we have

$$j_l(rr') \underset{rr'\to\infty}{=} \frac{1}{rr'}\frac{\mathrm{i}}{2}\left[\exp\left(-\mathrm{i}rr'+\frac{1}{2}\mathrm{i}\pi l\right)-\exp\left(\mathrm{i}rr'-\frac{1}{2}\mathrm{i}\pi l\right)\right]. \tag{A.5}$$

Substituting this expression in Eq. (A.1) and making use of the completeness relation for spherical harmonics (Jackson 1998, p. 108)

$$\sum_{l=0}^{\infty}\sum_{m=-l}^{l} Y_{lm}^*(\hat{\mathbf{r}})Y_{lm}(\hat{\mathbf{r}}') = \delta(\varphi-\varphi')\delta(\cos\vartheta-\cos\vartheta') = \delta(\hat{\mathbf{r}}-\hat{\mathbf{r}}') \tag{A.6}$$

and the symmetry relation

$$Y_{lm}(-\hat{\mathbf{r}}') = (-1)^l Y_{lm}(\hat{\mathbf{r}}'), \tag{A.7}$$

we derive, after simple algebra,

$$\exp(\mathrm{i}rr'\hat{\mathbf{r}}\cdot\hat{\mathbf{r}}') \underset{rr'\to\infty}{=} \frac{\mathrm{i}2\pi}{r'}\left[\delta(\hat{\mathbf{r}}+\hat{\mathbf{r}}')\frac{\mathrm{e}^{-\mathrm{i}rr'}}{r} - \delta(\hat{\mathbf{r}}-\hat{\mathbf{r}}')\frac{\mathrm{e}^{\mathrm{i}rr'}}{r}\right]. \tag{A.8}$$

# Appendix B

# Wigner functions, Jacobi polynomials, and generalized spherical functions

Jacobi polynomials, Wigner functions, and generalized spherical functions are closely related special functions which were introduced in classical analysis, the quantum theory of angular momentum, and the theory of representations of the rotation group, respectively (Szegő 1959; Wigner 1959; Gelfand *et al.* 1963). Because differences in notational conventions in various publications may lead to confusion, we present in this appendix a short consistent summary of the main properties of these functions.

Wigner *d*-functions are defined as

$$d^s_{mn}(\vartheta) = \sqrt{(s+m)!(s-m)!(s+n)!(s-n)!} \times \sum_k (-1)^k \frac{(\cos\frac{1}{2}\vartheta)^{2s-2k+m-n}(\sin\frac{1}{2}\vartheta)^{2k-m+n}}{k!(s+m-k)!(s-n-k)!(n-m+k)!}, \tag{B.1}$$

where $s$, $m$, and $n$ are integers, $0 \le \vartheta \le \pi$, and the sum is taken over all integer values of $k$ that lead to non-negative factorials. Thus the summation index runs from $k_{\min} = \max(0, m-n)$ to $k_{\max} = \min(s+m, s-n)$. Therefore, $d^s_{mn}(\vartheta) = 0$ unless $k_{\max} \ge k_{\min}$, which is equivalent to requiring that $s \ge 0$ and $-s \le m, n \le s$. Making the substitutions $k \to s-n-k$, $k \to s+m-k$, and $k \to m-n+k$, respectively, we derive the following alternative expressions:

$$d^s_{mn}(\vartheta) = (-1)^{s-n}\sqrt{(s+m)!(s-m)!(s+n)!(s-n)!} \times \sum_k (-1)^k \frac{(\cos\frac{1}{2}\vartheta)^{m+n+2k}(\sin\frac{1}{2}\vartheta)^{2s-m-n-2k}}{k!(s-m-k)!(s-n-k)!(m+n+k)!}, \tag{B.2}$$

$$d^s_{mn}(\vartheta) = (-1)^{s+m}\sqrt{(s+m)!(s-m)!(s+n)!(s-n)!}$$
$$\times\sum_k (-1)^k \frac{(\cos\frac{1}{2}\vartheta)^{2k-m-n}(\sin\frac{1}{2}\vartheta)^{2s+m+n-2k}}{k!(s+m-k)!(s+n-k)!(k-m-n)!}, \tag{B.3}$$

$$d^s_{mn}(\vartheta) = (-1)^{m-n}\sqrt{(s+m)!(s-m)!(s+n)!(s-n)!}$$
$$\times\sum_k (-1)^k \frac{(\cos\frac{1}{2}\vartheta)^{2s-2k-m+n}(\sin\frac{1}{2}\vartheta)^{2k+m-n}}{k!(s-m-k)!(s+n-k)!(m-n+k)!}. \tag{B.4}$$

Equations (B.1)–(B.4) imply that the $d$-functions are real and have the following symmetry properties:

$$d^s_{mn}(\vartheta) = (-1)^{m-n} d^s_{-m,-n}(\vartheta) = (-1)^{m-n} d^s_{nm}(\vartheta) = d^s_{-n,-m}(\vartheta), \tag{B.5}$$

$$d^s_{mn}(-\vartheta) = (-1)^{m-n} d^s_{mn}(\vartheta) = d^s_{nm}(\vartheta), \qquad d^s_{mn}(0) = \delta_{mn}, \tag{B.6}$$

$$d^s_{mn}(\pi-\vartheta) = (-1)^{s-n} d^s_{-mn}(\vartheta) = (-1)^{s+m} d^s_{m,-n}(\vartheta), \qquad d^s_{mn}(\pi) = (-1)^{s-n}\delta_{-mn}, \tag{B.7}$$

where $\delta_{mn}$ is the Kronecker delta:

$$\delta_{mn} = \begin{cases} 1 \text{ if } m = n, \\ 0 \text{ if } m \neq n. \end{cases}$$

Denoting $x = \cos\vartheta$ and requiring that $\vartheta \in [0, \pi]$ yields

$$\cos\tfrac{1}{2}\vartheta = \sqrt{\tfrac{1}{2}(1+x)}, \qquad \sin\tfrac{1}{2}\vartheta = \sqrt{\tfrac{1}{2}(1-x)}. \tag{B.8}$$

Substituting Eq. (B.8) in Eq. (B.1) and modifying the resulting formula, we obtain

$$d^s_{mn}(\vartheta) = A^s_{mn}(1-x)^{(m-n)/2}(1+x)^{-(m+n)/2}\sum_k (-1)^{s-n-k}$$
$$\times\frac{(s+m)!(s-m)!(s-n)!}{k!(s+m-k)!(s-n-k)!(k-m+n)!}(1+x)^{s+m-k}(1-x)^{k-m+n}, \tag{B.9}$$

where

$$A^s_{mn} = \frac{(-1)^{s-n}}{2^s}\left[\frac{(s+n)!}{(s+m)!(s-m)!(s-n)!}\right]^{1/2}. \tag{B.10}$$

Finally, recalling the Leibniz rule,

$$\left(\frac{\mathrm{d}}{\mathrm{d}x}\right)^N [f(x)g(x)] = \sum_{k=0}^{N}\frac{N!}{k!(N-k)!}\left[\left(\frac{\mathrm{d}}{\mathrm{d}x}\right)^k f(x)\right]\left[\left(\frac{\mathrm{d}}{\mathrm{d}x}\right)^{N-k} g(x)\right], \tag{B.11}$$

and applying it to $f(x) = (1+x)^{s+m}$, $g(x) = (1-x)^{s-m}$, and $N = s-n$, we can rewrite Eq. (B.9) in the form

$$d^s_{mn}(\vartheta) = A^s_{mn}(1-x)^{(m-n)/2}(1+x)^{-(m+n)/2}\left(\frac{\mathrm{d}}{\mathrm{d}x}\right)^{s-n}[(1+x)^{s+m}(1-x)^{s-m}]. \tag{B.12}$$

The Jacobi polynomial of degree $q$ is given by Eq. (4.3.1) of Szegő (1959):

$$P_q^{(a,b)}(x) = (1-x)^{-a}(1+x)^{-b}\frac{(-1)^q}{2^q q!}\left(\frac{\mathrm{d}}{\mathrm{d}x}\right)^q [(1+x)^{q+b}(1-x)^{q+a}], \tag{B.13}$$

where $q$ is a non-negative integer and $a > -1$ and $b > -1$ are real. Comparing Eq. (B.12) with Eq. (B.13), we obtain the following expression of the Wigner $d$-functions in terms of the Jacobi polynomials for $n \geq |m|$:

$$d_{mn}^s(\vartheta) = \frac{\xi_{mn}}{2^{(a+b)/2}}\left[\frac{q!(q+a+b)!}{(q+a)!(q+b)!}\right]^{1/2}(1-x)^{a/2}(1+x)^{b/2}P_q^{(a,b)}(x), \tag{B.14}$$

where $a = n-m$, $b = n+m$, $q = s-n$, and $\xi_{mn} = 1$. The condition $n \geq |m|$ ensures that $a \geq 0$ and $b \geq 0$, thereby preventing singularities for $x = \pm 1$. Using the symmetry relations of Eq. (B.5), it is straightforward to show that Eq. (B.14) can be used for arbitrary $m$ and $n$, provided that

$$a = |m-n|, \qquad b = |m+n|, \qquad q = s - \tfrac{1}{2}(a+b), \tag{B.15}$$

$$\xi_{mn} = \begin{cases} 1 & \text{for } n \geq m, \\ (-1)^{m-n} & \text{for } n < m. \end{cases} \tag{B.16}$$

The orthogonality property of the Jacobi polynomials (Eq. (4.3.3) of Szegő 1959) and Eqs. (B.14)–(B.16) lead to the following orthogonality property of the $d$-functions:

$$\int_0^\pi \mathrm{d}\vartheta \sin\vartheta\, d_{mn}^s(\vartheta) d_{mn}^{s'}(\vartheta) = \frac{2}{2s+1}\delta_{ss'}. \tag{B.17}$$

The completeness property of the Jacobi polynomials (Szegő 1959) and Eqs. (B.14) and (B.17) imply that functions $\sqrt{s+\frac{1}{2}}\, d_{mn}^s(\vartheta)$ with $s = s_{\min}, s_{\min}+1, \ldots$ form a complete orthonormal system of functions on $[0, \pi]$, where

$$s_{\min} = \max(|m|, |n|).$$

This means that if a real-valued function $f(\vartheta)$ defined on the closed interval $[0, \pi]$ is square integrable on this interval, i.e., if

$$\int_0^\pi \mathrm{d}\vartheta \sin\vartheta [f(\vartheta)]^2 < \infty, \tag{B.18}$$

then there exists a unique set of coefficients $\eta_s$ $(s \geq s_{\min})$ such that the series expansion

$$f(\vartheta) = \sum_{s=s_{\min}}^{\infty} \eta_s d_{mn}^s(\vartheta), \qquad \vartheta \in [0, \pi] \tag{B.19}$$

holds in the following sense:

$$\lim_{S\to\infty} \int_0^\pi \mathrm{d}\vartheta \sin\vartheta \left| f(\vartheta) - \sum_{s=s_{\min}}^{S} \eta_s d_{mn}^s(\vartheta)\right|^2 = 0. \tag{B.20}$$

Conversely, if a real-valued function $f(\vartheta)$ on $[0,\pi]$ admits the expansion of Eq. (B.19) in the sense of Eq. (B.20), then it is square integrable on $[0,\pi]$ and the expansion coefficients are given by

$$\eta_s = (s+\tfrac{1}{2})\int_0^\pi \mathrm{d}\vartheta \sin\vartheta\, f(\vartheta) d^s_{mn}(\vartheta). \tag{B.21}$$

The latter formula follows directly from Eqs. (B.19) and (B.17).

Using Eq. (4.5.1) of Szegő (1959) and Eq. (B.14), we obtain the following recurrence relation for the Wigner *d*-functions:

$$\begin{aligned} d^{s+1}_{mn}(\vartheta) = {} & \frac{1}{s\sqrt{(s+1)^2-m^2}\sqrt{(s+1)^2-n^2}}\{(2s+1)[s(s+1)x-mn]d^s_{mn}(\vartheta) \\ & -(s+1)\sqrt{s^2-m^2}\sqrt{s^2-n^2}\, d^{s-1}_{mn}(\vartheta)\}, \qquad s \ge s_{\min}. \end{aligned} \tag{B.22}$$

The simplest way to derive this formula is to consider first the case $n \ge |m|$, which corresponds to $a = n-m$, $b = n+m$, $q = s-n$, and then to use the symmetry relations of Eq. (B.5) in order to verify that Eq. (B.22) is correct for arbitrary *m* and *n*. The initial values are given by

$$d^{s_{\min}-1}_{mn}(\vartheta) = 0, \tag{B.23}$$

$$d^{s_{\min}}_{mn}(\vartheta) = \xi_{mn} 2^{-s_{\min}} \left[\frac{(2s_{\min})!}{(|m-n|)!(|m+n|)!}\right]^{1/2} (1-x)^{|m-n|/2}(1+x)^{|m+n|/2}, \tag{B.24}$$

where $\xi_{mn}$ is given by Eq. (B.16). Equation (B.24) follows directly from Eq. (B.12) if $n \ge |m|$, and it is extended to arbitrary *m* and *n* using Eq. (B.5). From Eq. (B.12), we easily derive

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}\vartheta} d^s_{mn}(\vartheta) &= \frac{m-n\cos\vartheta}{\sin\vartheta} d^s_{mn}(\vartheta) + \sqrt{(s+n)(s-n+1)}\, d^s_{mn-1}(\vartheta) \\ &= -\frac{m-n\cos\vartheta}{\sin\vartheta} d^s_{mn}(\vartheta) - \sqrt{(s-n)(s+n+1)}\, d^s_{mn+1}(\vartheta). \end{aligned} \tag{B.25}$$

Alternatively, we have from Eq. (4.5.5) of Szegő (1959) and Eq. (B.22)

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}\vartheta} d^s_{mn}(\vartheta) = \frac{1}{\sin\vartheta}\Bigg[ & -\frac{(s+1)\sqrt{(s^2-m^2)(s^2-n^2)}}{s(2s+1)} d^{s-1}_{mn}(\vartheta) \\ & -\frac{mn}{s(s+1)} d^s_{mn}(\vartheta) + \frac{s\sqrt{(s+1)^2-m^2}\sqrt{(s+1)^2-n^2}}{(s+1)(2s+1)} d^{s+1}_{mn}(\vartheta)\Bigg]. \end{aligned} \tag{B.26}$$

The Wigner *d*-functions with $m=0$ and $n=0$ are equivalent to the usual Legendre polynomials (cf. Eqs. (B.12) and (A.3)):

$$d^s_{00}(\vartheta) = P_s(x). \tag{B.27}$$

For $n = 0$, we obtain

$$d^s_{m0}(\vartheta) = \sqrt{\frac{(s-m)!}{(s+m)!}}\, P^m_s(x), \tag{B.28}$$

where $P^m_s(x)$ are associated Legendre functions defined by Eq. (A.3). Equations (B.22) and (B.28) give a simple recurrence relation for the associated Legendre functions:

$$(s-m+1)P^m_{s+1}(x) = (2s+1)xP^m_s(x) - (s+m)P^m_{s-1}(x). \tag{B.29}$$

Despite its simplicity, the use of this relation in computer calculations for large $s$ and $|m|$ results in overflows, whereas the original recurrence relation for the functions $d^s_{m0}(\vartheta)$ remains stable and accurate. Furthermore, the functions $d^s_{m0}(\vartheta)$ have simpler symmetry properties than the $P^m_s(x)$. It is, therefore, advisable to use the $d$-functions instead of the associated Legendre functions from both the analytical and the numerical standpoint.

