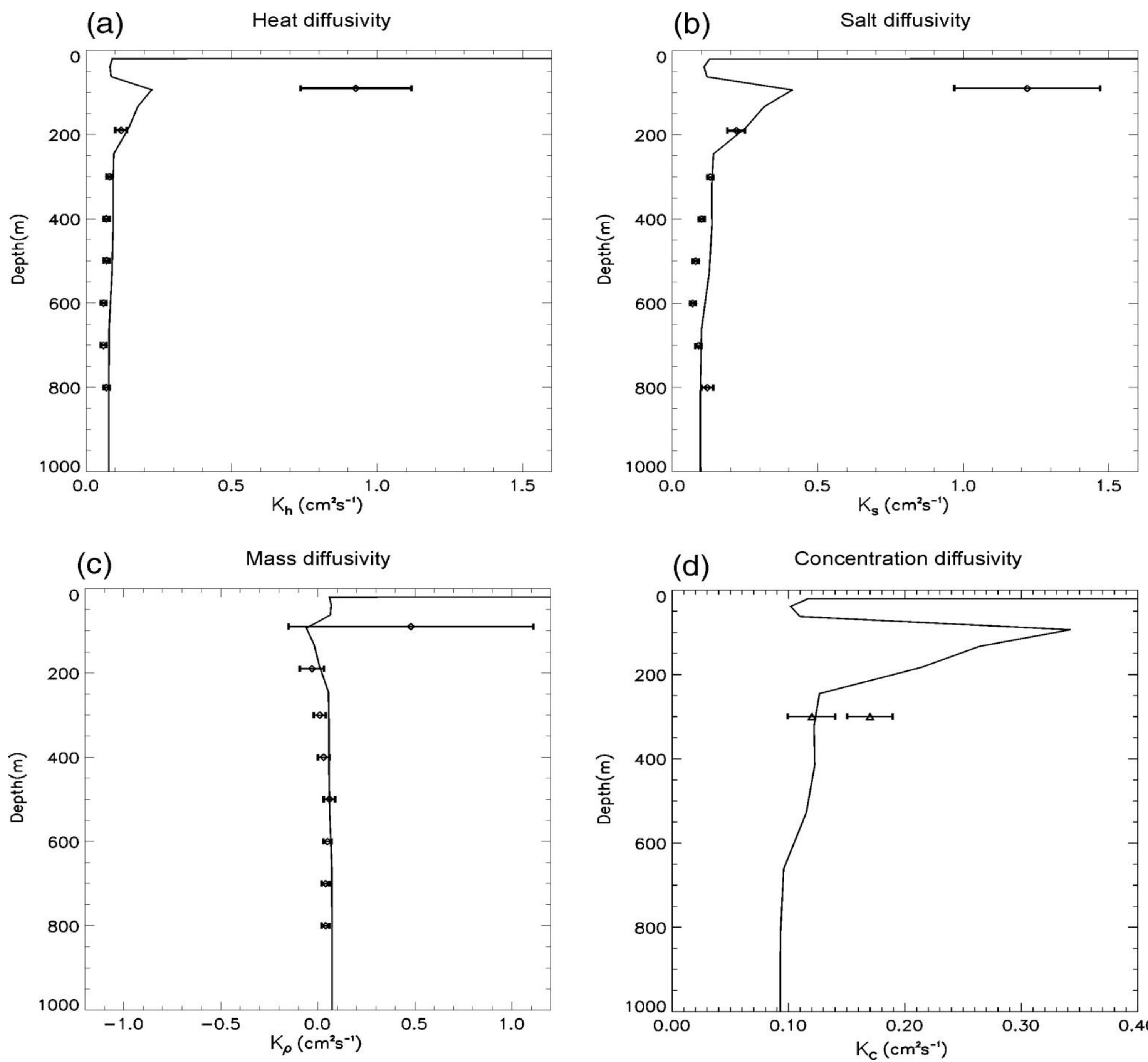
DC



SF

$$K_h = \Gamma \varepsilon N^{-2}$$



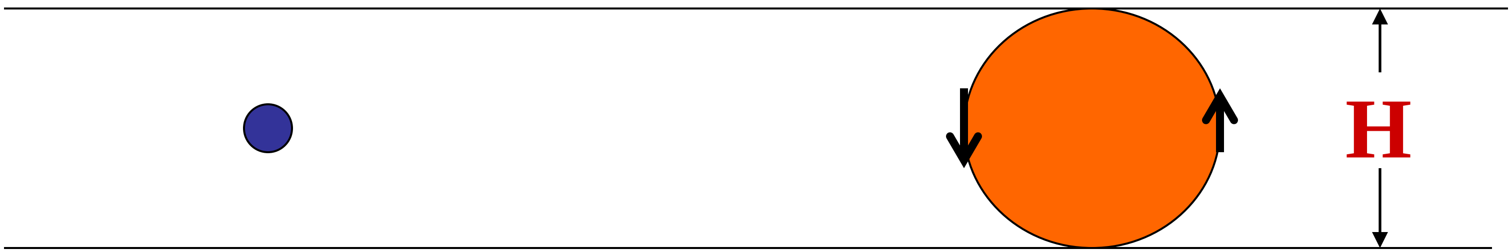

(a)
Heat diffusivity
Depth(m)
K_h (cm²s⁻¹)
(b)
Salt diffusivity
Depth(m)
K_s (cm²s⁻¹)
(c)
Mass diffusivity
Depth(m)
K_ρ (cm²s⁻¹)
(d)
Concentration diffusivity
Depth(m)
K_c (cm²s⁻¹)

DEEP CONVECTION

- Labrador Sea, Gulf of Lyon, Waddle Sea
- Strong winds lower the temperature
- Loss of Buoyancy
- Cooling from above
- Fast Process: Days, 1-2km,   w ~ a few cm/sec
- Labrador Data on the Depth of the Mixed layer
- **Rotation** ??

