Investigating Geometric Phase Sensitivity of an Array of Ring Interferometers as Inter-Ring Distance is Increased

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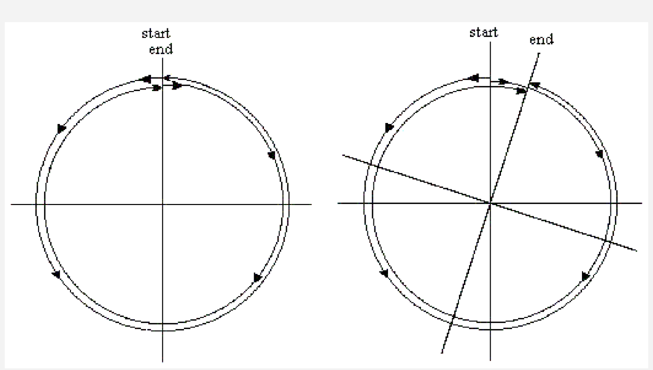
Abstract

The ability to detect inertial rotations via the Sagnac effect with an atom interferometer has been a strong stimulus for the development of atom interferometry because of the potential 10^{10} enhancement of the rotational phase shift in comparison to optical Sagnac gyroscopes of the same effective area. In our project we analyze ballistic transport of matter waves in a one dimensional chain of N coherently coupled quantum rings, our overall goal is to determine how the rotational sensitivity of the matter waves through the chain of rings changes as the distance between rings increases. We determine the rotational sensitivity of the chain by analyzing the transmission probability of atomic waves as they traverse the array. The transmission probability exhibits zero transmission stop gaps as a function of phase interspersed with regions of rapidly oscillating finite transmission. With increasing number of rings, the transition from zero transmission to the oscillatory regime becomes increasingly sharp, the slope of this transition indicates the region of the transmission that is most sensitive to phase shifts. We will parameterize the distance between rings as a product of the wavenumber and distance between ring, which will vary between 0 and 2π. We will investigate the effects of how changing this distance affects the phase sensitivity of atomic transmission through the array of rings as the number of rings increases. Although the results presented her on rotational phase shifts, it is significant to note that all of the results presented here will hold for any geometric phase shift, in other words interferometer paths separated in position space.

Introduction and Background

The Sagnac Effect

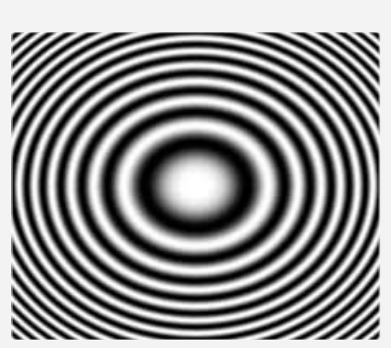


The Sagnac Effect. Digital image. *Math Pages*. N.p., n.d. Web. 28 June 2015. <mathpages.com>.

As the two waves propagates through the rotating ring in both the clockwise and counter clockwise direction one path becomes slightly longer and the other slightly shorter. This creates the path length difference required for an interference pattern to form with coherent sources

The Interference Pattern of an Interferometer

The white circles indicate constructive interference. The dark circles indicate destructive interference



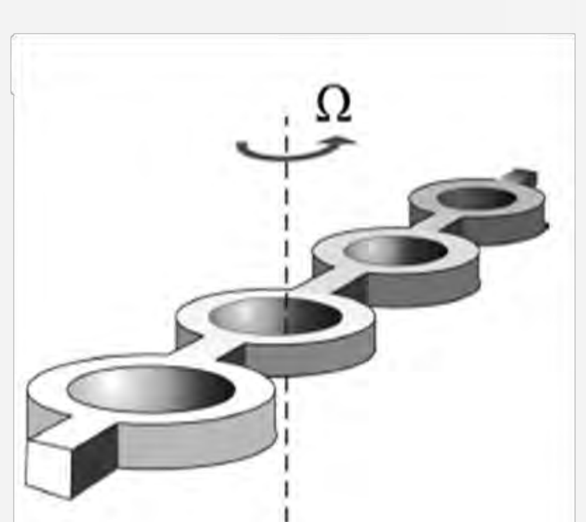
Fringe Pattern. Digital image. *Digital Library*. N.p., n.d. Web. 27 June 2015.<spiedigitallibrary.org>.