The generalized spherical functions $P^s_{mn}(x)$ are complex-valued functions related to the Wigner $d$-functions by (Gelfand *et al.* 1963; Hovenier and van der Mee 1983)

$$P^s_{mn}(x) = \mathrm{i}^{m-n} d^s_{mn}(\vartheta). \tag{B.30}$$

Using Eqs. (B.5)–(B.7), we easily derive

$$P^s_{mn}(x) = P^s_{nm}(x) = P^s_{-m,-n}(x) = (-1)^{m+n}[P^s_{mn}(x)]^*, \tag{B.31}$$

$$P^s_{mn}(-x) = (-1)^{s+m-n} P^s_{-mn}(x). \tag{B.32}$$

The corresponding orthogonality and normalization condition follows directly from Eq. (B.17):

$$\begin{aligned}\int_{-1}^{+1} \mathrm{d}x\, P^s_{mn}(x)P^{s'}_{mn}(x) &= (-1)^{m+n} \int_{-1}^{+1} \mathrm{d}x\, P^s_{mn}(x)[P^{s'}_{mn}(x)]^* \\ &= (-1)^{m+n} \frac{2}{2s+1}\delta_{ss'}.\end{aligned} \tag{B.33}$$

It is straightforward to show that the generalized spherical functions form a complete set of complex functions on the interval $x \in [-1, +1]$. This means that any complex-valued function $f(x)$, defined and square-integrable on the interval $x \in [-1, +1]$, can be uniquely expanded in the functions $P^s_{mn}(x)$ with $s = s_{\min}, s_{\min}+1, \ldots$. In other words, if

$$\int_{-1}^{+1} \mathrm{d}x\, |f(x)|^2 < \infty \tag{B.34}$$

then there exists a unique set of coefficients $\eta_s$ $(s \geq s_{\min})$ such that

$$\lim_{S\to\infty} \int_{-1}^{+1} \mathrm{d}x \left| f(x) - \sum_{s=s_{\min}}^{S} \eta_s P^s_{mn}(x) \right|^2 = 0. \tag{B.35}$$

Conversely, if a complex-valued function $f(x)$ on $[-1,+1]$ admits the expansion

$$f(x)=\sum_{s=s_{\min}}^{\infty}\eta_s P_{mn}^s(x) \tag{B.36}$$

in the sense of Eq. (B.35), then it is square integrable on $[-1,+1]$ and the expansion coefficients are given by

$$\eta_s=(-1)^{m+n}(s+\tfrac{1}{2})\int_{-1}^{+1}\mathrm{d}x\, f(x)P_{mn}^s(x) \tag{B.37}$$

(cf. Eqs. (B.33) and (B.36)).

The Wigner *D*-functions are defined as

$$D_{mm'}^n(\alpha,\beta,\gamma)=\mathrm{e}^{-\mathrm{i}m\alpha}d_{mm'}^n(\beta)\mathrm{e}^{-\mathrm{i}m'\gamma}, \tag{B.38}$$

where

$$0\le\alpha<2\pi,\quad 0\le\beta\le\pi,\quad 0\le\gamma<2\pi. \tag{B.39}$$

If the sets of Euler angles $(\alpha_1,\beta_1,\gamma_1)$ and $(\alpha_2,\beta_2,\gamma_2)$ (Section 2.4) describe two consecutive rotations of a coordinate system and the set $(\alpha,\beta,\gamma)$ describes the resulting rotation, then the addition theorem for the *D*-functions reads

$$D_{mm'}^n(\alpha,\beta,\gamma)=\sum_{m''=-n}^{n}D_{mm''}^n(\alpha_1,\beta_1,\gamma_1)D_{m''m'}^n(\alpha_2,\beta_2,\gamma_2) \tag{B.40}$$

(see Eq. (2) in Section 4.7 of Varshalovich *et al.* 1988). A direct consequence of the addition theorem is the unitarity condition

$$\sum_{m''=-n}^{n}D_{mm''}^n(\alpha,\beta,\gamma)D_{m''m'}^n(-\gamma,-\beta,-\alpha)=D_{mm'}^n(0,0,0)=\delta_{mm'} \tag{B.41}$$

(cf. Eq. (B.6)).

Using Eq. (B.40), we can derive the addition theorem for the Wigner *d*-functions. Consider the geometry shown in Fig. B.1, where the angles $\vartheta_1$, $\vartheta_2$, $\varphi_2-\varphi_1$, $\sigma_1$, $\sigma_2$, and $\Theta$ are non-negative and are related by

$$\cos\Theta=\cos\vartheta_2\cos\vartheta_1+\sin\vartheta_2\sin\vartheta_1\cos(\varphi_2-\varphi_1), \tag{B.42}$$

$$\cos\sigma_1=\frac{\cos\vartheta_2-\cos\vartheta_1\cos\Theta}{\sin\vartheta_1\sin\Theta}, \tag{B.43}$$

$$\cos\sigma_2=\frac{\cos\vartheta_1-\cos\vartheta_2\cos\Theta}{\sin\vartheta_2\sin\Theta} \tag{B.44}$$

(cf. Eqs. (4.17)–(4.19)). The reference frame formed by the unit vectors $(\hat{\mathbf{n}}_1,\hat{\boldsymbol{\vartheta}}_1,\hat{\boldsymbol{\varphi}}_1)$ can be transformed into the reference frame formed by the unit vectors $(\hat{\mathbf{n}}_2,\hat{\boldsymbol{\vartheta}}_2,\hat{\boldsymbol{\varphi}}_2)$ in two ways: (i) via a single rotation through Euler angles $(\pi-\sigma_1,\Theta,-\sigma_2)$, and (ii)

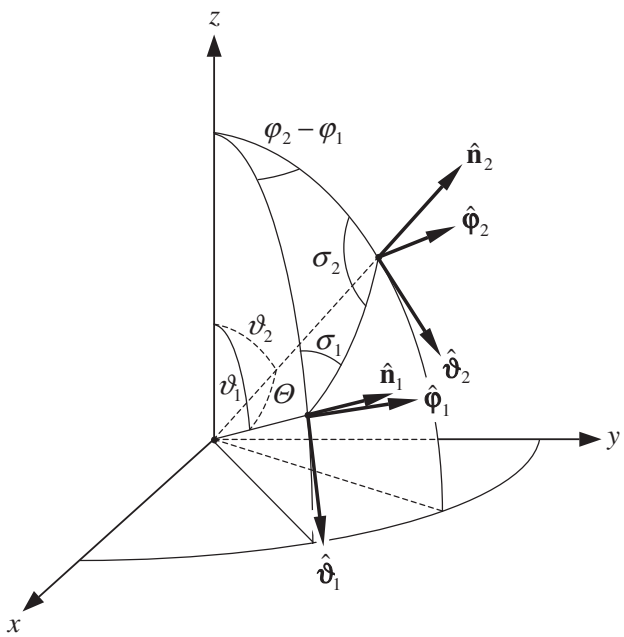


**Figure B.1.** Illustration of the addition theorem for Wigner *d*-functions.

via two consecutive rotations through Euler angles $(0, -\vartheta_1, \varphi_2 - \varphi_1)$ and $(0, \vartheta_2, 0)$. We, therefore, derive from Eqs. (B.38) and (B.40)

$$\mathrm{e}^{\mathrm{i}m\sigma_1} d^n_{mm'}(\Theta)\mathrm{e}^{\mathrm{i}m'\sigma_2} = \sum_{m''=-n}^{n} (-1)^{m''} d^n_{mm''}(\vartheta_1) d^n_{m''m'}(\vartheta_2)\mathrm{e}^{-\mathrm{i}m''(\varphi_2-\varphi_1)}. \tag{B.45}$$

Consider two special cases of Eq. (B.45). If $\varphi_2 - \varphi_1 = 0$ and $\vartheta_1 \geq \vartheta_2$ then $\sigma_1 = 0$, $\sigma_2 = \pi$, and $\Theta = \vartheta_1 - \vartheta_2$, and we obtain

$$d^n_{mm'}(\vartheta_1 - \vartheta_2) = \sum_{m''=-n}^{n} (-1)^{m''-m'} d^n_{mm''}(\vartheta_1) d^n_{m''m'}(\vartheta_2). \tag{B.46}$$

In particular, when $\vartheta_1 = \vartheta_2$, Eqs. (B.6) and (B.46) render the unitarity condition for the *d*-functions:

$$\begin{aligned}\sum_{m''=-n}^{n} (-1)^{m''-m'} d^n_{mm''}(\vartheta) d^n_{m''m'}(\vartheta) &= \sum_{m''=-n}^{n} d^n_{mm''}(\vartheta) d^n_{m''m'}(-\vartheta) \\ &= \sum_{m''=-n}^{n} d^n_{mm''}(\vartheta) d^n_{m'm''}(\vartheta) \\ &= \delta_{mm'}.\end{aligned} \tag{B.47}$$

This formula can also be derived directly from the unitarity condition for the *D*-functions, Eq. (B.41), by substituting $\alpha = \gamma = 0$. If $\varphi_2 - \varphi_1 = \pi$, then $\sigma_1 = \sigma_2 = 0$

and $\Theta = \vartheta_1 + \vartheta_2$, and we have

$$d^n_{mm'}(\vartheta_1 + \vartheta_2) = \sum_{m''=-n}^{n} d^n_{mm''}(\vartheta_1) d^n_{m''m'}(\vartheta_2). \tag{B.48}$$

The product of two $D$-functions can be expanded in the so-called Clebsch–Gordan series (Eq. (1) of Section 4.6 of Varshalovich *et al.* 1988):

$$D^{n_1}_{m_1m_1'}(\alpha,\beta,\gamma) D^{n_2}_{m_2m_2'}(\alpha,\beta,\gamma) = \sum_{n=|n_1-n_2|}^{n_1+n_2} C^{n,m_1+m_2}_{n_1m_1\ n_2m_2} D^n_{m_1+m_2,m_1'+m_2'}(\alpha,\beta,\gamma)\, C^{n,m_1'+m_2'}_{n_1m_1'\ n_2m_2'}, \tag{B.49}$$

where the $C^{nm}_{n_1m_1\ n_2m_2}$ are Clebsch–Gordan coefficients (Appendix D). Substituting $\alpha = \gamma = 0$ yields

$$d^{n_1}_{m_1m_1'}(\beta) d^{n_2}_{m_2m_2'}(\beta) = \sum_{n=|n_1-n_2|}^{n_1+n_2} C^{n,m_1+m_2}_{n_1m_1\ n_2m_2} d^n_{m_1+m_2,m_1'+m_2'}(\beta) C^{n,m_1'+m_2'}_{n_1m_1'\ n_2m_2'}. \tag{B.50}$$

## Further reading

Detailed accounts of Jacobi polynomials, Wigner $d$-functions, and generalized spherical functions are given in Szegő (1959), Gelfand *et al.* (1963), Vilenkin (1968), Biedenharn and Louck (1981), Varshalovich *et al.* (1988), Brink and Satchler (1993), Rose (1995), and Edmonds (1996). Our definition of the $d$-functions is consistent with that of Biedenharn and Louck (1981), Hovenier and van der Mee (1983), Varshalovich *et al.* (1988), Brink and Satchler (1993), and Rose (1995). Vilenkin (1968) uses functions $P^s_{mn}(x)$ related to the Wigner $d$-functions by $P^s_{mn}(x) = \mathrm{i}^{n-m} d^s_{mn}(\vartheta)$. Edmonds (1996) uses a function $d^{(s)}_{mn}(\vartheta)$, which is related to $d^s_{mn}(\vartheta)$ by $d^{(s)}_{mn}(\vartheta) = (-1)^{m+n} d^s_{mn}(\vartheta)$.

# Appendix C

# Scalar and vector spherical wave functions

Real-valued spherical Bessel functions of the first kind,

$$j_n(x) = x^n \left( -\frac{1}{x}\frac{\mathrm{d}}{\mathrm{d}x} \right)^n \left( \frac{\sin x}{x} \right),$$

and of the second kind,

$$y_n(x) = -x^n \left( -\frac{1}{x}\frac{\mathrm{d}}{\mathrm{d}x} \right)^n \left( \frac{\cos x}{x} \right),$$

as well as their complex-valued combinations such as Hankel functions of the first kind,

$$h_n^{(1)}(x) = j_n(x) + \mathrm{i}y_n(x), \tag{C.1}$$

and of the second kind,

$$h_n^{(2)}(x) = j_n(x) - \mathrm{i}y_n(x), \tag{C.2}$$

are solutions of the same differential equation,

$$\frac{\mathrm{d}}{\mathrm{d}x}\left( x^2 \frac{\mathrm{d}z_n(x)}{\mathrm{d}x} \right) + [x^2 - n(n+1)]z_n(x) = 0, \tag{C.3}$$

where $n$ is an integer (e.g., Abramowitz and Stegun 1964). Taking into account that

$$\frac{1}{\sin\vartheta}\frac{\mathrm{d}}{\mathrm{d}\vartheta}\left( \sin\vartheta \frac{\mathrm{d}P_n^m(\cos\vartheta)}{\mathrm{d}\vartheta} \right) + \left( n(n+1) - \frac{m^2}{\sin^2\vartheta} \right) P_n^m(\cos\vartheta) = 0, \tag{C.4}$$

we find that

$$\psi_{mn}(kr, \vartheta, \varphi) = h_n^{(1)}(kr)\, P_n^m(\cos\vartheta)\, \mathrm{e}^{\mathrm{i}m\varphi}, \qquad n = 0, 1, 2, \ldots, \qquad m = 0, \pm 1, \pm 2, \ldots, \pm n \tag{C.5}$$

is a solution of the scalar Helmholtz equation

$$(\nabla^2 + k^2)\psi_{mn}(kr, \vartheta, \varphi) = 0, \tag{C.6}$$

where, in spherical coordinates,

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\right) + \frac{1}{r^2 \sin\vartheta}\frac{\partial}{\partial \vartheta}\left(\sin\vartheta \frac{\partial}{\partial \vartheta}\right) + \frac{1}{r^2 \sin^2\vartheta}\left(\frac{\partial}{\partial \varphi}\right)^2. \tag{C.7}$$

Since

$$h_n^{(1)}(kr) \underset{\substack{kr\to\infty \\ kr \gg n^2}}{=} \frac{(-\mathrm{i})^{n+1}\mathrm{e}^{\mathrm{i}kr}}{kr} \tag{C.8}$$

and

$$h_n^{(1)}(kr) \underset{kr\to 0}{=} \frac{(kr)^n}{(2n+1)!!} - \mathrm{i}(2n-1)!!(kr)^{-n-1},$$

the functions $\psi_{mn}(kr, \vartheta, \varphi)$ behave as *outgoing scalar spherical waves* at infinity and diverge at the origin. The functions

$$\mathrm{Rg}\,\psi_{mn}(kr, \vartheta, \varphi) = j_n(kr)P_n^m(\cos\vartheta)\mathrm{e}^{\mathrm{i}m\varphi}, \quad n = 0, 1, 2, ..., \quad m = 0, \pm 1, \pm 2, ..., \pm n \tag{C.9}$$

also satisfy the Helmholtz equation but, unlike $\psi_{mn}(kr, \vartheta, \varphi)$, are *regular* (finite) at the origin, owing to

$$j_n(kr) \underset{kr\to 0}{=} \frac{(kr)^n}{(2n+1)!!}.$$

Since numerical computation of the associated Legendre functions having large $n$ and $|m|$ is problematic (see Appendix B), it is useful to rewrite Eqs. (C.5) and (C.9) in terms of Wigner $d$-functions:

$$\begin{matrix} \psi_{mn}(kr, \vartheta, \varphi) \\ \mathrm{Rg}\,\psi_{mn}(kr, \vartheta, \varphi) \end{matrix} = \begin{matrix} h_n^{(1)}(kr) \\ j_n(kr) \end{matrix} (-1)^m \sqrt{\frac{(n+m)!}{(n-m)!}}\, d_{0m}^n(\vartheta)\mathrm{e}^{\mathrm{i}m\varphi} \tag{C.10}$$

(cf. Eqs. (B.28) and (B.5)), where we have used a compact way to write two formulas (for $\psi_{mn}$ and $\mathrm{Rg}\,\psi_{mn}$) as a single equation.