⊕ **Atmospheric analogy:** Cloud Capped PBL

☆ **Stellar analogy:** cooling from the top due to change in opacity



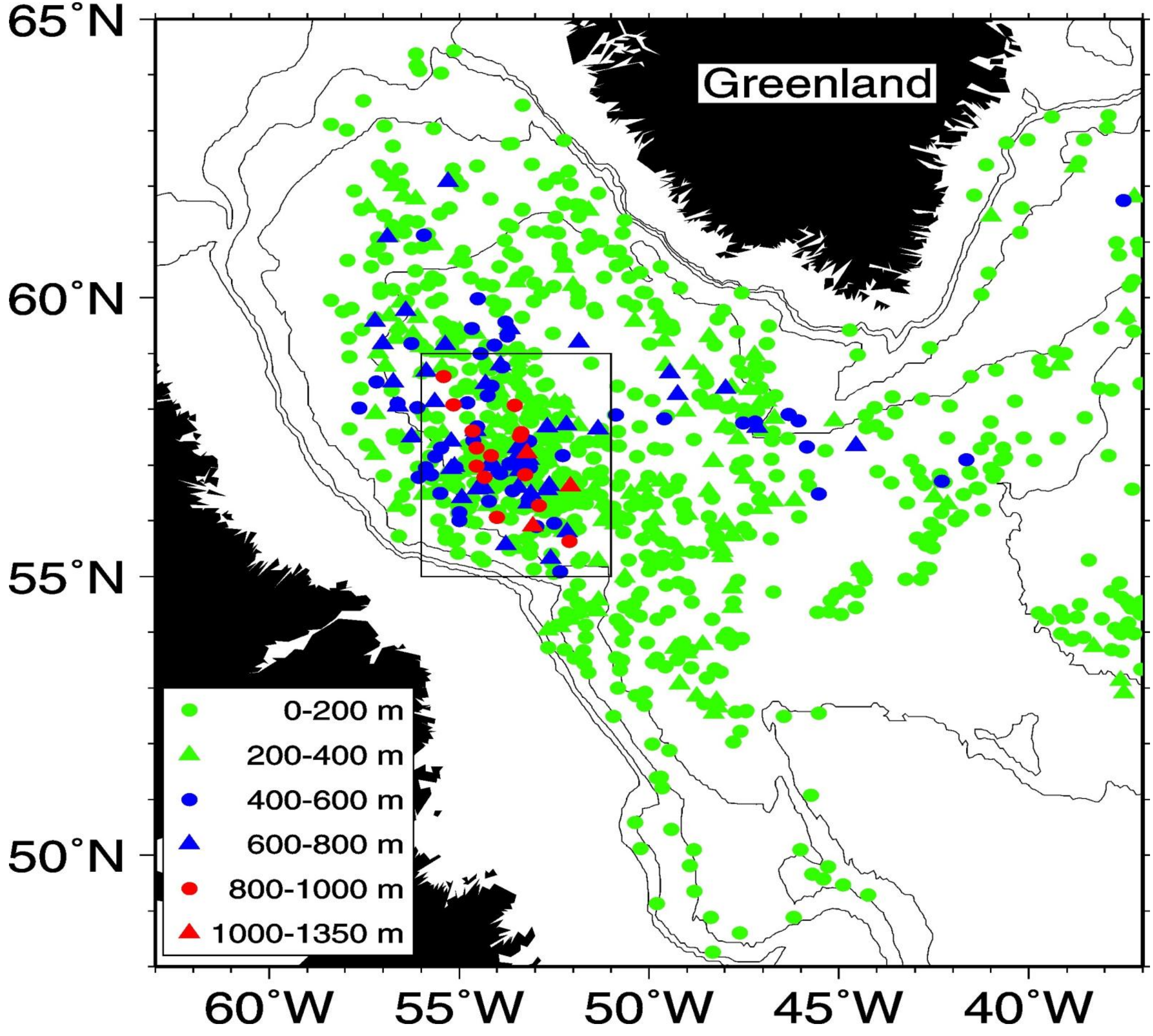
Small eddies | **Large eddies**

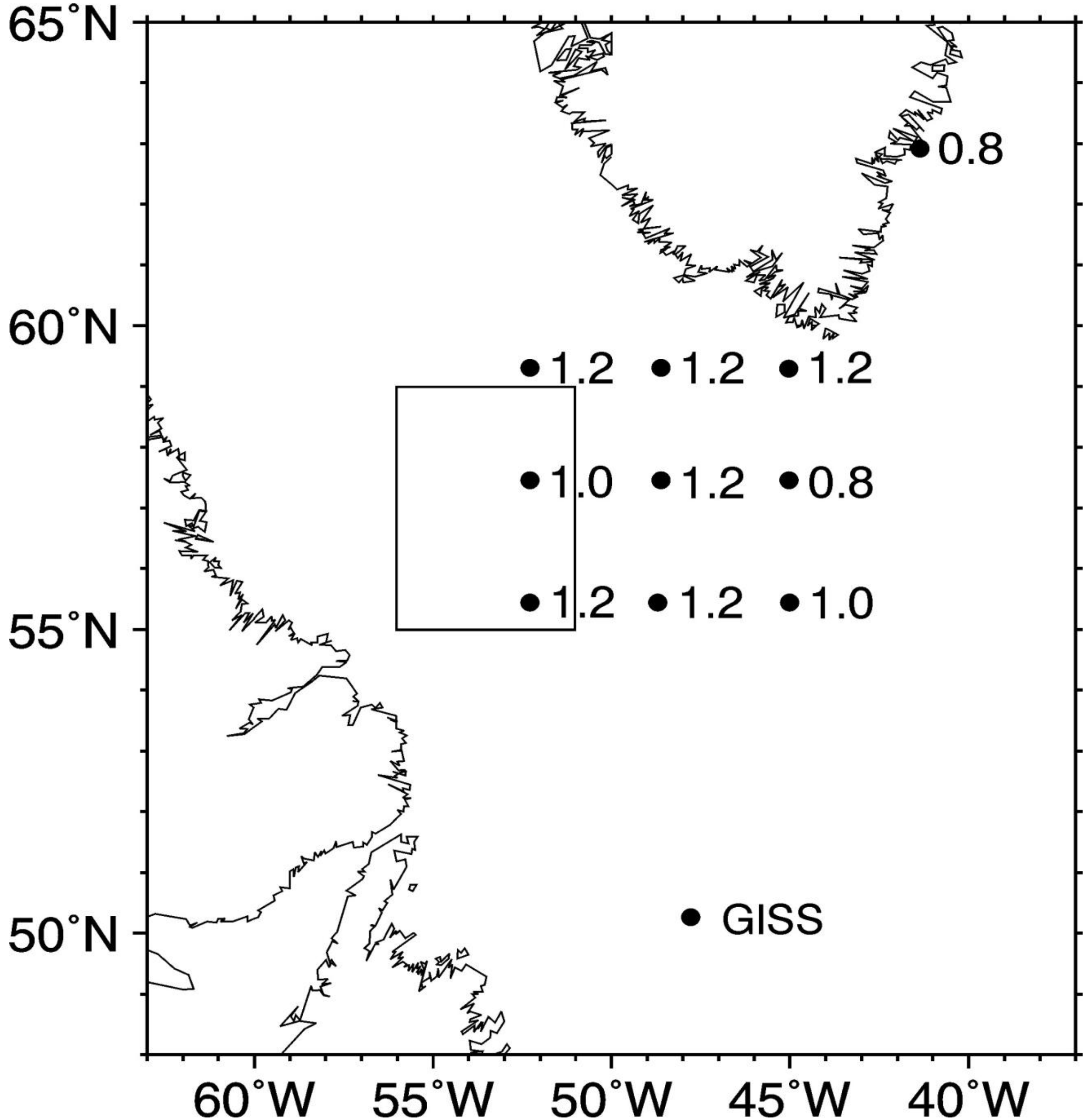
Local | **Non-Local**

$$J(z) = -K_h \frac{\partial T}{\partial z}$$

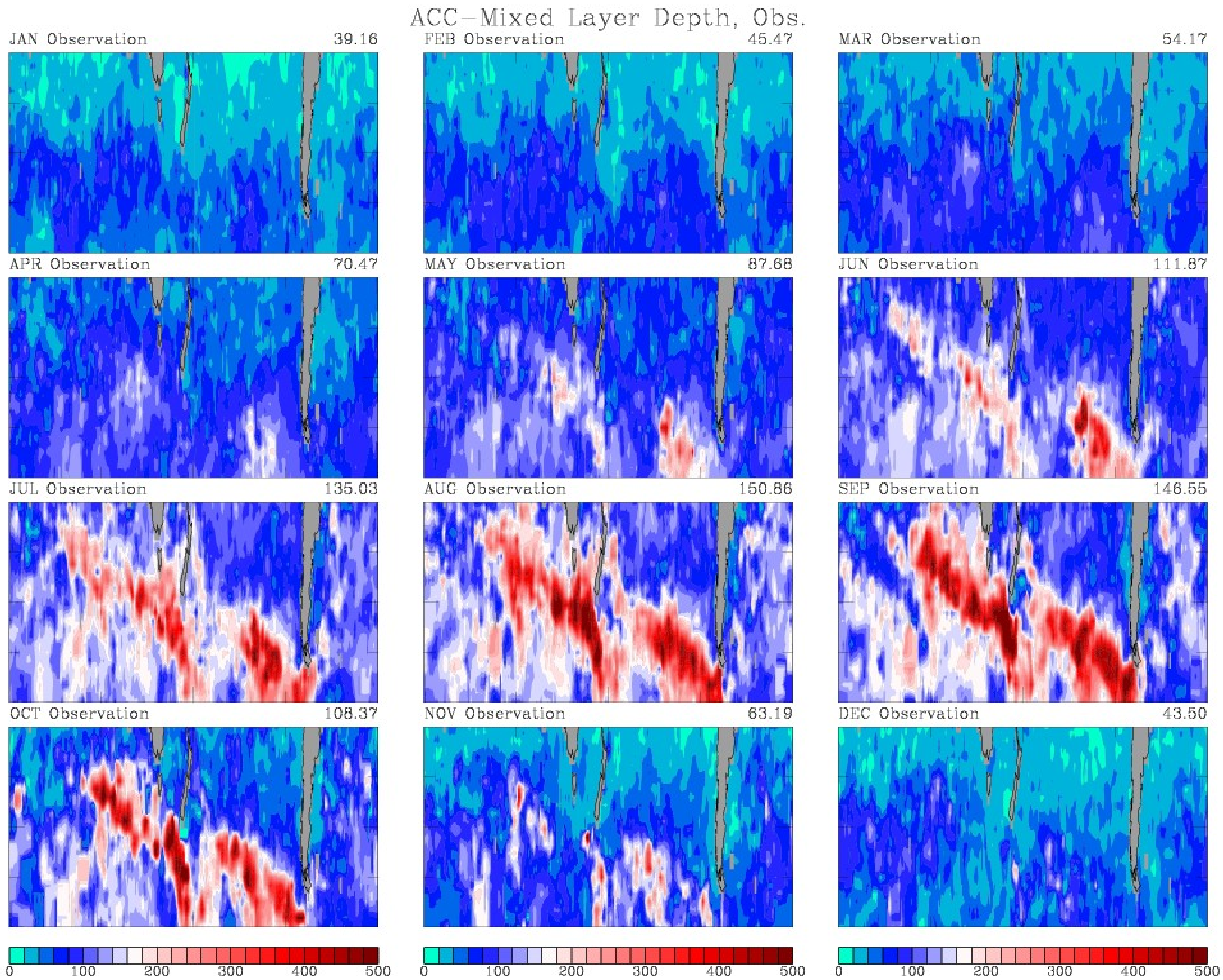
$$J(z) = \int_0^z K_h \frac{\partial T}{\partial z'} dz'$$

