Binning Photons

- Space (and time)
  - Position measurements

- Wavelength
  - Spectroscopy: narrow bins
  - Photometry: broad bands

- For example, SDSS
  - Spectro: \( r < 17.7 \) 1.6M sources
  - Photo: \( r < 22 + \) 360M sources
Information Content

- Incredible amount of information in the images
  - Traditionally very pragmatic approaches

**THERE IS MORE IN THE DATA!**

- Models are useful
  - Source catalogs are model-based extractions
  - Statistical and theoretical models of SEDs
Multicolor Challenges

- Cross-identification of sources
  - To assemble multicolor catalogs

- Drop-outs from sky coverage
  - To constrain fluxes not detected

- Constraining physical properties
  - To interpret the data
Photometric Redshifts

- Empirical methods
- Template fitting
Historical Overview

- Baum (1962)
  - Compared average SEDs (9 bands) of ellipticals in clusters

- Koo (1985)
  - Color-shape diagrams of and Bruzual-Charlot iso-z tracks (-U+J+F-N)

- Loh & Spillar (1986)
  - Template fitting with 6 non-standard filters and 34 known galaxies

- Connolly et al. (1995)
  - Redshift assumed to be a linear or quadratic function of magnitudes

- Steidel et al. (1996)
  - U dropout due to Ly-alpha blanketing (< 912Å) to select z > 2.25
Empirical Methods

- Regression $z = F(m)$
  - Polynomial fitting
  - Piecewise linear fits
  - Nearest neighbors
  - Neural Nets
  - Support Vector Machines

- Errors?
Template Fitting

☐ Compare reference spectra to measurements

☐ At different redshifts – MLE with Gaussian
Eigenspectra

\[ a_1 = 0.873 \]

\[ a_2 = -0.481 \]

\[ a_3 = -0.024 \]
Interpolation

- Two mixing angles encode the spectral type when using three eigenspectra

1D type parameter connects the CWW templates:

<table>
<thead>
<tr>
<th>Type</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ell</td>
<td>0</td>
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<tr>
<td>Sbc</td>
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<tr>
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<tr>
<td>Irr</td>
<td>0.56</td>
</tr>
<tr>
<td>blue</td>
<td>1</td>
</tr>
</tbody>
</table>
Consistent Redshift and Type
Traditional Methods

**Empirical Methods**
- Pure fit for $z$ over the color hyper-plane
  + fast redshift prediction
  - fitting functions and training sets needed for every survey
  - unreliable extrapolation

**Template Fitting**
- Comparison of known SEDs to photometry
  + physics is in templates, no training set required
  + more outcome: spectral type, reddening, etc.
  - templates used as come
Templates from Photometry?

![Graph showing oversampling in blueshifted passbands](attachment:graph.png)
Simple Repair

For each galaxy:

1. Compute type from known redshift
2. Derive estimate spectrum
3. Correct spectrum to photometry

Build new basis from corrected spectra
HDF/NICMOS in 1999
A Unified Framework
Photometric Redshifts

- Traditional ways
  - Empirical methods: fitting function, k-NN, neural nets
  - Template fitting: compare to model SEDs

- Very different techniques
  - Approx of low dimensional subspace
  - Single scalar estimates

- Many open questions
Photometric Inversion

- The general inversion problem
  - Constrain various properties consistently
  - Propagate uncertainties and correlations

- Assumption of functional relation is too simple
  - Probability density functions instead
  - Scientific analyses should use full PDFs
A Unified Framework

- Training and Query sets with different observables
  \[ T : \{ x_t, \xi_t \}_{t \in T} \]
  \[ Q : \{ y_q \}_{q \in Q} \]
  \[ M : \theta \]

- Model yields observables for given parameter
  - Prediction via \( p(x, y|\theta, M) \) and has prior \( p(\theta|M) \)
  - Also folds in the photometric accuracy