The functions $\psi_{mn}(kr, \vartheta, \varphi)$ and $\mathrm{Rg}\,\psi_{mn}(kr, \vartheta, \varphi)$ are called scalar spherical wave functions and form a complete set of expansion functions that can be used to represent any time-harmonic scalar wave. Furthermore, they can be used to create vector spherical wave functions suitable for expanding time-harmonic vector fields. According to Eq. (1.17), the electric field in a linear, isotropic, homogeneous medium is divergence-free:

$$\nabla \cdot \mathbf{E}(\mathbf{r}) = 0. \tag{C.11}$$

Therefore, it follows from Eqs. (2.3) and (2.4) and the vector identity

$$\nabla\times\nabla\times\mathbf{a}(\mathbf{r})=\nabla[\nabla\cdot\mathbf{a}(\mathbf{r})]-\nabla^2\mathbf{a}(\mathbf{r}) \tag{C.12}$$

that $\mathbf{E}(\mathbf{r})$ must satisfy the vector Helmholtz equation

$$\nabla^2\mathbf{E}(\mathbf{r})+k^2\,\mathbf{E}(\mathbf{r})=0. \tag{C.13}$$

Obviously, vector functions used to expand $\mathbf{E}(\mathbf{r})$ must also be divergence-free and must satisfy the vector Helmholtz equation. It can be shown (Stratton 1941; Morse and Feshbach 1953) that two sets of vector functions that meet these criteria are as follows:

$$\begin{aligned}\begin{matrix}\mathbf{M}_{mn}(kr,\vartheta,\varphi)\\ \mathrm{Rg}\mathbf{M}_{mn}(kr,\vartheta,\varphi)\end{matrix}&=\gamma_{mn}\nabla\times\left(\mathbf{r}\begin{matrix}\psi_{mn}(kr,\vartheta,\varphi)\\ \mathrm{Rg}\,\psi_{mn}(kr,\vartheta,\varphi)\end{matrix}\right)\\&=\gamma_{mn}\begin{matrix}h_n^{(1)}(kr)\\ j_n(kr)\end{matrix}\mathbf{C}_{mn}(\vartheta,\varphi)\\&=\frac{1}{k}\nabla\times\begin{matrix}\mathbf{N}_{mn}(kr,\vartheta,\varphi)\\ \mathrm{Rg}\mathbf{N}_{mn}(kr,\vartheta,\varphi)\end{matrix}\end{aligned} \tag{C.14}$$

and

$$\begin{aligned}\begin{matrix}\mathbf{N}_{mn}(kr,\vartheta,\varphi)\\ \mathrm{Rg}\mathbf{N}_{mn}(kr,\vartheta,\varphi)\end{matrix}&=\frac{1}{k}\nabla\times\begin{matrix}\mathbf{M}_{mn}(kr,\vartheta,\varphi)\\ \mathrm{Rg}\mathbf{M}_{mn}(kr,\vartheta,\varphi)\end{matrix}\\&=\gamma_{mn}\left\{\frac{n(n+1)}{kr}\begin{matrix}h_n^{(1)}(kr)\\ j_n(kr)\end{matrix}\mathbf{P}_{mn}(\vartheta,\varphi)+\frac{1}{kr}\frac{\mathrm{d}}{\mathrm{d}(kr)}\left(kr\begin{matrix}h_n^{(1)}(kr)\\ j_n(kr)\end{matrix}\right)\mathbf{B}_{mn}(\vartheta,\varphi)\right\},\end{aligned} \tag{C.15}$$

where

$$\begin{aligned}\mathbf{B}_{mn}(\vartheta,\varphi)&=r\nabla[P_n^m(\cos\vartheta)\mathrm{e}^{\mathrm{i}m\varphi}]\\&=\left[\hat{\boldsymbol{\vartheta}}\frac{\mathrm{d}}{\mathrm{d}\vartheta}P_n^m(\cos\vartheta)+\hat{\boldsymbol{\varphi}}\frac{\mathrm{i}m}{\sin\vartheta}P_n^m(\cos\vartheta)\right]\mathrm{e}^{\mathrm{i}m\varphi}\\&=(-1)^m\sqrt{\frac{(n+m)!}{(n-m)!}}\,\mathbf{B}_{mn}(\vartheta)\mathrm{e}^{\mathrm{i}m\varphi}\\&=\hat{\mathbf{r}}\times\mathbf{C}_{mn}(\vartheta,\varphi),\end{aligned} \tag{C.16}$$

$$\begin{aligned}\mathbf{C}_{mn}(\vartheta,\varphi)&=\nabla\times[\mathbf{r}P_n^m(\cos\vartheta)\mathrm{e}^{\mathrm{i}m\varphi}]\\&=\left[\hat{\boldsymbol{\vartheta}}\frac{\mathrm{i}m}{\sin\vartheta}P_n^m(\cos\vartheta)-\hat{\boldsymbol{\varphi}}\frac{\mathrm{d}}{\mathrm{d}\vartheta}P_n^m(\cos\vartheta)\right]\mathrm{e}^{\mathrm{i}m\varphi}\\&=(-1)^m\sqrt{\frac{(n+m)!}{(n-m)!}}\,\mathbf{C}_{mn}(\vartheta)\mathrm{e}^{\mathrm{i}m\varphi}\\&=\mathbf{B}_{mn}(\vartheta,\varphi)\times\hat{\mathbf{r}},\end{aligned} \tag{C.17}$$

$$\mathbf{P}_{mn}(\vartheta,\varphi)=\hat{\mathbf{r}}P_n^m(\cos\vartheta)\mathrm{e}^{\mathrm{i}m\varphi}$$

$$=(-1)^m\sqrt{\frac{(n+m)!}{(n-m)!}}\,\mathbf{P}_{mn}(\vartheta)\mathrm{e}^{\mathrm{i}m\varphi}, \tag{C.18}$$

$$\mathbf{B}_{mn}(\vartheta)=\hat{\boldsymbol{\vartheta}}\frac{\mathrm{d}}{\mathrm{d}\vartheta}d_{0m}^n(\vartheta)+\hat{\boldsymbol{\varphi}}\frac{\mathrm{i}m}{\sin\vartheta}d_{0m}^n(\vartheta)$$

$$=\hat{\boldsymbol{\vartheta}}\tau_{mn}(\vartheta)+\hat{\boldsymbol{\varphi}}\mathrm{i}\pi_{mn}(\vartheta), \tag{C.19}$$

$$\mathbf{C}_{mn}(\vartheta)=\hat{\boldsymbol{\vartheta}}\frac{\mathrm{i}m}{\sin\vartheta}d_{0m}^n(\vartheta)-\hat{\boldsymbol{\varphi}}\frac{\mathrm{d}}{\mathrm{d}\vartheta}d_{0m}^n(\vartheta)$$

$$=\hat{\boldsymbol{\vartheta}}\mathrm{i}\pi_{mn}(\vartheta)-\hat{\boldsymbol{\varphi}}\tau_{mn}(\vartheta), \tag{C.20}$$

$$\mathbf{P}_{mn}(\vartheta)=\hat{\mathbf{r}}d_{0m}^n(\vartheta), \tag{C.21}$$

$$\gamma_{mn}=\left[\frac{(2n+1)(n-m)!}{4\pi n(n+1)(n+m)!}\right]^{1/2}. \tag{C.22}$$

In Eqs. (C.19) and (C.20),

$$\pi_{mn}(\vartheta)=\frac{m}{\sin\vartheta}d_{0m}^n(\vartheta),$$

$$\tau_{mn}(\vartheta)=\frac{\mathrm{d}}{\mathrm{d}\vartheta}d_{0m}^n(\vartheta).$$

It is straightforward to verify that another class of solutions of the vector Helmholtz equation

$$\nabla^2\mathbf{E}(\mathbf{r})+k^2\,\mathbf{E}(\mathbf{r})=\nabla[\nabla\cdot\mathbf{E}(\mathbf{r})]-\nabla\times\nabla\times\mathbf{E}(\mathbf{r})+k^2\,\mathbf{E}(\mathbf{r})=0 \tag{C.23}$$

is the set of so-called longitudinal vector spherical wave functions

$$\begin{matrix}\mathbf{L}_{mn}(kr,\vartheta,\varphi)\\ \mathrm{Rg}\mathbf{L}_{mn}(kr,\vartheta,\varphi)\end{matrix}=\frac{\gamma'_{mn}}{k}\nabla\begin{matrix}\psi_{mn}(kr,\vartheta,\varphi)\\ \mathrm{Rg}\psi_{mn}(kr,\vartheta,\varphi)\end{matrix}$$

$$=\gamma'_{mn}\left\{\frac{\mathrm{d}}{\mathrm{d}(kr)}\frac{h_n^{(1)}(kr)}{j_n(kr)}\mathbf{P}_{mn}(\vartheta,\varphi)+\frac{1}{kr}\frac{h_n^{(1)}(kr)}{j_n(kr)}\mathbf{B}_{mn}(\vartheta,\varphi)\right\}, \tag{C.24}$$

where

$$\gamma'_{mn}=\left[\frac{(2n+1)(n-m)!}{4\pi(n+m)!}\right]^{1/2}. \tag{C.25}$$

These functions are not divergence-free. It is evident that the vector spherical harmonics $\mathbf{B}_{mn}(\vartheta,\varphi)$, $\mathbf{C}_{mn}(\vartheta,\varphi)$, and $\mathbf{P}_{mn}(\vartheta,\varphi)$ are mutually orthogonal:

$$\mathbf{B}_{mn}(\vartheta,\varphi)\cdot\mathbf{C}_{mn}(\vartheta,\varphi)=\mathbf{B}_{mn}(\vartheta,\varphi)\cdot\mathbf{P}_{mn}(\vartheta,\varphi)=\mathbf{C}_{mn}(\vartheta,\varphi)\cdot\mathbf{P}_{mn}(\vartheta,\varphi)=0. \tag{C.26}$$

Furthermore, **M** and Rg**M** are perpendicular to **N,** Rg**N**, **L**, Rg**L**, and the position

vector **r**. Recalling the symmetry relation (B.5), we derive the following symmetry property of the vector spherical harmonics:

$$\mathbf{B}_{-mn}(\vartheta, \varphi) = (-1)^m \frac{(n-m)!}{(n+m)!} \mathbf{B}^*_{mn}(\vartheta, \varphi), \tag{C.27}$$

and analogous relations hold for $\mathbf{C}_{mn}(\vartheta, \varphi)$ and $\mathbf{P}_{mn}(\vartheta, \varphi)$. The regular vector spherical wave functions obey a similar symmetry relation:

$$\mathrm{Rg}\mathbf{M}_{-mn}(kr, \vartheta, \varphi) = (-1)^m \mathrm{Rg}\mathbf{M}^*_{mn}(kr, \vartheta, \varphi), \tag{C.28}$$

and analogous relations again hold for Rg**N** and Rg**L**.