No H | **H**

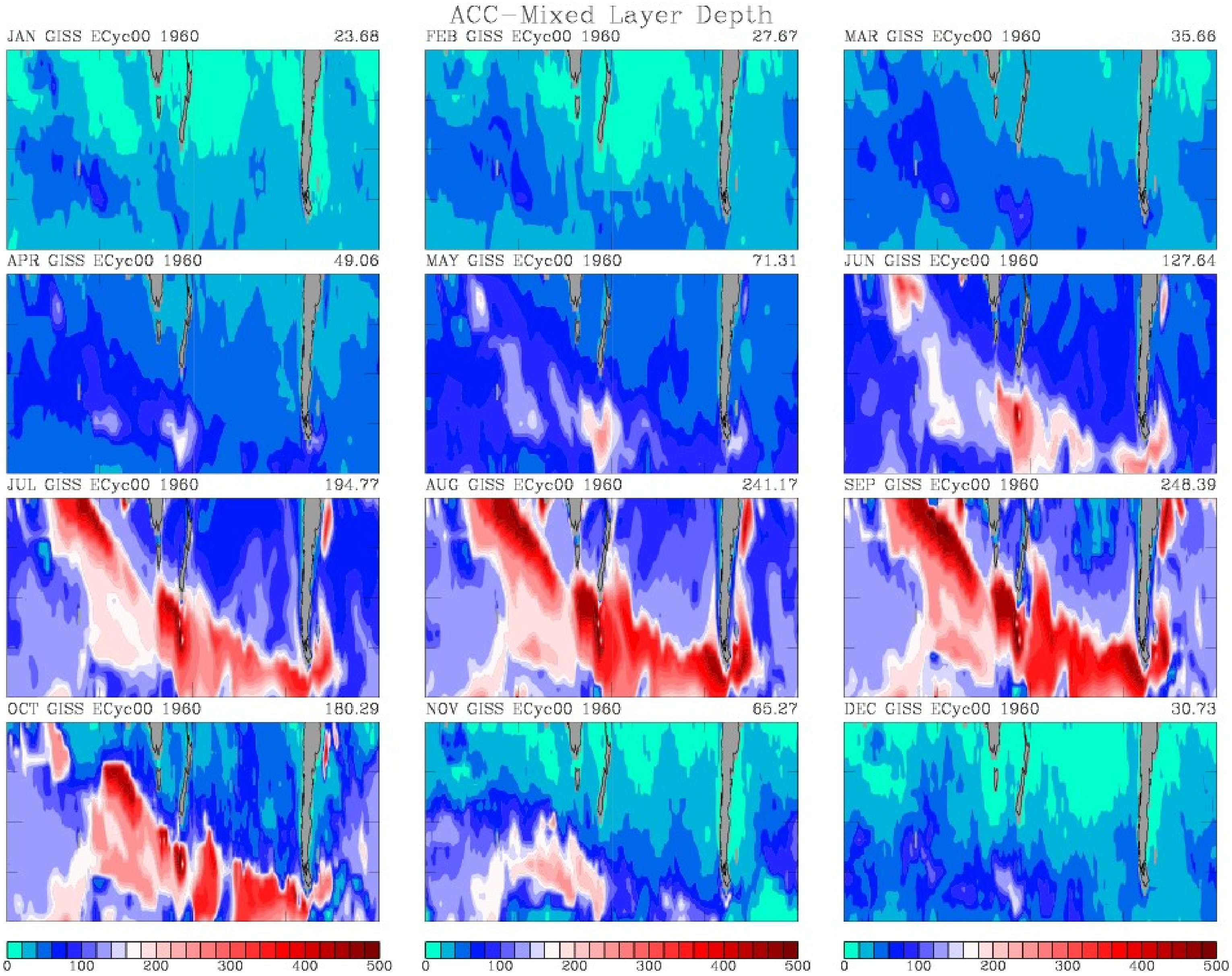

65°N
60°N
55°N
50°N
60°W
55°W
50°W
45°W
40°W
Greenland
0-200 m
200-400 m
400-600 m
600-800 m
800-1000 m
1000-1350 m


65°N
60°N
55°N
50°N
60°W
55°W
50°W
45°W
40°W
0.8
1.2
1.2
1.2
1.0
1.2
0.8
1.2
1.2
1.0
GISS

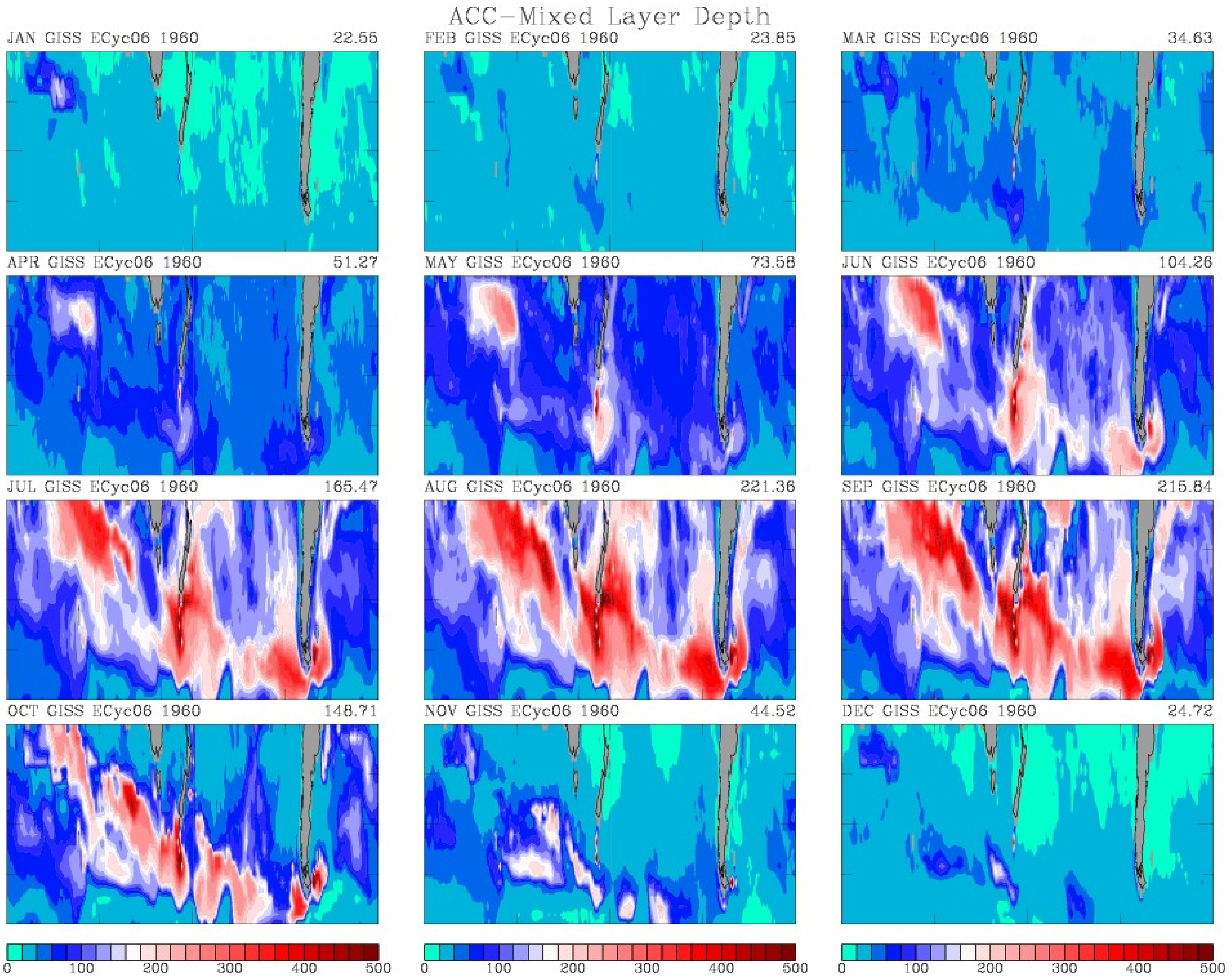
MLD at ACC: Observations (Dong et al. 2006)