$$\theta_S = \frac{2\pi M \Omega A}{\hbar}$$

Effective Mass of the Atom — Rotation Rate — Area of the Ring — Planck's Constant — Sagnac Phase Shift

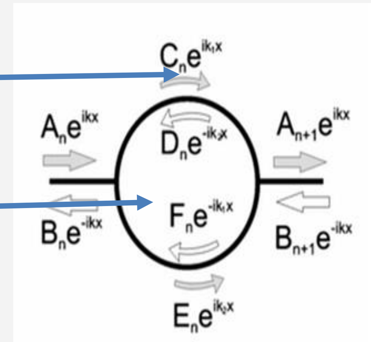
Array of Ring Interferometers

- This is the structure we will be modeling the transmission of atom waves through.
- The structure is rotating at some angular speed Ω
- We will be considering arrays of various size i.e. number of rings and consider the effects of increasing the length of the straight sections on both the transmission of atoms and the rotational sensitivity of these arrays.



Methods/Approach

Obtaining The Transmission Function



- The wave functions were found from solving the appropriate Schrodinger Equation (taking into consideration the kinetic energy and the energy associated with the magnetic vector potential in the Hamiltonian)
- Boundary conditions were set (wave continuity and current conservation) and a system of linear equations was solved to find the refection amplitude, r, and the transmission amplitude, t
- The reflection and transmission amplitudes were entered into the 2x2 transfer matrix

$$\begin{pmatrix} 1/t & r/t \\ (r/t)^* & 1/t^* \end{pmatrix}$$

- The transfer matrix of each ring in the system was multiplied to find the transfer matrix for the whole system

$$\mathcal{T}(N) = \prod_{n=1}^{N} \mathcal{T}_{n+1,n} = \mathcal{T}^N$$

- The Multiplication of N identical transfer matrices can be expressed as an analytic functions

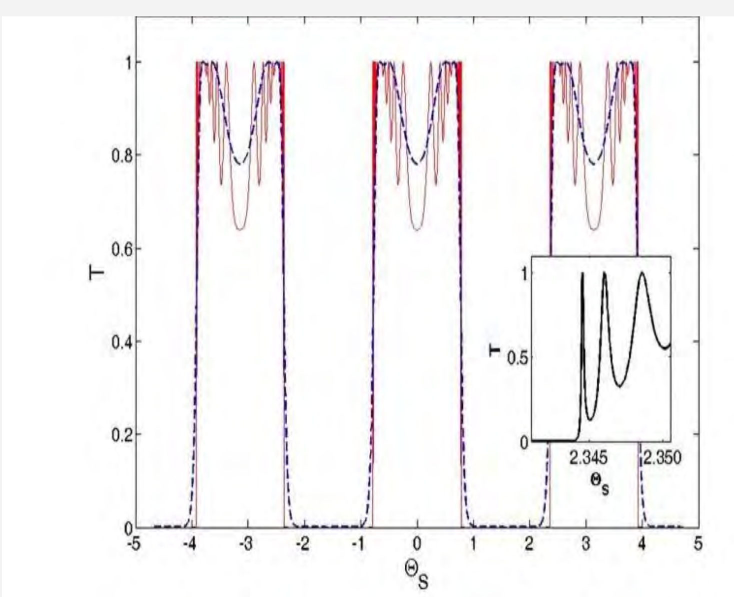
$$T(kL,\theta_s) = \frac{1}{1 + C(kL,\theta_s)\, u_{N-1}^2(\epsilon)}$$