As follows from Eqs. (C.14), (C.15), and (C.24), the functions Rg**M**, Rg**N**, and Rg**L** are regular at the origin $(r = 0)$. On the other hand, using Eq. (C.8) and the formula

$$\frac{\mathrm{d}h_n^{(1)}(x)}{\mathrm{d}x} = \frac{1}{2n+1}[n h_{n-1}^{(1)}(x) - (n+1)h_{n+1}^{(1)}(x)], \tag{C.29}$$

we find that at infinity, the functions **M** and **N** behave as outgoing transverse vector spherical waves:

$$\mathbf{M}_{mn}(kr, \vartheta, \varphi) \underset{\substack{kr\to\infty \\ kr \gg n^2}}{=} \frac{(-\mathrm{i})^{n+1}\mathrm{e}^{\mathrm{i}kr}}{kr} \gamma_{mn} \mathbf{C}_{mn}(\vartheta, \varphi), \tag{C.30}$$

$$\mathbf{N}_{mn}(kr, \vartheta, \varphi) \underset{\substack{kr\to\infty \\ kr \gg n^2}}{=} \frac{(-\mathrm{i})^{n}\mathrm{e}^{\mathrm{i}kr}}{kr} \gamma_{mn} \mathbf{B}_{mn}(\vartheta, \varphi), \tag{C.31}$$

whereas **L** behaves as an outgoing longitudinal vector spherical wave:

$$\mathbf{L}_{mn}(kr, \vartheta, \varphi) \underset{\substack{kr\to\infty \\ kr \gg n^2}}{=} \frac{(-\mathrm{i})^{n}\mathrm{e}^{\mathrm{i}kr}}{kr} \gamma'_{mn} \mathbf{P}_{mn}(\vartheta, \varphi). \tag{C.32}$$

The vector spherical wave functions defined by Eqs. (C.14), (C.15), and (C.24) and the vector spherical harmonics defined by Eqs. (C.16)–(C.18) are identical to those used by Tsang *et al.* (1985, 2000) and are directly related to the functions $\mathbf{M}^{1,3}_{\sigma mn}$, $\mathbf{N}^{1,3}_{\sigma mn}$, $\mathbf{L}^{1,3}_{\sigma mn}$, $\mathbf{B}^{\sigma}_{mn}$, $\mathbf{C}^{\sigma}_{mn}$, and $\mathbf{P}^{\sigma}_{mn}$ $(\sigma = e, o)$, where $e$ and $o$ stand for even and odd, introduced by Morse and Feshbach (1953). Taking into account that the definition of associated Legendre functions on p. 1325 of Morse and Feshbach (1953) lacks the factor $(-1)^m$ (cf. Eq. (A.3)) and using their Eqs. (13.3.67)–(13.3.69) as well as their table of vector spherical harmonics on pp. 1898 and 1899, we find for $m \geq 0$

$$\mathbf{B}_{mn}(\vartheta, \varphi) = (-1)^m \sqrt{n(n+1)}\,[\mathbf{B}^e_{mn}(\vartheta, \varphi) + \mathrm{i}\mathbf{B}^o_{mn}(\vartheta, \varphi)], \tag{C.33}$$

$$\mathbf{C}_{mn}(\vartheta, \varphi) = (-1)^m \sqrt{n(n+1)}\,[\mathbf{C}^e_{mn}(\vartheta, \varphi) + \mathrm{i}\mathbf{C}^o_{mn}(\vartheta, \varphi)], \tag{C.34}$$

$$\mathbf{P}_{mn}(\vartheta, \varphi) = (-1)^m [\mathbf{P}^e_{mn}(\vartheta, \varphi) + \mathrm{i}\mathbf{P}^o_{mn}(\vartheta, \varphi)], \tag{C.35}$$

$$\mathbf{M}_{mn}(kr, \vartheta, \varphi) = (-1)^m \gamma_{mn}\,[\mathbf{M}^3_{emn}(kr, \vartheta, \varphi) + \mathrm{i}\mathbf{M}^3_{omn}(kr, \vartheta, \varphi)], \tag{C.36}$$

$$\mathbf{N}_{mn}(kr, \vartheta, \varphi) = (-1)^m \gamma_{mn}[\mathbf{N}^3_{emn}(kr, \vartheta, \varphi) + \mathrm{i}\mathbf{N}^3_{omn}(kr, \vartheta, \varphi)], \tag{C.37}$$

$$\mathbf{L}_{mn}(kr, \vartheta, \varphi) = (-1)^m \gamma'_{mn}[\mathbf{L}^3_{emn}(kr, \vartheta, \varphi) + \mathrm{i}\mathbf{L}^3_{omn}(kr, \vartheta, \varphi)], \tag{C.38}$$

$$\mathrm{Rg}\mathbf{M}_{mn}(kr, \vartheta, \varphi) = (-1)^m \gamma_{mn}[\mathbf{M}^1_{emn}(kr, \vartheta, \varphi) + \mathrm{i}\mathbf{M}^1_{omn}(kr, \vartheta, \varphi)], \tag{C.39}$$

$$\mathrm{Rg}\mathbf{N}_{mn}(kr, \vartheta, \varphi) = (-1)^m \gamma_{mn}[\mathbf{N}^1_{emn}(kr, \vartheta, \varphi) + \mathrm{i}\mathbf{N}^1_{omn}(kr, \vartheta, \varphi)], \tag{C.40}$$

$$\mathrm{Rg}\mathbf{L}_{mn}(kr, \vartheta, \varphi) = (-1)^m \gamma'_{mn}[\mathbf{L}^1_{emn}(kr, \vartheta, \varphi) + \mathrm{i}\mathbf{L}^1_{omn}(kr, \vartheta, \varphi)]. \tag{C.41}$$

Conversely,

$$\begin{aligned}\mathbf{B}^{\substack{e\\o}}_{mn}(\vartheta, \varphi) &= \frac{1}{-\mathrm{i}}\frac{(-1)^m}{2\sqrt{n(n+1)}}[\mathbf{B}_{mn}(\vartheta, \varphi) \pm \mathbf{B}^*_{mn}(\vartheta, \varphi)] \\ &= \frac{1}{-\mathrm{i}}\frac{1}{2\sqrt{n(n+1)}}[(-1)^m \mathbf{B}_{mn}(\vartheta, \varphi) \pm \frac{(n+m)!}{(n-m)!}\mathbf{B}_{-mn}(\vartheta, \varphi)],\end{aligned} \tag{C.42}$$

$$\begin{aligned}\mathbf{C}^{\substack{e\\o}}_{mn}(\vartheta, \varphi) &= \frac{1}{-\mathrm{i}}\frac{(-1)^m}{2\sqrt{n(n+1)}}[\mathbf{C}_{mn}(\vartheta, \varphi) \pm \mathbf{C}^*_{mn}(\vartheta, \varphi)] \\ &= \frac{1}{-\mathrm{i}}\frac{1}{2\sqrt{n(n+1)}}[(-1)^m \mathbf{C}_{mn}(\vartheta, \varphi) \pm \frac{(n+m)!}{(n-m)!}\mathbf{C}_{-mn}(\vartheta, \varphi)],\end{aligned} \tag{C.43}$$

$$\begin{aligned}\mathbf{P}^{\substack{e\\o}}_{mn}(\vartheta, \varphi) &= \frac{1}{-\mathrm{i}}\frac{(-1)^m}{2}[\mathbf{P}_{mn}(\vartheta, \varphi) \pm \mathbf{P}^*_{mn}(\vartheta, \varphi)] \\ &= \frac{1}{-\mathrm{i}}\frac{1}{2}[(-1)^m \mathbf{P}_{mn}(\vartheta, \varphi) \pm \frac{(n+m)!}{(n-m)!}\mathbf{P}_{-mn}(\vartheta, \varphi)],\end{aligned} \tag{C.44}$$

$$\begin{aligned}\mathbf{M}^1_{\substack{e\\o}mn}(kr, \vartheta, \varphi) &= \frac{1}{-\mathrm{i}}\frac{(-1)^m}{2\gamma_{mn}}[\mathrm{Rg}\mathbf{M}_{mn}(kr, \vartheta, \varphi) \pm \mathrm{Rg}\mathbf{M}^*_{mn}(kr, \vartheta, \varphi)] \\ &= \frac{1}{-\mathrm{i}}\frac{1}{2\gamma_{mn}}[(-1)^m \mathrm{Rg}\mathbf{M}_{mn}(kr, \vartheta, \varphi) \pm \mathrm{Rg}\mathbf{M}_{-mn}(kr, \vartheta, \varphi)],\end{aligned} \tag{C.45}$$

$$\mathbf{M}^3_{\substack{e\\o}mn}(kr, \vartheta, \varphi) = \frac{1}{-\mathrm{i}}\frac{(-1)^m}{2\gamma_{mn}}[\mathbf{M}_{mn}(kr, \vartheta, \varphi) \pm \mathbf{M}^*_{mn}(kr, \vartheta, \varphi)], \tag{C.46}$$

$$\begin{aligned}\mathbf{N}^1_{\substack{e\\o}mn}(kr, \vartheta, \varphi) &= \frac{1}{-\mathrm{i}}\frac{(-1)^m}{2\gamma_{mn}}[\mathrm{Rg}\mathbf{N}_{mn}(kr, \vartheta, \varphi) \pm \mathrm{Rg}\mathbf{N}^*_{mn}(kr, \vartheta, \varphi)] \\ &= \frac{1}{-\mathrm{i}}\frac{1}{2\gamma_{mn}}[(-1)^m \mathrm{Rg}\mathbf{N}_{mn}(kr, \vartheta, \varphi) \pm \mathrm{Rg}\mathbf{N}_{-mn}(kr, \vartheta, \varphi)],\end{aligned} \tag{C.47}$$

$$\mathbf{N}^3_{\substack{e\\o}mn}(kr, \vartheta, \varphi) = \frac{1}{-\mathrm{i}}\frac{(-1)^m}{2\gamma_{mn}}[\mathbf{N}_{mn}(kr, \vartheta, \varphi) \pm \mathbf{N}^*_{mn}(kr, \vartheta, \varphi)], \tag{C.48}$$

$$\mathbf{L}^1_{\substack{e\\o}mn}(kr,\vartheta,\varphi)=\frac{1}{-\mathrm{i}}\frac{(-1)^m}{2\gamma'_{mn}}[\mathrm{Rg}\mathbf{L}_{mn}(kr,\vartheta,\varphi)\pm\mathrm{Rg}\mathbf{L}^*_{mn}(kr,\vartheta,\varphi)]$$

$$=\frac{1}{-\mathrm{i}}\frac{1}{2\gamma'_{mn}}[(-1)^m\,\mathrm{Rg}\mathbf{L}_{mn}(kr,\vartheta,\varphi)\pm\mathrm{Rg}\mathbf{L}_{-mn}(kr,\vartheta,\varphi)], \tag{C.49}$$

$$\mathbf{L}^3_{\substack{e\\o}mn}(kr,\vartheta,\varphi)=\frac{1}{-\mathrm{i}}\frac{(-1)^m}{2\gamma'_{mn}}[\mathbf{L}_{mn}(kr,\vartheta,\varphi)\pm\mathbf{L}^*_{mn}(kr,\vartheta,\varphi)]. \tag{C.50}$$

The equations on p. 1900 of Morse and Feshbach (1953) and Eqs. (C.33)–(C.35) yield the following orthogonality relations for the vector spherical harmonics:

$$\int_0^{2\pi}\mathrm{d}\varphi\int_0^{\pi}\mathrm{d}\vartheta\sin\vartheta\,\mathbf{B}_{mn}(\vartheta,\varphi)\mathbf{C}^*_{m'n'}(\vartheta,\varphi)$$

$$=\int_0^{2\pi}\mathrm{d}\varphi\int_0^{\pi}\mathrm{d}\vartheta\sin\vartheta\,\mathbf{B}_{mn}(\vartheta,\varphi)\mathbf{P}^*_{m'n'}(\vartheta,\varphi)$$

$$=\int_0^{2\pi}\mathrm{d}\varphi\int_0^{\pi}\mathrm{d}\vartheta\sin\vartheta\,\mathbf{C}_{mn}(\vartheta,\varphi)\mathbf{P}^*_{m'n'}(\vartheta,\varphi)$$

$$=0, \tag{C.51}$$

$$\int_0^{2\pi}\mathrm{d}\varphi\int_0^{\pi}\mathrm{d}\vartheta\sin\vartheta\,\mathbf{B}_{mn}(\vartheta,\varphi)\mathbf{B}^*_{m'n'}(\vartheta,\varphi)$$

$$=\int_0^{2\pi}\mathrm{d}\varphi\int_0^{\pi}\mathrm{d}\vartheta\sin\vartheta\,\mathbf{C}_{mn}(\vartheta,\varphi)\mathbf{C}^*_{m'n'}(\vartheta,\varphi)$$

$$=\frac{1}{(\gamma_{mn})^2}\delta_{mm'}\delta_{nn'}, \tag{C.52}$$

$$\int_0^{2\pi}\mathrm{d}\varphi\int_0^{\pi}\mathrm{d}\vartheta\sin\vartheta\,\mathbf{P}_{mn}(\vartheta,\varphi)\mathbf{P}^*_{m'n'}(\vartheta,\varphi)=\frac{1}{(\gamma'_{mn})^2}\delta_{mm'}\delta_{nn'}. \tag{C.53}$$

Using Eqs. (C.42)–(C.50), we can rewrite the series expansion of the dyadic $\overleftrightarrow{I}\mathrm{e}^{\mathrm{i}\mathbf{r}'\cdot\mathbf{r}}$ on p. 1866 of Morse and Feshbach (1953) as follows (Tsang *et al.* 1985, 2000):

$$\overleftrightarrow{I}\mathrm{e}^{\mathrm{i}\mathbf{r}'\cdot\mathbf{r}}=\sum_{n=0}^{\infty}\sum_{m=-n}^{n}(-1)^m\mathrm{i}^n\frac{2n+1}{n(n+1)}\left\{\frac{n(n+1)}{\mathrm{i}\gamma'_{mn}}\mathbf{P}_{-mn}(\vartheta',\varphi')\otimes\mathrm{Rg}\mathbf{L}_{mn}(r'r,\vartheta,\varphi)\right.$$

$$+\frac{1}{\gamma_{mn}}\mathbf{C}_{-mn}(\vartheta',\varphi')\otimes\mathrm{Rg}\mathbf{M}_{mn}(r'r,\vartheta,\varphi)$$

$$\left.+\frac{1}{\mathrm{i}\gamma_{mn}}\mathbf{B}_{-mn}(\vartheta',\varphi')\otimes\mathrm{Rg}\mathbf{N}_{mn}(r'r,\vartheta,\varphi)\right\}. \tag{C.54}$$

The expansion of the plane electromagnetic wave

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_0\, \mathrm{e}^{\mathrm{i}k\hat{\mathbf{r}}'\cdot\mathbf{r}}, \qquad \mathbf{E}_0 \cdot \hat{\mathbf{r}}' = 0, \tag{C.55}$$

in vector spherical wave functions is obtained by taking the dot product of $\mathbf{E}_0$ and $\overleftrightarrow{I}\mathrm{e}^{\mathrm{i}k\hat{\mathbf{r}}'\cdot\mathbf{r}}$:

$$\mathbf{E}_0\, \mathrm{e}^{\mathrm{i}k\hat{\mathbf{r}}'\cdot\mathbf{r}} = \sum_{n=1}^{\infty}\sum_{m=-n}^{n} [a_{mn}\mathrm{Rg}\mathbf{M}_{mn}(kr, \vartheta, \varphi) + b_{mn}\mathrm{Rg}\mathbf{N}_{mn}(kr, \vartheta, \varphi)], \tag{C.56}$$

where

$$a_{mn} = 4\pi(-1)^m \mathrm{i}^n d_n \mathbf{E}_0 \cdot \mathbf{C}^*_{mn}(\vartheta')\exp(-\mathrm{i}m\varphi'), \tag{C.57}$$

$$b_{mn} = 4\pi(-1)^m \mathrm{i}^{n-1} d_n \mathbf{E}_0 \cdot \mathbf{B}^*_{mn}(\vartheta')\exp(-\mathrm{i}m\varphi'), \tag{C.58}$$

$$d_n = \left[\frac{2n+1}{4\pi n(n+1)}\right]^{1/2}, \tag{C.59}$$

and we have taken into account that $a_{mn}$ and $b_{mn}$ vanish for $n = 0$. The free space dyadic Green's function given by

$$\overleftrightarrow{G}(\mathbf{r}, \mathbf{r}') = \left(\overleftrightarrow{I} + \frac{1}{k^2}\nabla\otimes\nabla\right)\frac{\exp(\mathrm{i}k|\mathbf{r}-\mathbf{r}'|)}{4\pi|\mathbf{r}-\mathbf{r}'|} \tag{C.60}$$

(cf. Eqs. (2.13) and (2.15)) can also be expressed in terms of vector spherical wave functions:

$$\overleftrightarrow{G}(\mathbf{r}, \mathbf{r}') = \mathrm{i}k\sum_{n=1}^{\infty}\sum_{m=-n}^{n}(-1)^m \begin{cases} \mathbf{M}_{-mn}(kr, \vartheta, \varphi)\otimes\mathrm{Rg}\mathbf{M}_{mn}(kr', \vartheta', \varphi') \\ +\mathbf{N}_{-mn}(kr, \vartheta, \varphi)\otimes\mathrm{Rg}\mathbf{N}_{mn}(kr', \vartheta', \varphi') \quad \text{for } r > r', \\ \\ \mathrm{Rg}\mathbf{M}_{-mn}(kr, \vartheta, \varphi)\otimes\mathbf{M}_{mn}(kr', \vartheta', \varphi') \\ +\mathrm{Rg}\mathbf{N}_{-mn}(kr, \vartheta, \varphi)\otimes\mathbf{N}_{mn}(kr', \vartheta', \varphi') \quad \text{for } r' > r, \end{cases} \tag{C.61}$$

where $\mathbf{r} \neq \mathbf{r}'$ (Tsang *et al.* 1985, p. 183).