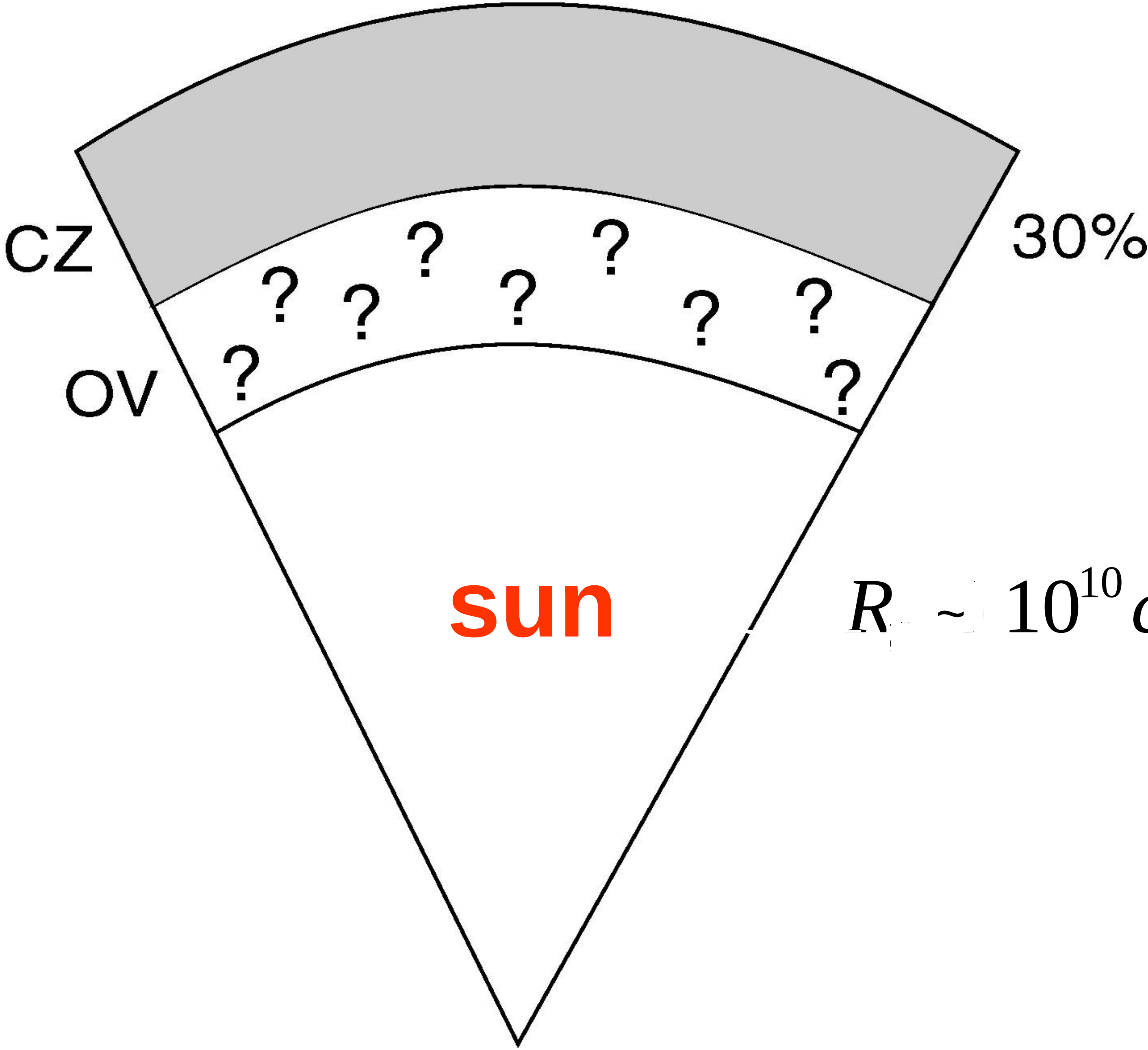


MLD at ACC: KPP Model



MLD at ACC: GISS Model 2011





$R \sim 10^{10}\,cm$

STARS (e.g. Sun)

- Last 30% is governed by CONVECTION
- Heated from Below, Cooled from Above
- Unstable Stratification. Large Eddies
- Non-Local Model

OVERSHOOTING. Unsolved problem: **OV = ?H_p**

$$\frac{\partial K}{\partial t} + \frac{\partial}{\partial z} F(K) = J - \varepsilon$$

- In the OV, stratification is stable, there is no local source of mixing
- RHS < 0
- LHS < 0
- F(K), flux of turbulent kinetic energy, acts like a Non-Local Source

F(K) is a TOM (third-order moment)

- **TOM can be derived from FOM (4-th order moments) and so on….**

- **New aircraft measurements on FOM have changed the scenario**

- **New FOM mean new TOM and thus a Non-Local Model can be constructed**

New FOM Model (JAS, 2005)

$$\overline{w^4} = \overline{w^4}\Big|_{QN} + p_1 \int_0^z \frac{\overline{w^3}}{\tau} dz$$

$$\overline{w^3\theta} = \overline{w^3\theta}\Big|_{QN} + \int_0^z (p_2 \frac{\overline{w^2\theta}}{\tau} + d_1 \beta \overline{w^3}) dz$$

$$\overline{w\theta^3} = \overline{w\theta^3}\Big|_{QN} + \int_0^z (p_4 \frac{\overline{\theta^3}}{\tau} + d_3 \beta \overline{w\theta^2}) dz$$

$$\overline{w^2\theta^2} = \overline{w^2\theta^2}\Big|_{QN} + \int_0^z (p_3 \frac{\overline{w\theta^2}}{\tau} + d_2 \beta \overline{w^2\theta}) dz$$

$$\overline{u^2w^2} = \overline{u^2w^2}\Big|_{QN} + p_5 \int_0^z \frac{\overline{u^2w}}{\tau} dz$$

$$\overline{u^2w\theta} = \overline{u^2w\theta}\Big|_{QN} + \int_0^z (p_6 \frac{\overline{u^2\theta}}{\tau} + d_4 \beta \overline{u^2w}) dz$$

Fourth order moments in a convective PBL

Circles: aircraft data of Hartmann et al. (1999)