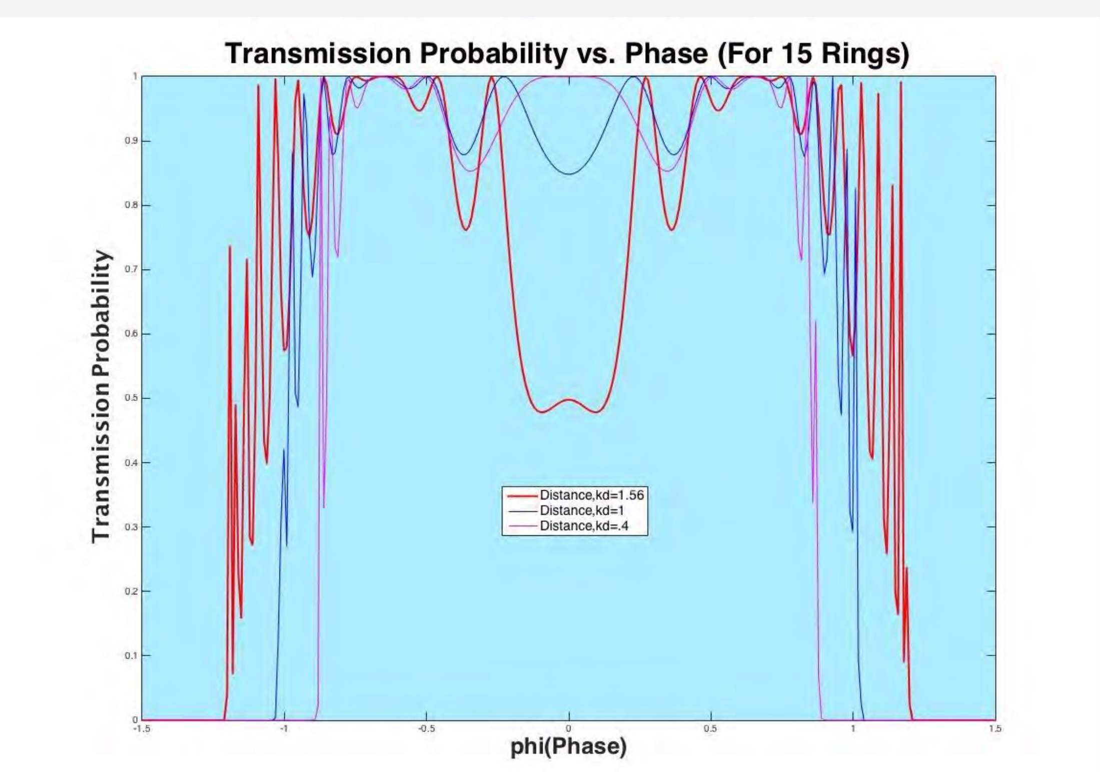
$$C(kL,\theta_s) = \frac{[4\sin^2(\theta_s) - 3\sin^2(kL)]^2}{16\sin^2(kL)\cos^2(\theta_s)}$$

$$\epsilon = \frac{\cos(kL)}{\cos\theta_s}\cos(kd) + \frac{4\sin^2(\theta_s) - 5\sin^2(kL)}{4\cos\theta_s\sin(kL)}\sin(kd)$$

Transmission vs Saganc Phase of a 50 ring (Solid Red) and a 5 ring array (Dashed Blue) with the product of the wave number and the circumference(kL) equal to $\pi/4$ and the interring distance (d) and wave number product equal to zero.

The Inset shows transmission resonance that separates the stop gap region from the oscillatory region. Previous studies into these types of systems have shown that this first resonance is the steepest and thus the most sensitive to phase shifts.

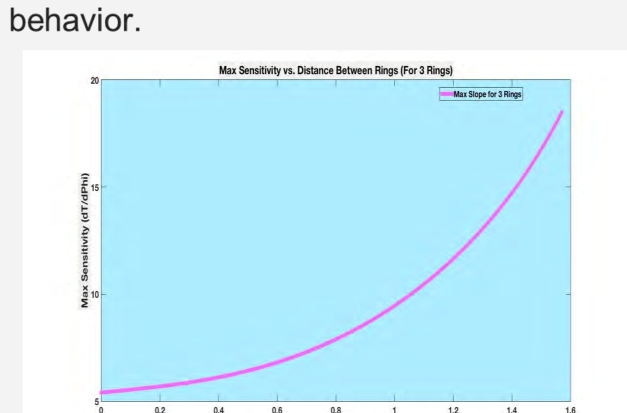




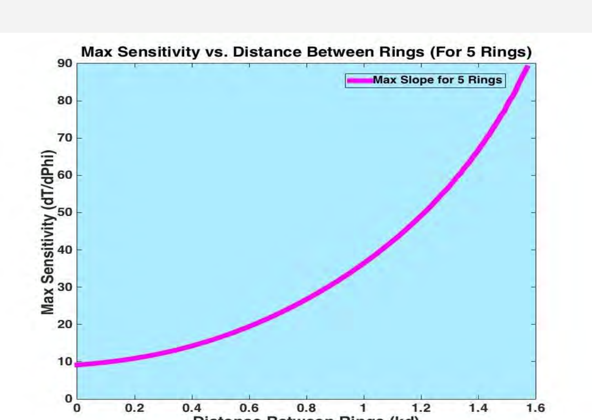
Left Picture: This plot illustrates the broadening of the transmission and the narrowing of the stop gap as the distance between rings is increased, this is parameterized by the product of the wave number and the inter-ring distance kd. The values of kd illustrated here are 1.56 (Red), 1 (Blue), and .4 (Magenta)