Let $(\vartheta_1, \varphi_1)$ and $(\vartheta_2, \varphi_2)$ be the spherical angles of the same position vector $\mathbf{r}$ in coordinate systems 1 and 2, respectively. Both coordinate systems have the same origin. Coordinate system 2 is obtained by rotating coordinate system 1 though Euler angles $(\alpha, \beta, \gamma)$ (Section 2.4). From Eq. (1) of Section 5.2 and Eq. (1) of Section 5.5 of Varshalovich *et al.* (1988) and Eqs. (C.5) and (C.9) above we have

$$\gamma_{mn}\begin{matrix}\psi_{mn}(kr, \vartheta_2, \varphi_2) \\ \mathrm{Rg}\,\psi_{mn}(kr, \vartheta_2, \varphi_2)\end{matrix} = \sum_{m'=-n}^{n}\gamma_{m'n}\begin{matrix}\psi_{m'n}(kr, \vartheta_1, \varphi_1) \\ \mathrm{Rg}\,\psi_{m'n}(kr, \vartheta_1, \varphi_1)\end{matrix} D^n_{m'm}(\alpha, \beta, \gamma), \tag{C.62}$$

where

$$D^n_{m'm}(\alpha, \beta, \gamma) = \mathrm{e}^{-\mathrm{i}m'\alpha} d^n_{m'm}(\beta)\mathrm{e}^{-\mathrm{i}m\gamma} \tag{C.63}$$

are Wigner *D*-functions and $d^n_{m'm}(\beta)$ are Wigner *d*-functions (Appendix B). Substituting Eq. (C.62) in Eqs. (C.14) and (C.15) yields

$$\begin{matrix}\mathbf{M}_{mn}(kr, \vartheta_2, \varphi_2)\\ \mathrm{Rg}\mathbf{M}_{mn}(kr, \vartheta_2, \varphi_2)\end{matrix} = \sum_{m'=-n}^{n} \begin{matrix}\mathbf{M}_{m'n}(kr, \vartheta_1, \varphi_1)\\ \mathrm{Rg}\mathbf{M}_{m'n}(kr, \vartheta_1, \varphi_1)\end{matrix} D^n_{m'm}(\alpha, \beta, \gamma), \tag{C.64}$$

and analogously for **N** and Rg**N**. Conversely,

$$\begin{matrix}\mathbf{M}_{mn}(kr, \vartheta_1, \varphi_1)\\ \mathrm{Rg}\mathbf{M}_{mn}(kr, \vartheta_1, \varphi_1)\end{matrix} = \sum_{m'=-n}^{n} \begin{matrix}\mathbf{M}_{m'n}(kr, \vartheta_2, \varphi_2)\\ \mathrm{Rg}\mathbf{M}_{m'n}(kr, \vartheta_2, \varphi_2)\end{matrix} D^n_{m'm}(-\gamma, -\beta, -\alpha). \tag{C.65}$$

Consider now coordinate systems 1 and 2 having the same spatial orientation and denote by $\mathbf{r}_{12}$ the vector connecting the origin of coordinate system 1 with the origin of coordinate system 2 (cf. Fig. 5.2). The translation addition theorem allows one to expand the vector spherical wave functions centered at the origin of coordinate system 1 in terms of the vector spherical wave functions centered at the origin of coordinate system 2 (Tsang *et al.* 1985; Boström *et al.* 1991). Specifically, if vectors $\mathbf{r}_1$, $\mathbf{r}_2$, and $\mathbf{r}_{12}$ form a triangle such that $\mathbf{r}_1 = \mathbf{r}_{12} + \mathbf{r}_2$ (Fig. 5.2) and $(r_1, \vartheta_1, \varphi_1)$, $(r_2, \vartheta_2, \varphi_2)$, and $(r_{12}, \vartheta_{12}, \varphi_{12})$ are their respective spherical coordinates, then

$$\mathrm{Rg}\begin{matrix}\mathbf{M}_{mn}\\ \mathbf{N}_{mn}\end{matrix}(kr_1, \vartheta_1, \varphi_1) = \sum_{\nu=1}^{\infty}\sum_{\mu=-\nu}^{\nu}\left[\mathrm{Rg}\begin{matrix}A_{\mu\nu mn}\\ B_{\mu\nu mn}\end{matrix}(kr_{12}, \vartheta_{12}, \varphi_{12})\mathrm{Rg}\mathbf{M}_{\mu\nu}(kr_2, \vartheta_2, \varphi_2)\right.$$
$$\left. + \mathrm{Rg}\begin{matrix}B_{\mu\nu mn}\\ A_{\mu\nu mn}\end{matrix}(kr_{12}, \vartheta_{12}, \varphi_{12})\mathrm{Rg}\mathbf{N}_{\mu\nu}(kr_2, \vartheta_2, \varphi_2)\right], \tag{C.66}$$

$$\begin{matrix}\mathbf{M}_{mn}\\ \mathbf{N}_{mn}\end{matrix}(kr_1, \vartheta_1, \varphi_1) = \sum_{\nu=1}^{\infty}\sum_{\mu=-\nu}^{\nu}\left[\mathrm{Rg}\begin{matrix}A_{\mu\nu mn}\\ B_{\mu\nu mn}\end{matrix}(kr_{12}, \vartheta_{12}, \varphi_{12})\mathbf{M}_{\mu\nu}(kr_2, \vartheta_2, \varphi_2)\right.$$
$$\left. + \mathrm{Rg}\begin{matrix}B_{\mu\nu mn}\\ A_{\mu\nu mn}\end{matrix}(kr_{12}, \vartheta_{12}, \varphi_{12})\mathbf{N}_{\mu\nu}(kr_2, \vartheta_2, \varphi_2)\right], \quad r_2 > r_{12}, \tag{C.67}$$

$$\begin{matrix}\mathbf{M}_{mn}\\ \mathbf{N}_{mn}\end{matrix}(kr_1, \vartheta_1, \varphi_1) = \sum_{\nu=1}^{\infty}\sum_{\mu=-\nu}^{\nu}\left[\begin{matrix}A_{\mu\nu mn}\\ B_{\mu\nu mn}\end{matrix}(kr_{12}, \vartheta_{12}, \varphi_{12})\mathrm{Rg}\mathbf{M}_{\mu\nu}(kr_2, \vartheta_2, \varphi_2)\right.$$
$$\left. + \begin{matrix}B_{\mu\nu mn}\\ A_{\mu\nu mn}\end{matrix}(kr_{12}, \vartheta_{12}, \varphi_{12})\mathrm{Rg}\mathbf{N}_{\mu\nu}(kr_2, \vartheta_2, \varphi_2)\right], \quad r_2 < r_{12}, \tag{C.68}$$

where

$$A_{\mu\nu mn}(kr_{12}, \vartheta_{12}, \varphi_{12}) = \frac{\gamma_{mn}}{\gamma_{\mu\nu}}(-1)^{\mu}\sum_{p=|n-\nu|}^{n+\nu} a(m, n|-\mu, \nu|p)a(n, \nu, p)h_p^{(1)}(kr_{12})$$
$$\times P_p^{m-\mu}(\cos\vartheta_{12})\exp[\mathrm{i}(m-\mu)\varphi_{12}], \tag{C.69}$$

$$B_{\mu\nu mn}(kr_{12}, \vartheta_{12}, \varphi_{12}) = \frac{\gamma_{mn}}{\gamma_{\mu\nu}}(-1)^{\mu+1} \sum_{p=|n-\nu|}^{n+\nu} a(m, n|-\mu, \nu|p, p-1)b(n, \nu, p)h_p^{(1)}(kr_{12})$$
$$\times P_p^{m-\mu}(\cos\vartheta_{12})\exp[\mathrm{i}(m-\mu)\varphi_{12}] \tag{C.70}$$

are translation coefficients. Here

$$a(m, n|\mu, \nu|p) = (-1)^{m+\mu}(2p+1)\left[\frac{(n+m)!(\nu+\mu)!(p-m-\mu)!}{(n-m)!(\nu-\mu)!(p+m+\mu)!}\right]^{1/2}$$
$$\times\begin{pmatrix} n & \nu & p \\ m & \mu & -(m+\mu) \end{pmatrix}\begin{pmatrix} n & \nu & p \\ 0 & 0 & 0 \end{pmatrix}, \tag{C.71}$$

$$a(m, n|\mu, \nu|p, q) = (-1)^{m+\mu}(2p+1)\left[\frac{(n+m)!(\nu+\mu)!(p-m-\mu)!}{(n-m)!(\nu-\mu)!(p+m+\mu)!}\right]^{1/2}$$
$$\times\begin{pmatrix} n & \nu & p \\ m & \mu & -(m+\mu) \end{pmatrix}\begin{pmatrix} n & \nu & q \\ 0 & 0 & 0 \end{pmatrix}, \tag{C.72}$$

$$a(n, \nu, p) = \frac{\mathrm{i}^{\nu-n+p}(2\nu+1)}{2\nu(\nu+1)}[n(n+1)+\nu(\nu+1)-p(p+1)], \tag{C.73}$$

$$b(n, \nu, p) = -\frac{\mathrm{i}^{\nu-n+p}(2\nu+1)}{2\nu(\nu+1)}[(n+\nu+1+p)(n+\nu+1-p)$$
$$\times(p+n-\nu)(p-n+\nu)]^{1/2}, \tag{C.74}$$

and the coefficients

$$\begin{pmatrix} n & \nu & p \\ m & \mu & -(m+\mu) \end{pmatrix}$$

are Wigner 3$j$ symbols (Appendix D). As usual, the formulas for $\mathrm{Rg}A_{\mu\nu mn}$ and $\mathrm{Rg}B_{\mu\nu mn}$ are obtained by replacing $h_p^{(1)}$ by $j_p$ in Eqs. (C.69) and (C.70). Efficient numerical computation of the translation coefficients $A_{\mu\nu mn}$, $B_{\mu\nu mn}$, $\mathrm{Rg}A_{\mu\nu mn}$, and $\mathrm{Rg}B_{\mu\nu mn}$ is discussed by Fuller and Mackowski (2000).

# Appendix D

# Clebsch–Gordan coefficients and Wigner 3*j* symbols

The real-valued coefficients appearing in the Clebsch–Gordan expansion, Eq. (B.49), are defined as an algebraic sum (Varshalovich *et al.* 1988; Rose 1995):

$$
\begin{aligned}
&C^{nm}_{n_1m_1\,n_2m_2} \\
&= \delta_{m,m_1+m_2}\left[\frac{(n_1+n_2-n)!(n_1-n_2+n)!(-n_1+n_2+n)!}{(n_1+n_2+n+1)!}\right]^{1/2} \\
&\times[(n_1+m_1)!(n_1-m_1)!(n_2+m_2)!(n_2-m_2)!(n+m)!(n-m)!(2n+1)]^{1/2} \\
&\times\sum_p \frac{(-1)^p}{p!(n_1+n_2-n-p)!(n_1-m_1-p)!(n_2+m_2-p)!(n-n_2+m_1+p)!(n-n_1-m_2+p)!},
\end{aligned}
\tag{D.1}
$$

where

- $n_1$, $n_2$, and $n$ are non-negative integers or half-integers;
- $m_1$, $m_2$, and $m$ are integers or half-integers;
- $|m_1| \le n_1$, $|m_2| \le n_2$, and $|m| \le n$;
- $m_1+n_1$, $m_2+n_2$, $m+n$, and $n_1+n_2+n$ are non-negative integers.

The Clebsch–Gordan coefficients are nonzero only when

$$|n_1-n_2| \le n \le n_1+n_2 \tag{D.2}$$

and

$$m = m_1+m_2. \tag{D.3}$$

The so-called unitarity relations for the Clebsch–Gordan coefficients read as

$$\sum_{m_1m_2} C^{nm}_{n_1m_1\,n_2m_2} C^{n'm'}_{n_1m_1\,n_2m_2} = \delta_{nn'}\delta_{mm'}, \tag{D.4}$$

$$\sum_{nm} C^{nm}_{n_1m_1\,n_2m_2} C^{nm}_{n_1m_1'\,n_2m_2'} = \delta_{m_1m_1'}\,\delta_{m_2m_2'}. \tag{D.5}$$

The Clebsch–Gordan coefficients have the following symmetry properties:

$$\begin{aligned} C^{nm}_{n_1m_1\,n_2m_2} &= (-1)^{n_1+n_2-n} C^{nm}_{n_2m_2\,n_1m_1} = (-1)^{n_1-m_1}\sqrt{\frac{2n+1}{2n_2+1}}\, C^{n_2,-m_2}_{n_1m_1\,n,-m} \\ &= (-1)^{n_1-m_1}\sqrt{\frac{2n+1}{2n_2+1}}\, C^{n_2m_2}_{nm\,n_1,-m_1} = (-1)^{n_2+m_2}\sqrt{\frac{2n+1}{2n_1+1}}\, C^{n_1,-m_1}_{n,-m\,n_2m_2} \\ &= (-1)^{n_2+m_2}\sqrt{\frac{2n+1}{2n_1+1}}\, C^{n_1m_1}_{n_2,-m_2\,nm}, \end{aligned} \tag{D.6}$$

$$C^{nm}_{n_1m_1\,n_2m_2} = (-1)^{n_1+n_2-n} C^{n,-m}_{n_1,-m_1\,n_2,-m_2}. \tag{D.7}$$

The so-called Wigner 3*j* symbols are defined in terms of the Clebsch–Gordan coefficients as

$$\begin{pmatrix} n_1 & n_2 & n \\ m_1 & m_2 & m \end{pmatrix} = (-1)^{n+m+2n_1}\frac{1}{\sqrt{2n+1}} C^{nm}_{n_1,-m_1\,n_2,-m_2}. \tag{D.8}$$

Conversely,

$$C^{nm}_{n_1m_1\,n_2m_2} = (-1)^{n_1-n_2+m}\sqrt{2n+1}\begin{pmatrix} n_1 & n_2 & n \\ m_1 & m_2 & -m \end{pmatrix}. \tag{D.9}$$

The symmetry properties of the 3*j* symbols follow from Eqs. (D.6)–(D.9) and are simpler than those for the Clebsch–Gordan coefficients:

$$\begin{aligned} &\begin{pmatrix} n_1 & n_2 & n \\ m_1 & m_2 & m \end{pmatrix} = \begin{pmatrix} n_2 & n & n_1 \\ m_2 & m & m_1 \end{pmatrix} = \begin{pmatrix} n & n_1 & n_2 \\ m & m_1 & m_2 \end{pmatrix} \\ &= (-1)^{n_1+n_2+n}\begin{pmatrix} n_1 & n & n_2 \\ m_1 & m & m_2 \end{pmatrix} = (-1)^{n_1+n_2+n}\begin{pmatrix} n_2 & n_1 & n \\ m_2 & m_1 & m \end{pmatrix} \\ &= (-1)^{n_1+n_2+n}\begin{pmatrix} n & n_2 & n_1 \\ m & m_2 & m_1 \end{pmatrix}, \end{aligned} \tag{D.10}$$

$$\begin{pmatrix} n_1 & n_2 & n \\ m_1 & m_2 & m \end{pmatrix} = (-1)^{n_1+n_2+n}\begin{pmatrix} n_1 & n_2 & n \\ -m_1 & -m_2 & -m \end{pmatrix}. \tag{D.11}$$

The numerical computation of the Clebsch–Gordan coefficients is based on the following formulas, which are either listed by Varshalovich *et al.* (1988) or can be easily derived from equations therein (Mishchenko 1991a).