- We are after \( p(\xi|y_q, M) \)
Mapping Observables

- The model provides the transformation rule

\[ p(x|y_q, M) = \int d\theta \ p(x|\theta, M) \ p(\theta|y_q, M) \]

with

\[ p(\theta|y_q, M) = \frac{p(\theta|M) p(y_q|\theta, M)}{p(y_q|M)} \]

- Think empirical conversion formulas but better
  - For example, from UJFN to ugriz with errors
Empirical Relation

- Usually just assume a function $\xi = \hat{\xi}(x)$
  - Wrong! We know there are degeneracies: $p(\xi|x)$

- There is a more general relation
  - Usual restriction is $p(\xi|x) = \delta(|\xi - \hat{\xi}(x)|)$
  - A better estimation:
    $$p(\xi|x) = \frac{p(\xi, x)}{p(x)}$$
Properties of Interest

- The final distribution is

\[ p(\xi | y_q, M) = \int d\mathbf{x} \ p(\xi | \mathbf{x}) p(\mathbf{x} | y_q, M) \]

- Estimate by the mean
  - If the result is unimodal (no guarantee)
Properties of Samples

- Often we need subsamples
  - Redshift slices and/or color cuts
  - Complicated interaction with photo-z errors
  - Variable kernel deconvolution

- Natural with the PDFs
  - \( p(\xi|Q, M) = \langle p(\xi|y_q, M) \rangle_{q \in Q} \)
  - Selection boundaries in \( Q \)
Measure of reliability is the prob of $q$ making the cuts
- Using the window function

$$P(W|y_q, M) = \int dx \ P(W|x) \ p(x|y_q, M)$$

Measured relation may be biased
- Include all observables in the selection!

For instance
- Cannot use just colors, if there was a magnitude cut
- Cannot use just fluxes, if cut on morphology
Template Fitting

- Artificial training set \( \{x_t, \xi_t\} = \{\bar{x}(\theta_t), \bar{\xi}(\theta_t)\} \)
  - From a grid of model points
  - No errors \( p(x|\theta, M) = \delta(|x - \bar{x}(\theta)|) \)

- Analytic result

\[
p(\xi|y_q, M) \propto \sum_{t \in T} \delta(|\xi - \xi_t|) p(\theta_t|M) N(y_q|\bar{y}(\theta_t), C_q)
\]
Empirical Estimates

- Simplest assumption
- Generalized kernel regression
  - Average
  - Weighting
    - Local lin, RF, Wz

- Need representative training sets without $\Delta x$
Minimalist model

Normal distributions, same quantities: $\bar{x}(\theta) = \theta$ and $\bar{y}(\theta) = \theta$

With simple prior, mapping is analytic, e.g., for flat

$$p(x_t|y_q, M) = \int d\theta \ N(x_t|\theta, C_t) \ N(\theta|y_q, C_q)$$

Empirical relation

Ratio: KDE of joint and marginalized

Numerical summation

$$p(\xi_r|y_q, T, M) \propto \sum_{t \in T} p(\xi_r|x_t, T) \frac{p(x_t|y_q, M)}{p(x_t|T)}$$
Red Galaxies
Blue Galaxies
Advanced Methods

- Mapping observables via models
  - Any complete basis on wavelength range
  - Physics is in the prior!

- Relation of properties
  - Conditional densities

- Empirical but with templates
  - Unified framework at its best
Concerning Priors

- Measured densities in the Query set
  - KDE, Voronoi?

- Consistent models should match that
  - Equation

- Deconvolution yields empirical prior
  - Cf. naïve repair

\[ p(y|M) = \int d\theta p(y|\theta, M) p(\theta|M) \]

\[ p(y|M) = p(y|Q) \]
Summary

- **Bayesian approach** places former heuristics on a firm statistical basis
  - Existing photo-z methods are special cases of the more general approach
- Enables us to properly include physical priors
  - E.g., fluxes for xmatch, SEDs and positions for photoz
- Opens the door for optimal, next generation techniques
  - Missing ingredients: fast algorithms, indexing, new hardware, GPUs
- Full inversion of photometry is just ahead of us