Dotted lines: QN

Dashed lines: Gryanik-Hartman

Solid lines: New Model

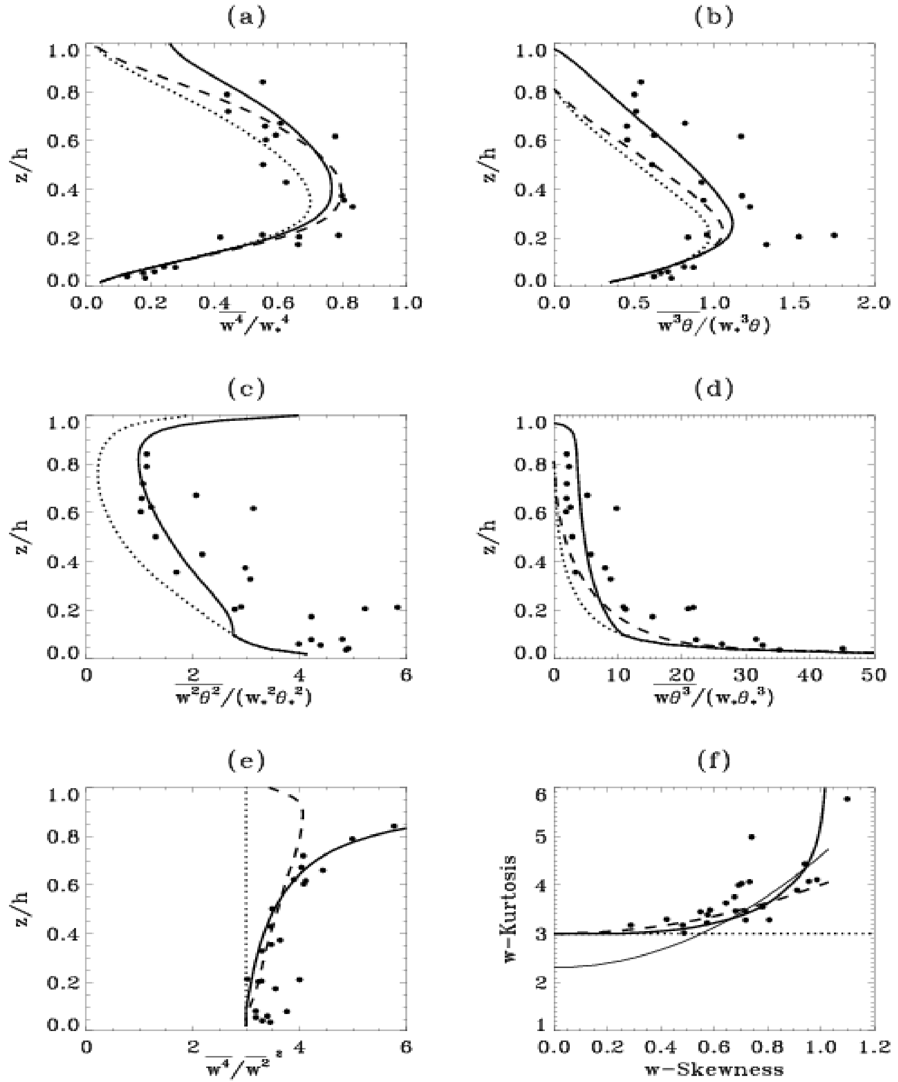


Fig.2

Third order moments in a convective PBL

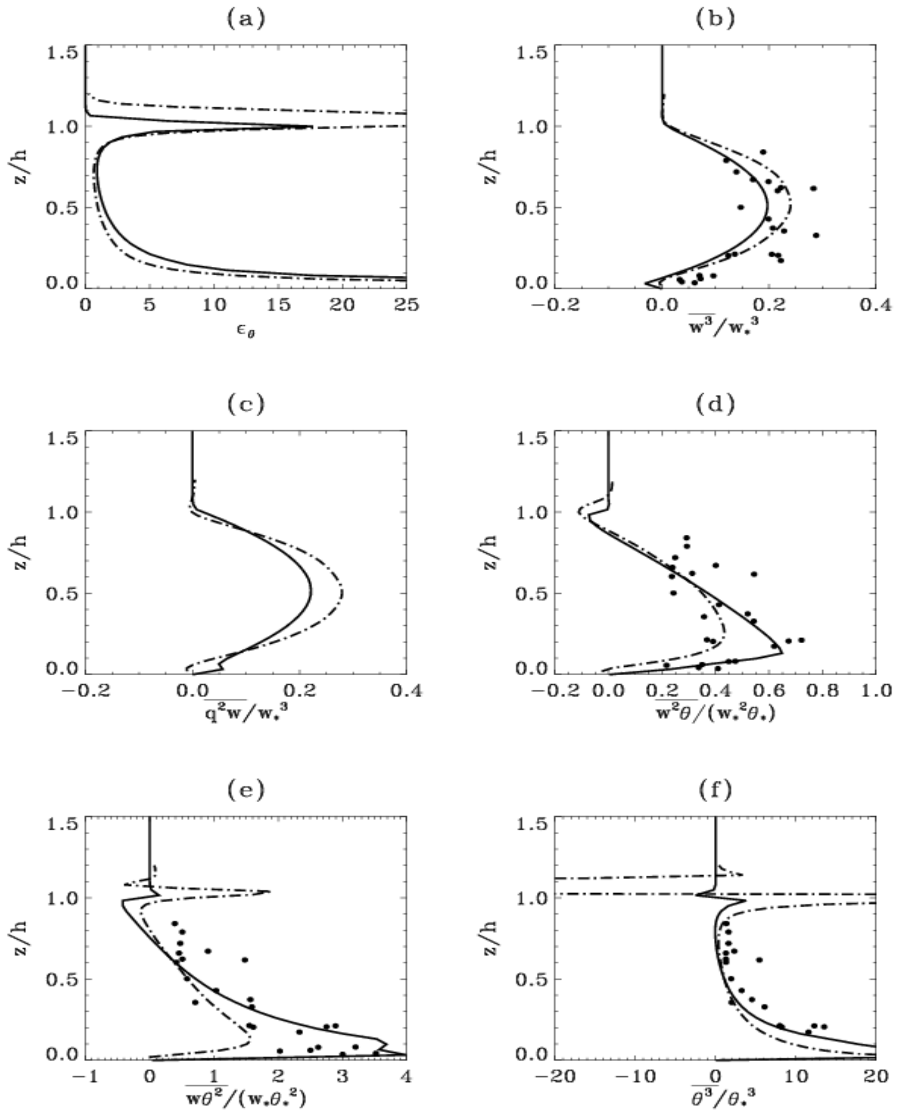


Fig.4

Filled Circles: aircraft data

Dash-Dotted lines: QN

Solid lines: New Model

MIXING AND STIRRING

$$\frac{\partial \Phi}{\partial t} + \overline{\mathbf{U}} \cdot \nabla \Phi = \frac{\partial}{\partial x_j}(D_{ij}\frac{\partial \Phi}{\partial x_i})$$

$$D_{ij} = D_{ij}(s)+D_{ji}(a), \quad D_{ij}(s) = \frac{1}{2}(D_{ij}+D_{ji}), \quad D_{ij}(s) = \frac{1}{2}(D_{ij} - D_{ji})$$

$$\frac{\partial}{\partial x_j}[D_{ij}(a)\frac{\partial \Phi}{\partial x_i}] = \frac{\partial D_{ij}(a)}{\partial x_j}\frac{\partial \Phi}{\partial x_i} + D_{ij}(a)\frac{\partial^2 \Phi}{\partial x_i \partial x_j}$$

$$\frac{\partial D_{ij}(a)}{\partial x_j} = -U_{*i} \quad \Rightarrow \quad \partial_i U_{*i} = 0$$

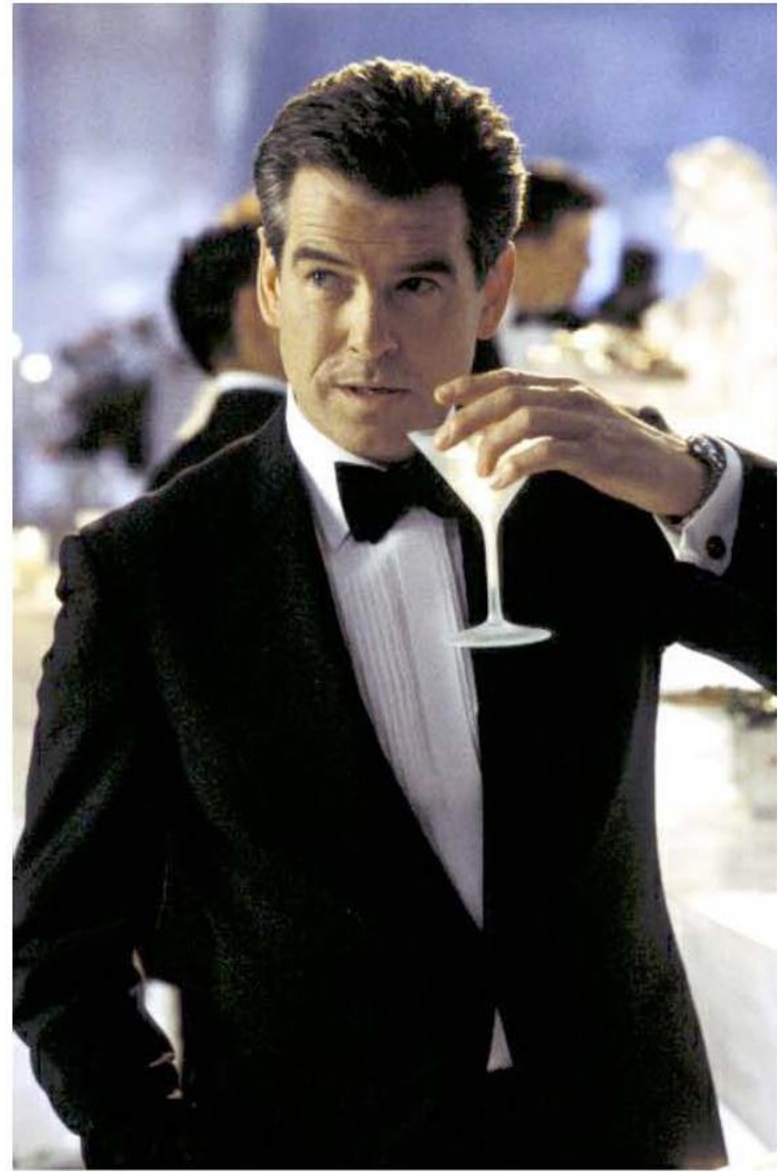
$$\frac{\partial \Phi}{\partial t} + (\underbrace{\overline{\mathbf{U}} + \mathbf{U}_*}_{stirring}) \cdot \nabla \Phi = \frac{\partial}{\partial x_j}\underbrace{[D_{ij}(s)\frac{\partial \Phi}{\partial x_i}]}_{mixing}$$