- For $n' < N_{\min} = \max(|n-n_1|, |m'|)$,

$$C^{n'm'}_{nm\,n_1,m'-m} = 0. \tag{D.12}$$

- For $n' > N_{\min}$,

$$C^{n'm'}_{nm\,n_1,m'-m} = A(BC^{n'-1,m'}_{nm\,n_1,m'-m} - CC^{n'-2,m'}_{nm\,n_1,m'-m}), \tag{D.13}$$

where

$$A = \left[\frac{4n'^2(2n'+1)(2n'-1)}{(n'+m')(n'-m')(n_1-n+n')(n-n_1+n')(n+n_1-n'+1)(n+n_1+n'+1)}\right]^{1/2}, \tag{D.14}$$

$$B = \frac{(2m-m')n'(n'-1)-m'n(n+1)+m'n_1(n_1+1)}{2n'(n'-1)}, \tag{D.15}$$

$$C = \left[\frac{(n'-m'-1)(n'+m'-1)(n_1-n+n'-1)(n-n_1+n'-1)(n+n_1-n'+2)(n+n_1+n')}{4(n'-1)^2(2n'-3)(2n'-1)}\right]^{1/2}. \tag{D.16}$$

- For $n' = N_{\min}$, the following four particular cases must be considered.

(1) For $|n-n_1| \geq |m'|$ and $n \geq n_1$,

$$C^{n-n_1,m'}_{nm\,n_1,m'-m} = (-1)^{n_1+m'+m}\left[\frac{(n+m)!(n-m)!(2n_1)!(2n-2n_1+1)!}{(2n+1)!(n_1+m'-m)!(n_1-m'+m)!(n-n_1+m')!(n-n_1-m')!}\right]^{1/2}. \tag{D.17}$$

(2) For $|n-n_1| \geq |m'|$ and $n < n_1$, we use the symmetry relation (cf. Eq. (D.6))

$$C^{n_1-n,m'}_{nm\,n_1,m'-m} = C^{n_1-n,m'}_{n_1,m'-m\,nm} \tag{D.18}$$

along with Eq. (D.17).

(3) For $|n-n_1| < |m'|$ and $m' \geq 0$,

$$C^{m'm'}_{nm\,n_1,m'-m} = (-1)^{n+m}\left[\frac{(2m'+1)!(n+n_1-m')!(n+m)!(n_1+m'-m)!}{(n+n_1+m'+1)!(n-n_1+m')!(n_1-n+m')!(n-m)!(n_1-m'+m)!}\right]^{1/2}. \tag{D.19}$$

(4) For $|n-n_1| < |m'|$ and $m' < 0$, we use the formula (cf. Eq. (D.7))

$$C^{-m',m'}_{nm\,n_1,m'-m} = (-1)^{n+n_1+m'}C^{-m',-m'}_{n,-m\,n_1,m-m'} \tag{D.20}$$

along with Eq. (D.19).

We have found that the numerical scheme based on the upward recursion (D.13) produces stable results only for $n'$ smaller than approximately 60 if double-precision (REAL*8) floating-point FORTRAN variables are used and smaller than approximately 105 if the scheme is implemented using extended-precision (REAL*16) variables. The instability occurs in those cases when the use of the upward recursion causes $n'$ to reach what Schulten and Gordon (1975) call the classical domain of

$n'$ - values centered at $n' \approx N = (N_{\min} + N_{\max})/2$, where $N_{\max} = n + n_1$, and then the non-classical domain with $n' \gg N$. Clebsch–Gordan coefficients first increase in absolute value with increasing $n'$, then oscillate with a slowly varying amplitude in the classical domain, and finally rapidly decay in absolute value as $n'$ reaches the non-classical domain. This behavior results in catastrophic loss of numerical accuracy in the classical domain followed by overflows in the non-classical domain.

To stabilize the computation of the Clebsch–Gordan coefficients, we have implemented a modified version of the procedure proposed by Schulten and Gordon (1975). Specifically, the upward recursion of Eq. (13) is used only for $n' \le N$, whereas the Clebsch–Gordan coefficients with $n' > N$ are computed using the downward analog of Eq. (13),

$$C^{n'm'}_{nm\,n_1,m'-m} = D(EC^{n'+1,m'}_{nm\,n_1,m'-m} - FC^{n'+2,m'}_{nm\,n_1,m'-m}), \tag{D.21}$$

where

$$D = \left[\frac{4(n'+1)^2(2n'+1)(2n'+3)}{(n'+m'+1)(n'-m'+1)(n_1-n+n'+1)(n-n_1+n'+1)(n+n_1-n')(n+n_1+n'+2)}\right]^{1/2}, \tag{D.22}$$

$$E = \frac{(2m-m')(n'+2)(n'+1) - m'n(n+1) + m'n_1(n_1+1)}{2(n'+2)(n'+1)}, \tag{D.23}$$

$$F = \left[\frac{(n'-m'+2)(n'+m'+2)(n_1-n+n'+2)(n-n_1+n'+2)(n+n_1-n'-1)(n+n_1+n'+3)}{4(n'+2)^2(2n'+5)(2n'+3)}\right]^{1/2}, \tag{D.24}$$

supplemented by the initial values

$$C^{N_{\max}+1,m'}_{nm\,n_1,m'-m} = 0, \tag{D.25}$$

$$C^{N_{\max},m'}_{nm\,n_1,m'-m} = \left[\frac{(2n)!(2n_1)!(n+n_1+m')!(n+n_1-m')!}{(2n+2n_1)!(n+m)!(n-m)!(n_1+m'-m)!(n_1-m'+m)!}\right]^{1/2}. \tag{D.26}$$

Since the two-directional recursion scheme always works towards the center of the classical domain $(n' \cong N)$, it produces highly accurate and stable results. We have found that it works well for $n'$ exceeding 150 even when the numerical procedure is implemented using double-precision variables (Wielaard *et al.* 1997).

Equations (D.17), (D.19), and (D.26) involve ratios of products of several factorials. Although these ratios are finite numbers with quite limited values, the factorials themselves can cause computer overflows in actual computer calculations. In order to avoid overflows, a useful approach is to store an array of natural logarithms of factorials of integers ranging from 0 to an appropriate maximal value. Then the ratio of two factorials is computed as $m!/n! = \exp[\ln(m!) - \ln(n!)]$. The logarithms of the factorials should be computed using the upward recursion $\ln[(m+1)!] = \ln(m!) + \ln(m+1)$.

# Appendix E

## Système International units

The system of physical units adopted in this book is the internationally accepted form of the metric system known as the Système International (SI). The SI is formed by base units, supplementary units, and units derived from the base units. The table below lists only those derived SI units that are used in this book.

| Quantity | Unit | Symbol | Definition |
|---|---|---|---|
| Base units | | | |
| length | meter | m | |
| time | second | s | |
| mass | kilogram | kg | |
| electric current | ampere | A | |
| temperature | kelvin | K | |
| amount of substance | mole | mol | |
| Supplementary units | | | |
| plane angle | radian | rad | |
| solid angle | steradian | sr | |
| Derived units | | | |
| energy | joule | J | $\mathrm{N\,m = kg\,m^2\,s^{-2}}$ |
| electric charge | coulomb | C | $\mathrm{A\,s}$ |
| electric potential | volt | V | $\mathrm{W\,A^{-1} = kg\,m^2\,s^{-3}\,A^{-1}}$ |
| electric capacitance | farad | F | $\mathrm{C\,V^{-1} = kg^{-1}\,m^{-2}\,s^4\,A^2}$ |
| electric resistance | ohm | Ω | $\mathrm{V\,A^{-1} = kg\,m^2\,s^{-3}\,A^{-2}}$ |
| magnetic flux | weber | Wb | $\mathrm{V\,s = kg\,m^2\,s^{-2}\,A^{-1}}$ |
| inductance | henry | H | $\mathrm{Wb\,A^{-1} = kg\,m^2\,s^{-2}\,A^{-2}}$ |
| frequency | hertz | Hz | $\mathrm{s^{-1}}$ |
| power | watt | W | $\mathrm{J\,s^{-1} = kg\,m^2\,s^{-3}}$ |
| force | newton | N | $\mathrm{kg\,m\,s^{-2}}$ |

# Abbreviations and symbols

The list includes only those abbreviations and symbols that are encountered in two or more sections or are used to denote two or more quantities.

| Abbreviation or symbol | Definition and dimension in SI units | Introduced in Section |
|---|---|---|
| **English abbreviations and symbols** | | |
| $a$ | semi-axis of an ellipsoid [m] | 3.2 |
| $a$ | equivalent-sphere radius [m] | 3.2 |
| $a$ | radius of a sphere [m] | 3.3 |
| $a$ | typical particle dimension [m] | 5.8.2 |
| $a$ | parameter of the gamma size distribution [m] | 5.10.1 |
| $a$ | horizontal semi-axis of a spheroid [m] | 5.11.2 |
| $a$ | semi-major particle dimension [m] | 7.1 |
| $\mathbf{a}$ | column vector of expansion coefficients of the incident electric field [V m$^{-1}$] | 5.1 |
| $a_n$ | Lorenz–Mie coefficients [–] | 5.2.2 |
| $a_{mn}$ | expansion coefficients of the incident electric field [V m$^{-1}$] | 5.1 |
| $a_j(\Theta)$ | diagonal elements of the normalized Stokes scattering matrix [–] | 4.10 |
| $\mathbf{A}$ | $4\times4$ transformation matrix [–] | 1.3 |
| $\mathbf{A}$ | matrix formed by the translation coefficients $A_{\mu\nu mn}$ [–] | 5.2.4 |
| $\mathbf{A}$ | Mueller matrix of the analyzer [–] | 8.1 |
| $A_{\mu\nu mn}$ | translation coefficients [–] | Appendix C |
| $\overleftrightarrow{A}$ | scattering dyadic [m] | 2.2 |
| $b$ | semi-axis of an ellipsoid [m] | 3.2 |
| $b$ | parameter of the gamma size distribution [–] | 5.10.1 |

| Abbreviation or symbol | Definition and dimension in SI units | Introduced in Section |
|---|---|---|
| $b$ | vertical (rotational) semi-axis of a spheroid [m] | 5.11.2 |
| $\mathbf{b}$ | column vector of the expansion coefficients of the incident electric field [V m$^{-1}$] | 5.1 |
| $b_n$ | Lorenz–Mie coefficients [–] | 5.2.2 |
| $b_{mn}$ | expansion coefficients of the incident electric field [V m$^{-1}$] | 5.1 |
| $b_j(\Theta)$ | off-diagonal elements of the normalized Stokes scattering matrix [–] | 4.10 |
| $\mathbf{B}$ | magnetic induction [Wb m$^{-2}$] | 1.1 |
| $\mathbf{B}$ | $4\times 4$ transformation matrix [–] | 1.3 |
| $\mathbf{B}$ | matrix formed by the translation coefficients $B_{\mu\nu mn}$ [–] | 5.2.4 |
| BCGM | bi-conjugate gradient method | 6.5 |
| $B_{\mu\nu mn}$ | translation coefficients [–] | Appendix C |
| $\mathbf{B}_{mn}(\vartheta)$ | vector angular functions [–] | Appendix C |
| $\mathbf{B}_{mn}(\vartheta, \varphi)$ | vector angular functions [–] | Appendix C |
| $c$ | speed of light in a vacuum [m s$^{-1}$] | 1.2 |
| $c$ | semi-axis of an ellipsoid [m] | 3.2 |
| $\langle\cos\Theta\rangle$ | asymmetry parameter [–] | 2.8 |
| $\mathbf{C}$ | circular-polarization amplitude scattering matrix [m] | 4.12 |
| CGM | conjugate gradient method | 6.2 |
| $C_{++}, C_{+-}, C_{-+}, C_{--}$ | elements of the circular-polarization amplitude scattering matrix [m] | 4.12 |
| $C_{\mathrm{abs}}$ | absorption cross section [m$^2$] | 2.8 |
| $C_{\mathrm{ext}}$ | extinction cross section [m$^2$] | 2.8 |
| $C_{\mathrm{pr}}$ | radiation-pressure cross section [m$^2$] | 2.9 |
| $C_{\mathrm{sca}}$ | scattering cross section [m$^2$] | 2.8 |
| $C^{nm}_{n_1m_1\,n_2m_2}$ | Clebsch–Gordan coefficients [–] | Appendix D |
| $\mathbf{C}_{mn}(\vartheta)$ | vector angular functions [–] | Appendix C |
| $\mathbf{C}_{mn}(\vartheta, \varphi)$ | vector angular functions [–] | Appendix C |
| $\langle C_{\mathrm{abs}}\rangle$ | average absorption cross section per particle [m$^2$] | 3.1 |
| $\langle C_{\mathrm{b}}\rangle$ | average backscattering cross section per particle [m$^2$] | 9.5 |
| $\langle C_{\mathrm{ext}}\rangle$ | average extinction cross section per particle [m$^2$] | 3.1 |
| $\langle C_{\mathrm{pr}}\rangle$ | average radiation-pressure cross section per particle [m$^2$] | 3.1 |