The number of rings in this plot is N= 15
The kl value (circumference)is kL= .25π

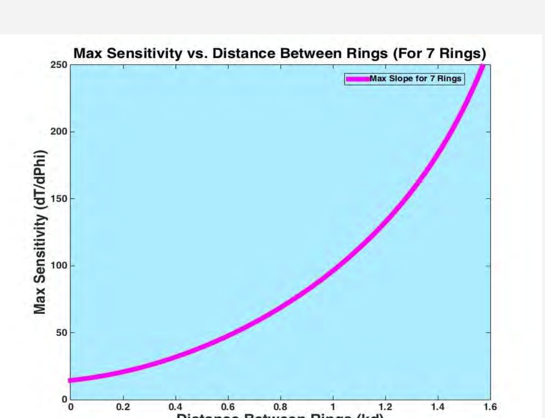
The plots below illustrate how the slope of the first transmission resonance varies as the inter-ring distance (kd) is increased from 0 to $\pi/2$ for A N=3,B N=5,C N=7 where N is the number of rings. While the behavior appears similar in all three cases, we still need to quantify this behavior.



A



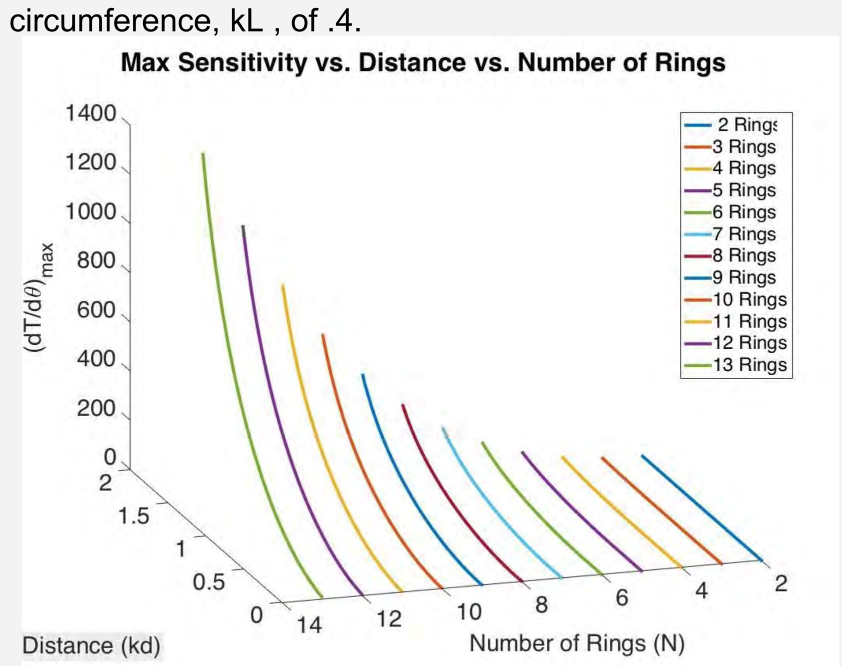
B



C

Discussion and Conclusion:

The 3D Figure below allows the maximum slope as the distance, kd ,varies from 0 to pi/2 to be directly compared for arrays ranging in size from two rings to thirteen rings with a circumference, kL , of .4.



We find that the sensitivity of the interferometer varies nonlinearly with respect to the distance between the rings. Additionally, we see that this trend remains as the number of rings is increased.

Next Steps:

Fit Max Slope Vs Kd to quantify behavior

Extend current analysis to 50 rings

Generalize model to transfer matrices so that inter-ring sizes can be individually modified and combined with non-uniform array geometries

Include effects of velocity broadening of incident atomic waves

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