Advection & Mixing

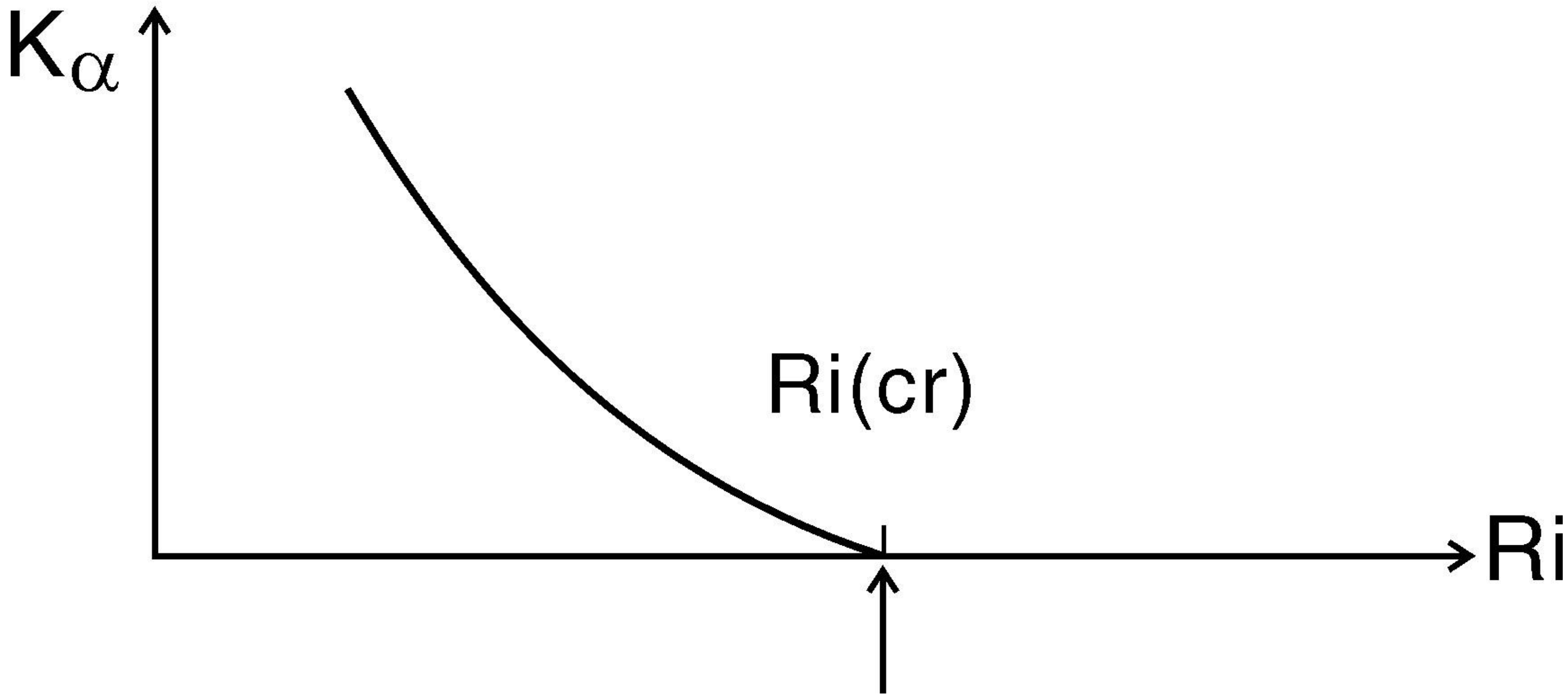
=

Stirring & Shaking

Shaken not stirred, please.



Stable Stratification



PLUME MODEL

Atmospheric LES

Skinny downflows (Twiggy-like), fat upflows

Plume Model is suggestive, **WHO's?**

Only one:Taylor, Turner and Morton. **TTM model**

Can be applied to stars?

TTM model assumptions:

A. Quiescent Environment

B. Isolated Plume

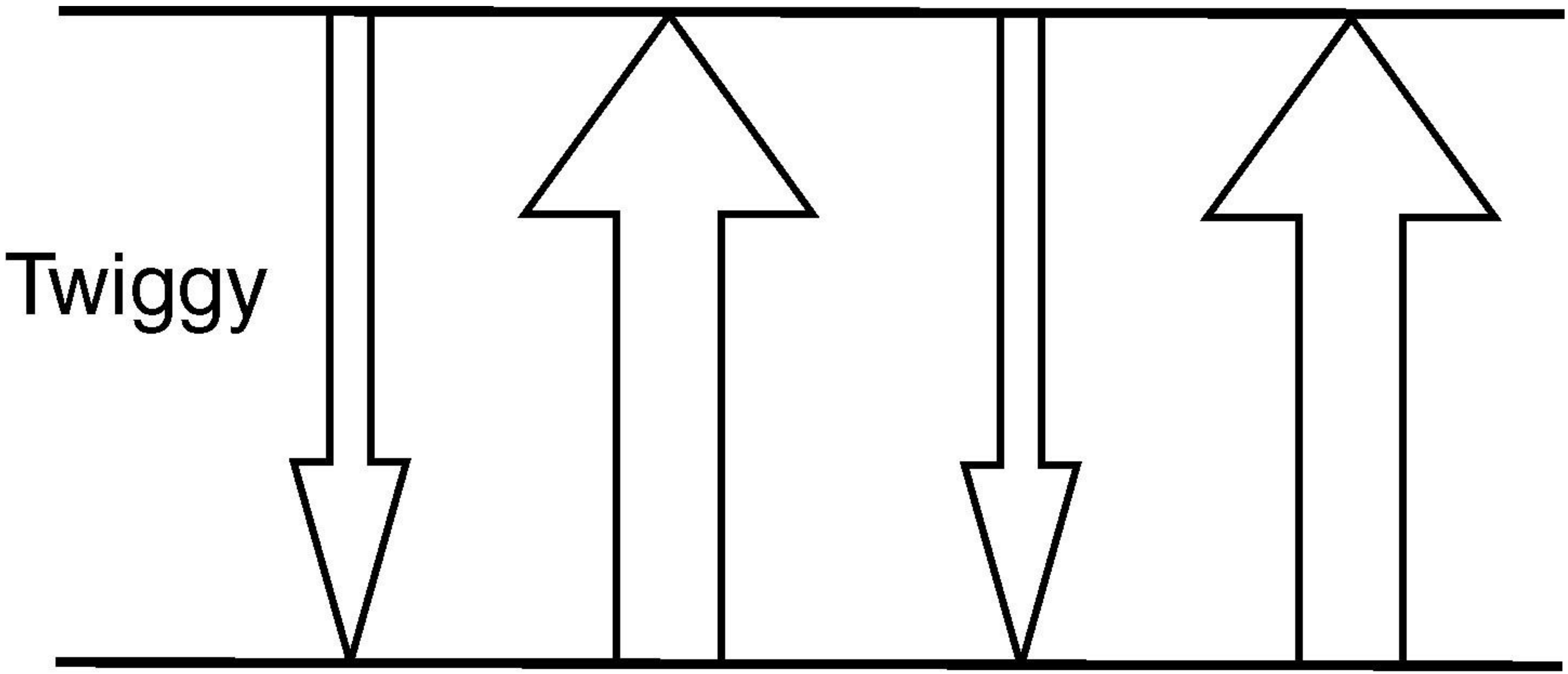
C. Entrainment is an adjustable parameter

Mass conservation law:

$$\lambda w_p + (1\text{-}\lambda) w_e = 0 \qquad \lambda = \frac{\Sigma_p}{\Sigma_p + \Sigma_e}$$

A) means that w_e tends to zero. Thus:

Isolated plume, **B)** above $\qquad \lambda << 1$



Plumology

Is this correct?

- As Twiggy descends into a stably stratified medium, w_p must slow down.
- "Entrainment" of surrounding material makes Twiggy fatter

➢ Thus, λ increases with time until the Plume merges with Environment
λ and 1- λ become the same and thus:

$$\lambda = 1/2$$

- TTM model demands:

$$\lambda \ll 1$$

➢ ***The TTM model is valid only for the initial phases of the Plume's life***

- A PM must account for the whole process. It must be invariant under:

$$\lambda \rightarrow 1 - \lambda, \quad w(p) \rightarrow w(e)$$

- TTM is not invariant under such transformation. *Can it be made invariant?*

Entrainment, E

- TTM: E is an adjustable parameter

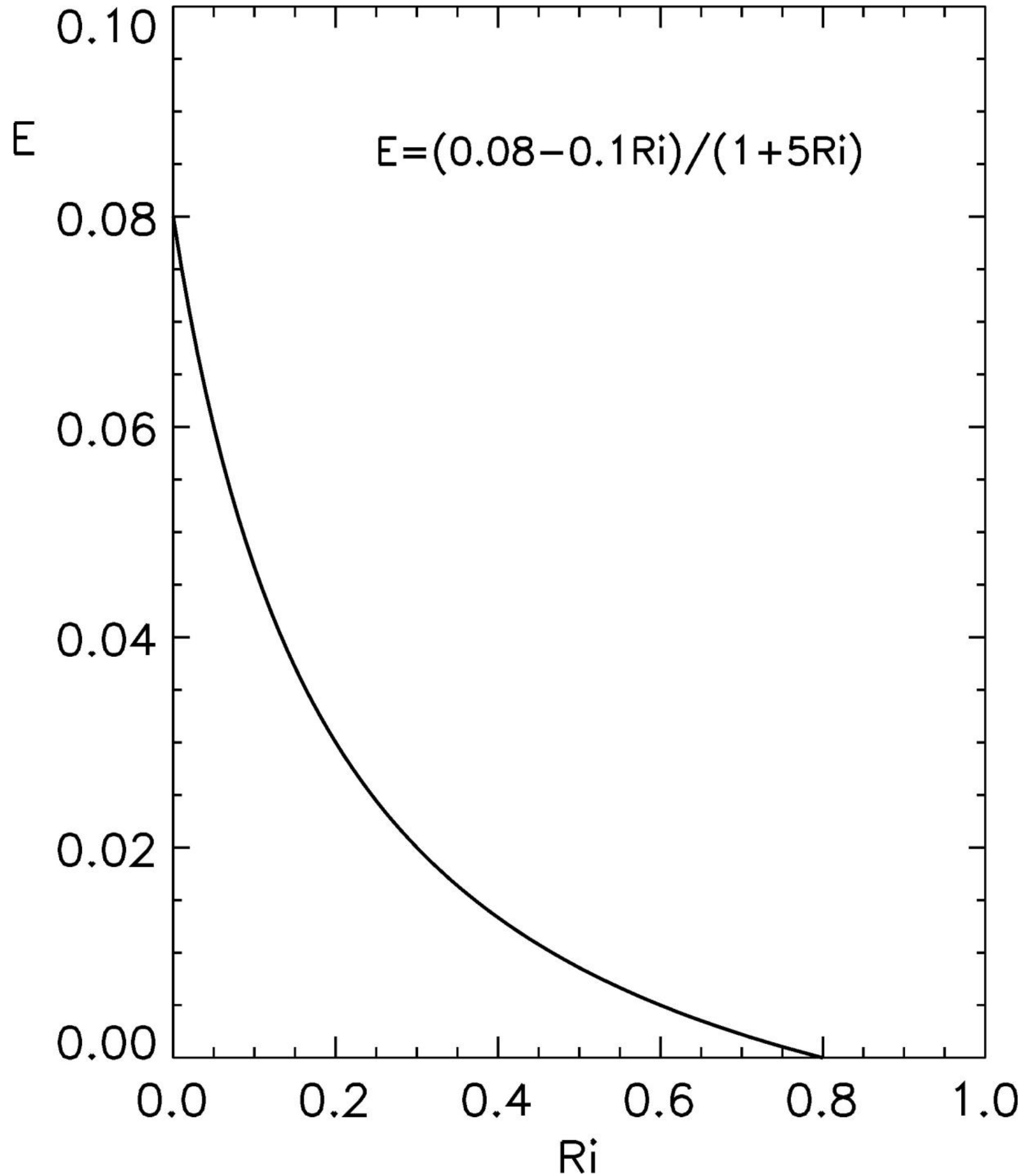
- Ellison-Turner (1959): E=E(Ri), lab experiment