| Abbreviation or symbol | Definition and dimension in SI units | Introduced in Section |
|---|---|---|
| $\langle C_{\text{sca}} \rangle$ | average scattering cross section per particle [m$^2$] | 3.1 |
| $\dfrac{\mathrm{d}C_{\text{sca}}}{\mathrm{d}\Omega}$ | differential scattering cross section [m$^2$] | 2.8 |
| $d_n$ | coefficient [–] | Appendix C |
| $d^s_{mn}(\vartheta)$ | Wigner $d$-functions [–] | Appendix B |
| $D$ | diameter of a circular cylinder [m] | 5.11.2 |
| $D$ | diameter of the host sphere [m] | 10.10 |
| $\mathbf{D}$ | electric displacement [C m$^{-2}$] | 1.1 |
| $\mathsf{D}$ | $4\times4$ transformation matrix [–] | 1.3 |
| DDA | discrete dipole approximation | 6.5 |
| $D^s_{mn}(\vartheta)$ | Wigner $D$-functions [–] | Appendix B |
| e | base of natural logarithms [–] | |
| $\mathbf{E}$ | electric field [V m$^{-1}$] | 1.1 |
| $\mathsf{E}$ | unit matrix [–] | 5.2.3 |
| EBCM | extended boundary condition method | 5.8.1 |
| $E_\vartheta$, $E_\varphi$ | spherical coordinate components of the electric field vector [V m$^{-1}$] | 1.3 |
| $E_+$, $E_-$ | circular components of the electric field vector [V m$^{-1}$] | 4.12 |
| $\mathbf{F}$ | radiation force [N] | 2.9 |
| $\mathsf{F}$ | Stokes scattering matrix [m$^2$] | Introduction to Chapter 4 |
| FDM | finite-difference method | 6.2 |
| FDTDM | finite-difference time-domain method | 6.3 |
| FEM | finite-element method | 6.2 |
| FFT | fast Fourier transform | 6.5 |
| FIEM | Fredholm integral equation method | 6.5 |
| $F_{ij}$ | elements of the Stokes scattering matrix [m$^2$] | 4.1 |
| $\tilde{\mathsf{F}}$ | normalized Stokes scattering matrix [–] | 4.10 |
| $\tilde{\mathsf{F}}^{\text{CP}}$ | normalized circular-polarization scattering matrix [–] | 4.12 |
| $\langle \mathbf{F} \rangle$ | average radiation force per particle [N] | 4.6 |
| $\langle \mathsf{F} \rangle$ | average Stokes scattering matrix per particle [m$^2$] | 4.2 |
| $g(\mathbf{r}, \mathbf{r}')$ | scalar Green's function [m$^{-1}$] | 2.1 |
| $g^s_{pq}$ | expansion coefficients [–] | 4.12 |
| $G$ | area of the particle geometrical projection [m$^2$] | 2.8 |
| GE | Gaussian elimination | 6.2 |

| Abbreviation or symbol | Definition and dimension in SI units | Introduced in Section |
|---|---|---|
| GPMM | generalized PMM | 6.4 |
| $\overleftrightarrow{G}$ | free space dyadic Green's function [$m^{-1}$] | 2.1 |
| $\langle G \rangle$ | average area of the geometrical projection per particle [$m^2$] | 4.6 |
| $h_n^{(1)}$ | Hankel functions of the first kind [–] | Appendix C |
| $h_n^{(2)}$ | Hankel functions of the second kind [–] | Appendix C |
| $\mathbf{H}$ | magnetic field [A $m^{-1}$] | 1.1 |
| i | $\sqrt{-1}$ [–] | 1.1 |
| $I$ | intensity (irradiance) [W $m^{-2}$] | 1.2 |
| $I$ | monochromatic specific intensity (radiance) [W $rad^{-1}$ s $m^{-2}$ $sr^{-1}$] | 3.4 |
| $I$ | first Stokes parameter [W $m^{-2}$] | 1.3 |
| $\mathbf{I}$ | $4\times1$ Stokes column vector [W $m^{-2}$] | 1.3 |
| $\mathbf{I}$ | $4\times1$ monochromatic specific intensity column vector [W $rad^{-1}$ s $m^{-2}$ $sr^{-1}$] | 3.4 |
| Im | imaginary part | |
| $I_2, I_0, I_{-0}, I_2$ | elements of the circular-polarization column vector [W $m^{-2}$] | 1.3 |
| $I_\mathrm{b}$ | Planck blackbody energy distribution [W $rad^{-1}$ s $m^{-2}$ $sr^{-1}$] | 2.10 |
| $\mathbf{I}_\mathrm{b}$ | $4\times1$ blackbody Stokes column vector [W $rad^{-1}$ s $m^{-2}$ $sr^{-1}$] | 2.10 |
| $I_\mathrm{v}, I_\mathrm{h}$ | first and second elements of the modified Stokes column vector [W $m^{-2}$] | 1.3 |
| $\mathbf{I}^\mathrm{CP}$ | $4\times1$ circular-polarization column vector [W $m^{-2}$] | 1.3 |
| $\mathbf{I}^\mathrm{MS}$ | $4\times1$ modified Stokes column vector [W $m^{-2}$] | 1.3 |
| $\overleftrightarrow{I}$ | identity dyadic [–] | 2.1 |
| $j_l$ | spherical Bessel functions of the first kind [–] | Appendix A |
| $\mathbf{J}$ | current density [A $m^{-2}$] | 1.1 |
| $\mathbf{J}$ | $4\times1$ coherency column vector [W $m^{-2}$] | 1.3 |
| $\mathbf{J}_S$ | surface current density [A $m^{-1}$] | 1.1 |
| $k = k_\mathrm{R} + \mathrm{i}k_\mathrm{I}$ | (complex) wave number [$m^{-1}$] | 1.2 |
| $\mathbf{k} = \mathbf{k}_\mathrm{R} + \mathrm{i}\mathbf{k}_\mathrm{I}$ | (complex) wave vector [$m^{-1}$] | 1.2 |
| $k_1$ | wave number in the exterior region [$m^{-1}$] | 2.1 |
| $k_2$ | wave number in the interior region [$m^{-1}$] | 2.1 |
| $L$ | length of a cylinder [m] | 5.11.2 |
| $\mathbf{L}, \mathbf{L}^\mathrm{MS}, \mathbf{L}^\mathrm{CP}$ | $4\times4$ rotation matrices [–] | 1.5 |
| $m = m_\mathrm{R} + \mathrm{i}m_\mathrm{I}$ | (complex) refractive index relative to vacuum or surrounding medium [–] | 1.2, 2.1 |

| Abbreviation or symbol | Definition and dimension in SI units | Introduced in Section |
|---|---|---|
| $\mathbf{M}$ | magnetization [A m$^{-1}$] | 1.1 |
| $\mathsf{M}$ | Mueller matrix of the modulator [–] | 8.1 |
| ME-GPMM | multiple-expansion GPMM | 6.4 |
| MOM | method of moments | 6.5 |
| $\mathbf{M}_{mn}$ | vector spherical wave functions [–] | Appendix C |
| $n(a)$ | size distribution function [m$^{-1}$] | 3.2 |
| $n_0$ | particle number density [m$^{-3}$] | 3.4 |
| $n_{\max}$ | parameter specifying the size of the $T$ matrix [–] | 5.8.4 |
| $n_{\max}$ | maximal order of Lorenz–Mie coefficients [–] | 5.10.1 |
| $\hat{\mathbf{n}}$ | unit vector [–] | 1.1 |
| $\hat{\mathbf{n}}^{\text{inc}}$ | unit vector in the incidence direction [–] | 2.2 |
| $\hat{\mathbf{n}}^{\text{sca}}$ | unit vector in the scattering direction [–] | 2.2 |
| $N$ | number of particles [–] | 3.1 |
| $N_{\text{G}}$ | number of Gaussian division points in Eq. (5.202) [–] | 5.8.3 |
| $\mathbf{N}_{mn}$ | vector spherical wave functions [–] | Appendix C |
| $p$ | phase function [–] | 2.8 |
| $p(x)$ | probability density function [dimension is that of $x^{-1}$] | 3.2 |
| $\mathsf{p}$ | column vector of the expansion coefficients of the scattered electric field [V m$^{-1}$] | 5.1 |
| $p_{mn}$ | expansion coefficients of the scattered electric field [V m$^{-1}$] | 5.1 |
| $P$ | degree of (elliptical) polarization [–] | 1.6 |
| $\mathbf{P}$ | electric polarization [C m$^{-2}$] | 1.1 |
| $\mathsf{P}$ | Mueller matrix of the polarizer [–] | 8.1 |
| PMM | point-matching method | 6.4 |
| $P_{\text{C}}$ | degree of circular polarization [–] | 1.6 |
| $P_{\text{L}}$ | degree of linear polarization [–] | 1.6 |
| $P_Q$ | degree of linear polarization [–] | 1.6 |
| $P_l(x)$ | Legendre polynomials [–] | Appendix A |
| $P_l^m(x)$ | associated Legendre functions [–] | Appendix A |
| $P_{mn}^s(x)$ | generalized spherical functions [–] | Appendix B |
| $P_q^{(a,b)}(x)$ | Jacobi polynomials [–] | Appendix B |
| $\mathbf{P}_{mn}(\vartheta)$ | vector angular functions [–] | Appendix C |
| $\mathbf{P}_{mn}(\vartheta, \varphi)$ | vector angular functions [–] | Appendix C |
| $\mathsf{q}$ | $2\times 2$ transformation matrix [–] | 4.12 |
| $\mathsf{q}$ | column vector of the expansion coefficients of the scattered electric field [V m$^{-1}$] | 5.1 |

| Abbreviation or symbol | Definition and dimension in SI units | Introduced in Section |
|---|---|---|
| $q_{mn}$ | expansion coefficients of the scattered electric field [V m$^{-1}$] | 5.1 |
| $Q$ | second Stokes parameter [W m$^{-2}$] | 1.3 |
| $Q$ | second element of the monochromatic specific intensity column vector [W rad$^{-1}$ s m$^{-2}$ sr$^{-1}$] | 3.4 |
| $\mathbf{Q}$ | $Q$ matrix [–] | 5.8.1 |
| $\mathbf{Q}$ | Mueller matrix of the quarter-wave plate [–] | 8.1 |
| $Q_{\mathrm{abs}}$ | efficiency factor for absorption [–] | 2.8 |
| $Q_{\mathrm{b}}$ | backscattering efficiency factor [–] | 9.5 |
| $Q_{\mathrm{ext}}$ | efficiency factor for extinction [–] | 2.8 |
| $Q_{\mathrm{pr}}$ | efficiency factor for radiation pressure [–] | 2.9 |
| $Q_{\mathrm{sca}}$ | efficiency factor for scattering [–] | 2.8 |
| $r$ | distance from the origin of a coordinate system [m] | 2.2 |
| $r$ | radius of a spherical particle [m] | 5.10.1 |
| $\mathbf{r}$ | radius (position) vector [m] | 1.1 |
| $r_0$ | parameter specifying the size of Chebyshev and generalized Chebyshev particles [m] | 5.11.2, 5.12 |
| $r_1, r_2$ | parameters of the power law and modified power law size distributions [m] | 5.10.1 |
| $r_{12}$ | distance between two coordinate system origins [m] | 2.11 |
| $r_{12}$ | distance between the components of a two-sphere cluster [m] | 5.13 |
| $r_c$ | parameter of the modified gamma size distribution [m] | 5.10.1 |
| $r_{\mathrm{eff}}$ | effective radius of a size distribution [m] | 5.10.1 |
| $r_g$ | parameter of the log normal size distribution [m] | 5.10.1 |
| $r_{g1}, r_{g2}$ | parameters of the modified bimodal log normal size distribution [m] | 5.10.1 |
| $r_{\mathrm{max}}$ | maximal radius of a size distribution [m] | 5.10.1 |
| $r_{\mathrm{min}}$ | minimal radius of a size distribution [m] | 5.10.1 |
| $r_{\mathrm{s}}$ | surface-equivalent-sphere radius [m] | 5.8.4 |
| $r_{\mathrm{v}}$ | volume-equivalent-sphere radius [m] | 5.11.2 |
| $r_>$ | radius of the smallest sphere circumscribing a particle [m] | 5.1 |
| $r_<$ | radius of the largest inscribed sphere of a nonspherical particle [m] | 5.8.1 |

| Abbreviation or symbol | Definition and dimension in SI units | Introduced in Section |
|---|---|---|
| $\hat{\mathbf{r}}$ | unit vector in the direction of **r** [–] | 2.2 |
| Re | real part | |
| Rg | stands for "regular" | Appendix C |
| RH | Rayleigh hypothesis | 6.4 |
| $R_{\mathrm{be}}$ | backscatter-to-extinction ratio [–] | 9.5 |
| $R_{\mathrm{eb}}$ | extinction-to-backscatter ratio [–] | 9.5 |
| $S$ | surface area [m$^2$] | 1.1 |
| $\mathbf{S}$ | Poynting vector [W m$^{-2}$] | 1.1 |
| $\mathsf{S}$ | amplitude scattering matrix [m] | 2.2 |
| $\mathsf{S}$ | $S$ matrix [–] | 5.2.3 |
| $S_{ij}$ | elements of the amplitude scattering matrix [m] | 2.2 |
| SIEM | surface integral equation method | 6.5 |
| SVM | separation of variables method | 6.1 |
| $\overleftrightarrow{S}$ | scattering tensor [–] | 2.3 |
| $\langle\mathbf{S}\rangle$ | time-averaged Poynting vector [W m$^{-2}$] | 1.1 |
| $t$ | time [s] | 1.1 |
| $t$ | crystal distortion parameter [–] | 10.7 |
| $\mathsf{t}$ | $2\times 2$ transformation matrix [–] | 2.4 |
| $T$ | absolute temperature [K] | 2.10 |
| $\mathsf{T}$ | $T$ matrix [–] | 5.1 |
| $T^{kl}_{mnm'n'}$ | elements of the $T$ matrix [–] | 5.1 |
| $T_n(\cos\vartheta)$ | Chebyshev polynomial of degree $n$ [–] | 5.11.2 |
| $\overleftrightarrow{T}$ | dyadic transition operator [m$^{-5}$] | 2.1 |
| $\overleftrightarrow{T}_{\mathrm{M}}$ | Maxwell stress tensor [N m$^{-2}$] | 2.9 |
| $U$ | third Stokes parameter [W m$^{-2}$] | 1.3 |
| $U$ | third element of the monochromatic specific intensity column vector [W rad$^{-1}$ s m$^{-2}$ sr$^{-1}$] | 3.4 |
| $v$ | phase velocity [m s$^{-1}$] | 1.2 |
| $\mathbf{v}$ | velocity vector [m s$^{-1}$] | 1.1 |
| $v_{\mathrm{eff}}$ | effective variance of a size distribution [–] | 5.10.1 |
| $V$ | volume [m$^3$] | 1.1 |
| $V$ | fourth Stokes parameter [W m$^{-2}$] | 1.3 |
| $V$ | fourth element of the monochromatic specific Intensity column vector [W rad$^{-1}$ s m$^{-2}$ sr$^{-1}$] | 3.4 |
| VIEM | volume integral equation method | 6.5 |
| $V_{\mathrm{EXT}}$ | exterior region [m$^3$] | 2.1 |
| $V_{\mathrm{INT}}$ | interior region [m$^3$] | 2.1 |
| $W$ | power [W] | 1.1 |