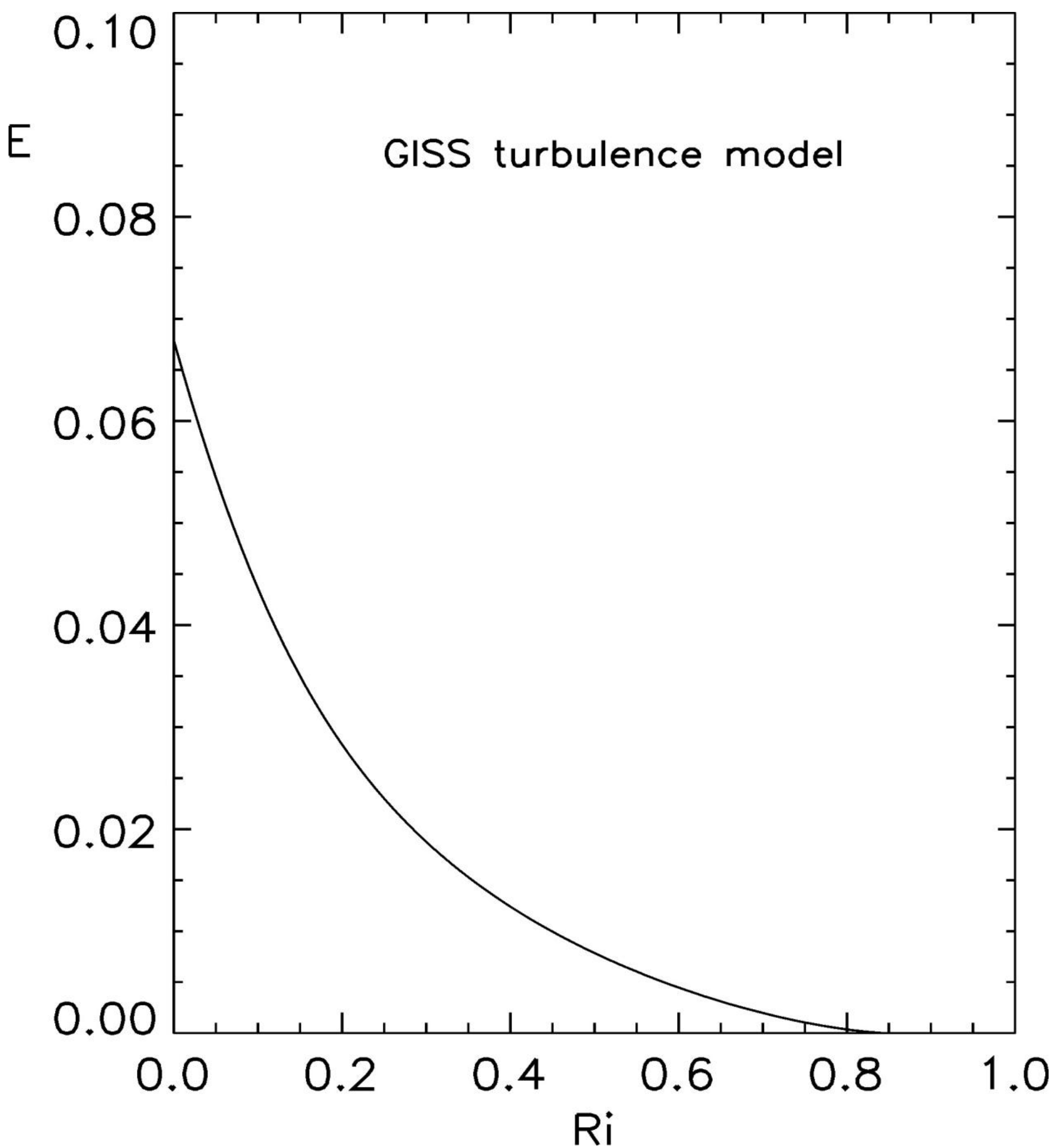
- GISS Model (2004): reproduced it

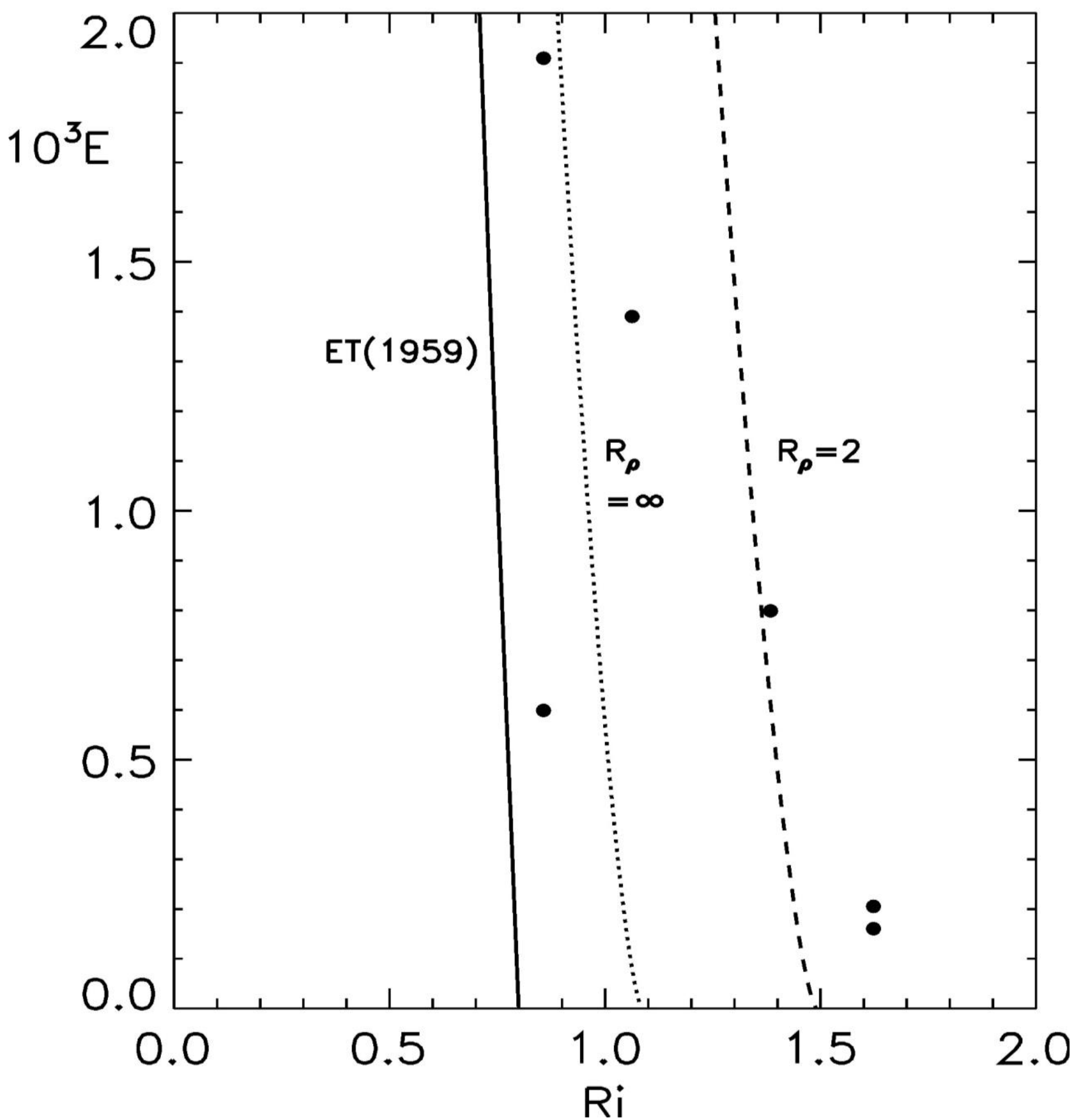
➢Comparison with the data: Rather poor

✓GISS Model: E=E(Ri, R_ρ)

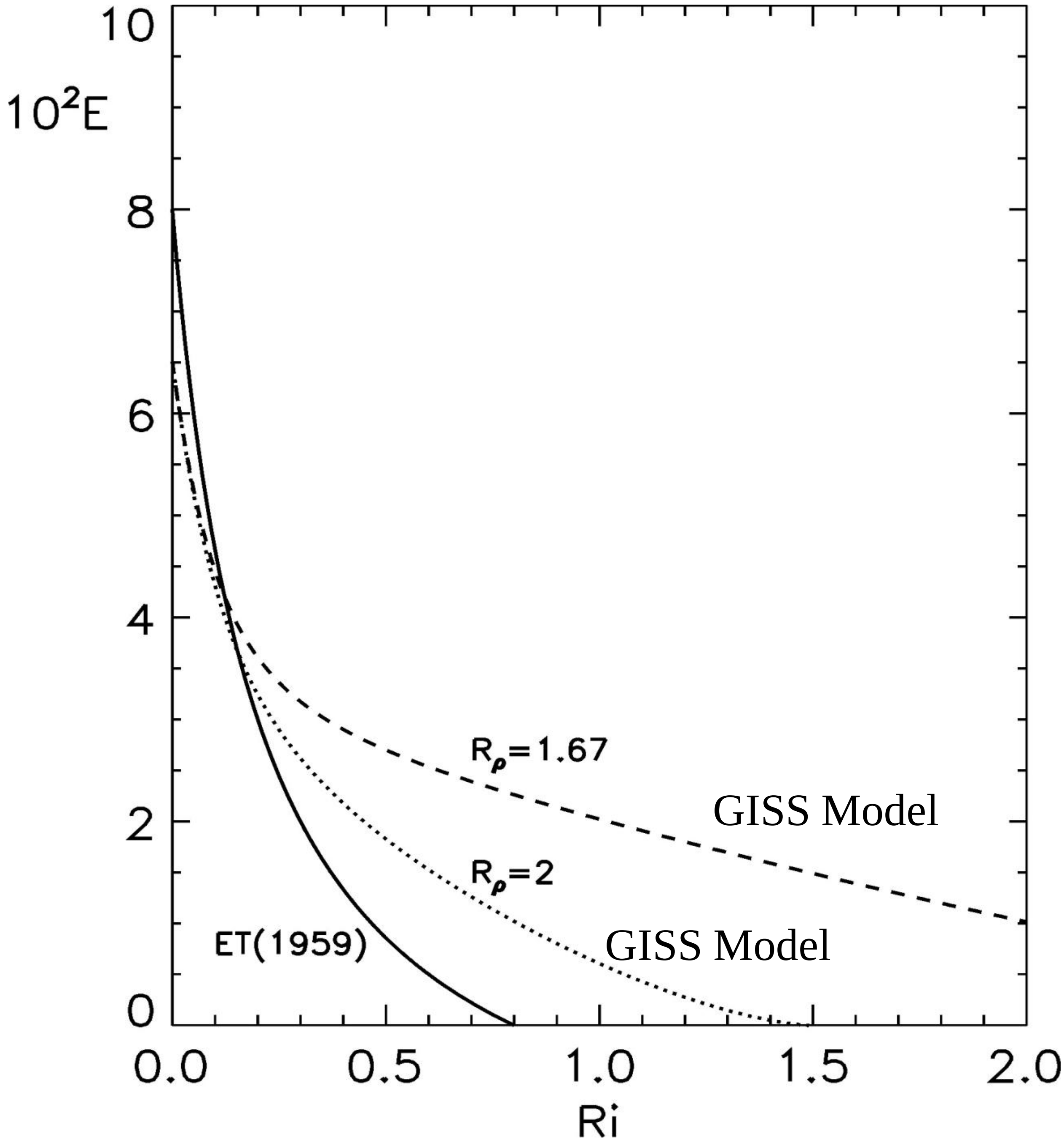
✓Better fit to the data


E=(0.08−0.1Ri)/(1+5Ri)
E
0.10
0.08
0.06
0.04
0.02
0.00
0.0
0.2
0.4
0.6
0.8
1.0
Ri


GISS turbulence model
E
0.10
0.08
0.06
0.04
0.02
0.00
0.0
0.2
0.4
0.6
0.8
1.0
Ri



2.0
1.5
1.0
0.5
0.0
10^3E
ET(1959)
R_ρ
$=\infty$
$R_\rho=2$
0.0
0.5
1.0
1.5
2.0
Ri


10
10^2E
8
6
4
2
0
$R_\rho=1.67$
GISS Model
$R_\rho=2$
ET(1959)
GISS Model
0.0
0.5
1.0
1.5
2.0
Ri

Back to the OV Problem

- How do you define the extent of the OV?
- J(z) vanishes
- w(z) vanishes, w(p) vanishes

$$\lambda w_p + (1\text{-}\lambda) w_e = 0, \quad \lambda \to 1$$

Plumes take up the whole space. *Not very likely*

- Suggestion for the OV extent:

$$\lambda \to 1/2$$

✓ Plumes and Environment become indistinguishable

$$\mathbf{OV}(\lambda=1/2) < \mathbf{OV}\ (\lambda=1)$$

- **TTM: only *Advective*; Local TM: only *Diffusive***
- **How about having both Advection and Diffusion?**
- **A Non-Local Turbulence Model? Automatically invariant**
- **Can be written in Plume language assuming: up and down flows.**

$$B = -\overbrace{\underbrace{A_1(\lambda) K_b N^2}_{\textbf{LOCAL}}}^{\textit{Diffusion}} - A_2(\lambda) \overbrace{\underbrace{\ell S_w \frac{\partial B}{\partial z}}_{\textbf{NON-LOCAL}}}^{\textit{Advection}}$$

$$K_b = \ell w_p, \quad S_w = \frac{\overline{w^3}}{\overline{w^2}^{3/2}} = (2\lambda-1)[\lambda(1-\lambda)]^{-1/2}$$

- Buoyancy flux is contributed by both **Diffusion** and **Advection**

$$\lambda \to 1/2, \quad S_w \to 0, \quad \text{Advection} \to 0$$

$$\frac{\partial T}{\partial t} + w\frac{\partial T}{\partial z} = -\frac{\partial B}{\partial z}$$

$$\frac{\partial T}{\partial t} + (w + w_*)\frac{\partial T}{\partial z} = \frac{\partial}{\partial z}(A_1 K_b N^2)$$

$$w_* = A_1 A_2 S_w \frac{\partial K_b}{\partial z}$$

Table

Stage	**λ**	**S_w**	**P vs E**	**Adv**	**Diff**
Initial	Small	<0	$w_p >> w_e$	**Yes**	No
Final	1/2	≈0	$w_p \approx w_e$	No	**Yes**

Thus :

$$TM \Rightarrow PM$$

$$PM \not\Rightarrow TM$$

BOTTOM

Data show an increase in diffusivity

Source of energy:

TIDES 3.75Tw

Example: $E=1/2\ I\Omega^2$

E(dot) =I Ω Ω (dot)

Astronomical data P(dot)/P= 1-2 millisec/year

E(dot) few TW

ε:

$$E(dot)/\text{mass ocean} = 10\text{-}2\ cm^2s^{-3}$$

$$K = \gamma\varepsilon/N^2$$

Barotropic Tides transform into Baroclinic ones which then scatter

Fragment against rough topography and generate mixing