| Abbreviation or symbol | Definition and dimension in SI units | Introduced in Section |
|---|---|---|
| $x$ | size parameter [–] | 5.8.2 |
| $x_a, x_b$ | size parameters along spheroid semi-axes [m] | 5.11.7 |
| $x_{\text{eff}}$ | effective size parameter of a size distribution [–] | 9.2 |
| $x_{\text{s}}$ | surface-equivalent-sphere size parameter [–] | 5.8.4 |
| $x_{\text{v}}$ | volume-equivalent-sphere size parameter [–] | 10.4 |
| $y_l$ | spherical Bessel functions of the second kind [–] | Appendix C |
| $Y_{lm}(\hat{\mathbf{r}})$ | spherical harmonics [–] | Appendix A |
| $\mathbf{Z}$ | Stokes phase matrix [m$^2$] | 2.6 |
| $Z_{ij}$ | elements of the Stokes phase matrix [m$^2$] | 2.6 |
| $\mathbf{Z}^{\text{CP}}$ | circular-polarization phase matrix [m$^2$] | 2.6 |
| $\mathbf{Z}^{J}$ | coherency phase matrix [m$^2$] | 2.6 |
| $\mathbf{Z}^{\text{MS}}$ | modified Stokes phase matrix [m$^2$] | 2.6 |
| $\tilde{\mathbf{Z}}$ | normalized Stokes phase matrix [–] | 4.10 |
| $\tilde{\mathbf{Z}}^{\text{CP}}$ | normalized circular-polarization phase matrix [–] | 4.12 |
| $\langle\mathbf{Z}\rangle$ | average Stokes phase matrix per particle [m$^2$] | 3.1 |
| **Greek symbols** | | |
| $\alpha$ | absorption coefficient [m$^{-1}$] | 1.2 |
| $\alpha$ | first Euler angle [rad] | 2.4 |
| $\alpha$ | parameter of the modified gamma size distribution [–] | 5.10.1 |
| $\alpha$ | exponent of the modified power law size distribution [–] | 5.10.1 |
| $\alpha$ | local incidence angle [rad] | 9.4 |
| $\boldsymbol{\alpha}$ | $3\times2$ transformation matrix [–] | 2.4 |
| $\alpha_j^s$ | expansion coefficients [–] | 4.11 |
| $\beta$ | ellipticity angle of the polarization ellipse [rad] | 1.4 |
| $\beta$ | second Euler angle [rad] | 2.4 |
| $\boldsymbol{\beta}$ | $3\times3$ transformation matrix [–] | 2.4 |
| $\beta_j^s$ | expansion coefficients [–] | 4.11 |
| $\bar{\beta}$ | backscattered fraction for isotropically incident radiation [–] | 10.3 |
| $\gamma$ | third Euler angle [rad] | 2.4 |
| $\gamma$ | parameter of the modified gamma size distribution [–] | 5.10.1 |
| $\gamma$ | parameter of the modified bimodal log normal size distribution [–] | 5.10.1 |

| Abbreviation or symbol | Definition and dimension in SI units | Introduced in Section |
|---|---|---|
| $\gamma_{mn}$ | coefficient [–] | Appendix C |
| $\gamma'_{mn}$ | coefficient [–] | Appendix C |
| $\mathbf{\Gamma}$ | radiation torque [N m] | 2.9 |
| $\delta(x)$ | Dirac delta function [dimension is that of $x^{-1}$] | 2.1 |
| $\delta(\mathbf{r})$ | three-dimensional Dirac delta function [m$^{-3}$] | 2.1 |
| $\delta(\hat{\mathbf{n}})$ | solid-angle Dirac delta function [–] | 2.3 |
| $\delta_{\mathrm{C}}$ | circular backscattering depolarization ratio [–] | 10.2 |
| $\delta_{\mathrm{L}}$ | linear backscattering depolarization ratio [–] | 4.9 |
| $\delta_{mn}$ | Kronecker delta [–] | Appendix B |
| $\mathbf{\Delta}$ | $4\times4$ unit matrix [–] | 4.4 |
| $\mathbf{\Delta}_3$ | $4\times4$ transformation matrix [–] | 2.6 |
| $\mathbf{\Delta}_{23}$ | $4\times4$ transformation matrix [–] | 2.6 |
| $\mathbf{\Delta}_{34}$ | $4\times4$ transformation matrix [–] | 4.3 |
| $\mathbf{\Delta}^{\mathrm{CP}}$ | $4\times4$ transformation matrix [–] | 4.12 |
| $\mathbf{\Delta}^{\mathrm{MS}}$ | $4\times4$ transformation matrix [–] | 2.6 |
| $\varepsilon$ | electric permittivity [F m$^{-1}$] | 1.1 |
| $\varepsilon$ | aspect ratio of a nonspherical particle [–] | 3.2 |
| $\varepsilon_0$ | electric permittivity of free space [F m$^{-1}$] | 1.1 |
| $\varepsilon_1$ | electric permittivity of the surrounding medium [F m$^{-1}$] | 2.1 |
| $\zeta$ | orientation angle of the polarization ellipse [rad] | 1.4 |
| $\eta$ | rotation angle [rad] | 1.5 |
| $\vartheta$ | polar (zenith) angle [rad] | 1.3 |
| $\hat{\boldsymbol{\vartheta}}$ | unit vector in the $\vartheta$ direction [–] | 1.3 |
| $\Theta$ | scattering angle [rad] | 2.8 |
| $\mathbf{K}$ | Stokes extinction matrix [m$^2$] | 2.7 |
| $K_{ij}$ | elements of the Stokes extinction matrix [m$^2$] | 2.7 |
| $\mathbf{K}^{\mathrm{CP}}$ | circular-polarization extinction matrix [m$^2$] | 2.7 |
| $\mathbf{K}_{\mathrm{e}}$ | $4\times1$ Stokes emission column vector [W rad$^{-1}$ s sr$^{-1}$] | 2.10 |
| $\mathbf{K}^J$ | coherency extinction matrix [m$^2$] | 2.7 |
| $\mathbf{K}^{\mathrm{MS}}$ | modified Stokes extinction matrix [m$^2$] | 2.7 |
| $\langle\mathbf{K}\rangle$ | average Stokes extinction matrix per particle [m$^2$] | 3.1 |
| $\langle\mathbf{K}_{\mathrm{e}}\rangle$ | average Stokes emission column vector per particle [W rad$^{-1}$ s sr$^{-1}$] | 3.1 |
| $\lambda$ | free-space wavelength [m] | 1.2 |
| $\lambda_1$ | wavelength in the surrounding medium [m] | 3.4 |

| Abbreviation or symbol | Definition and dimension in SI units | Introduced in Section |
|---|---|---|
| $\mu$ | magnetic permeability [H m$^{-1}$] | 1.1 |
| $\mu_0$ | magnetic permeability of free space [H m$^{-1}$] | 1.1 |
| $\xi$ | deformation parameter for Chebyshev particles [–] | 5.11.2 |
| $\xi_{mn}$ | coefficient [–] | Appendix B |
| $\pi$ | pi [–] | |
| $\varpi$ | single-scattering albedo [–] | 2.8 |
| $\pi_n(\vartheta)$ | angular functions [–] | 5.7 |
| $\pi_{mn}(\vartheta)$ | angular functions [–] | 5.1 |
| $\rho$ | charge density [C m$^{-3}$] | 1.1 |
| $\rho$ | phase shift [–] | 9.1 |
| $\rho$ | phase function ratio [–] | 10.2 |
| $\boldsymbol{\rho}$ | $2\times2$ coherency (density) matrix [W m$^{-2}$] | 1.3 |
| $\rho_S$ | surface charge density [C m$^{-2}$] | 1.1 |
| $\sigma$ | conductivity [$\Omega^{-1}$ m$^{-1}$] | 1.1 |
| $\sigma$ | surface roughness parameter [–] | 10.7 |
| $\sigma_1, \sigma_2$ | rotation angles [rad] | 4.3 |
| $\sigma_g$ | parameter of the log normal size distribution [–] | 5.10.1 |
| $\sigma_{g1}, \sigma_{g2}$ | parameters of the modified bimodal log normal size distribution [–] | 5.10.1 |
| $\langle\sigma_\mathrm{b}\rangle$ | average radar backscattering cross section per particle [m$^2$] | 9.5 |
| $\tau$ | optical path length [–] | 4.13 |
| $\tau$ | optical thickness of the host particle [–] | 10.10 |
| $\tau_n(\vartheta)$ | angular functions [–] | 5.7 |
| $\tau_{mn}(\vartheta)$ | angular functions [–] | 5.1 |
| $\varphi$ | azimuth angle [rad] | 1.3 |
| $\hat{\boldsymbol{\varphi}}$ | unit vector in the $\varphi$ direction [–] | 1.3 |
| $\chi$ | electric susceptibility [–] | 1.1 |
| $\omega$ | angular frequency [rad s$^{-1}$] | 1.1 |
| $\Omega$ | solid angle [sr] | 2.10 |

**Miscellaneous symbols**

| | | |
|---|---|---|
| $x^*$ | complex-conjugate value of $x$ | 1.1 |
| $\langle x\rangle$ | average of $x$ | 1.1 |
| $\lvert x\rvert$ | absolute value of $x$ | |
| $\mathbf{a}\cdot\mathbf{b}$ | dot (scalar) product of vectors **a** and **b** | |
| $\mathbf{a}\times\mathbf{b}$ | vector product of vectors **a** and **b** | |
| $\mathbf{a}\otimes\mathbf{b}$ | dyadic product of vectors **a** and **b** | 2.1 |

| Abbreviation or symbol | Definition and dimension in SI units | Introduced in Section |
|---|---|---|
| $\begin{pmatrix} l & m & n \\ p & q & r \end{pmatrix}$ | Wigner $3j$ symbols | Appendix D |
| $\begin{bmatrix} a & \dots & b \\ \vdots & \ddots & \vdots \\ c & \cdots & d \end{bmatrix}$ | matrix | |
| $\mathbf{A}^{-1}$ | inverse of $\mathbf{A}$ | 1.3 |
| $\mathbf{A}^{\mathrm{T}}$ | transpose of $\mathbf{A}$ | 1.4 |
| $\vec{S}^{\mathrm{T}}$ | transpose of $\vec{S}$ | 2.3 |
| $\nabla$ | gradient | |
| $\in$ | element of | |
| $\cup$ | union of sets | |

# References


Abramowitz, M., and Stegun, I. A., eds. (1964). *Handbook of Mathematical Functions with Formulas, Graphs and Mathematical Tables* (National Bureau of Standards, Washington, D. C.).

Acquista, C. (1976). Light scattering by tenuous particles: A generalization of the Rayleigh–Gans–Rocard approach. *Appl. Opt.* **15**, 2932–2936.

Aden, A. L., and Kerker, M. (1951). Scattering of electromagnetic waves from two concentric spheres. *J. Appl. Phys.* **22**, 1242–1246.

Agarwal, G. S. (1976). Relation between Waterman's extended boundary condition and the generalized extinction theorem. *Phys. Rev. D* **14**, 1168–1171.

Alpers, M., Gerding, M., Höffner, J., and Schneider, J. (2001). Multiwavelength lidar observation of a strange noctilucent cloud at Kühlungsborn, Germany (54º N). *J. Geophys. Res.* **106**, 7945–7953.

Al-Rizzo, H. M., and Tranquilla, J. M. (1995a). Electromagnetic scattering from dielectrically coated axisymmetric objects using the generalized point-matching technique. I. Theoretical formulation. *J. Comput. Phys.* **119**, 342–355.

Al-Rizzo, H. M., and Tranquilla, J. M. (1995b). Electromagnetic scattering from dielectrically coated axisymmetric objects using the generalized point-matching technique (GPMT). II. Numerical results and comparisons. *J. Comput. Phys.* **119**, 356–373.

Al-Rizzo, H. M., and Tranquilla, J. M. (1995c). Electromagnetic wave scattering by highly elongated and geometrically composite objects of large size parameters: The generalized multipole technique. *Appl. Opt.* **34**, 3502–3521.

Amic, E., Luck, J. M., and Nieuwenhuizen, Th. M. (1997). Multiple Rayleigh scattering of electromagnetic waves. *J. Phys. I (Paris)* **7**, 445–483.

Anderson, R. (1992). Measurement of Mueller matrices. *Appl. Opt.* **31**, 11–13.

Anderson, T. L., Covert, D. S., Marshall, S. F., *et al*. (1996). Performance characteristics of a

high-sensitivity, three-wavelength, total scatter/backscatter nephelometer. *J. Atmos. Oceanic Technol.* **13**, 967–986.

Apresyan, L. A., and Kravtsov, Yu. A. (1996). *Radiation Transfer. Statistical and Wave Aspects* (Gordon and Breach, Basel).

Arfken, G. B., and Weber, H. J. (1995). *Mathematical Methods for Physicists* (Academic Press, San Diego).

Arnott, W. P., and Marston, P. L. (1991). Unfolded optical glory of spheroids: backscattering of laser light from freely rising spheroidal air bubbles in water. *Appl. Opt.* **30**, 3429–3442.

Asano, S. (1979). Light scattering properties of spheroidal particles. *Appl. Opt.* **18**, 712–723.

Asano, S. (1983). Light scattering by horizontally oriented spheroidal particles. *Appl. Opt.* **22**, 1390–1396.

Asano, S., and Sato, M. (1980). Light scattering by randomly oriented spheroidal particles. *Appl. Opt.* **19**, 962–974.

Asano, S., and Yamamoto, G. (1975). Light scattering by a spheroidal particle. *Appl. Opt.* **14**, 29–49.

Ashkin, A. (2000). History of optical trapping and manipulation of small-neutral particle, atoms, and molecules. *IEEE J. Select. Topics Quant. Electron.* **6**, 841–856.

Ashkin, A., and Dziedzic, J. (1980). Observation of light scattering from nonspherical particles using optical levitation. *Appl. Opt.* **19**, 660–668.

Astafieva, L. G., and Babenko, V. A. (1999). Heating of a spheroidal particle by intense laser radiation. *J. Quant. Spectrosc. Radiat. Transfer* **63**, 459–468.

Auger, J.-C., Stout, B., Barrera, R. G., and Curiel, F. (2001). Scattering properties of rutile pigments located eccentrically within microvoids. *J. Quant. Spectrosc. Radiat. Transfer* **70**, 675–695.

Aydin, K. (2000). Centimeter and millimeter wave scattering from nonspherical hydrometeors. In *Light Scattering by Nonspherical Particles: Theory, Measurements, and Applications*, eds. M. I. Mishchenko, J. W. Hovenier, and L. D. Travis, pp. 451–479 (Academic Press, San Diego).

Aydin, K., and Tang, C. (1997a). Millimeter wave radar scattering from model ice crystal distributions. *IEEE Trans. Geosci. Remote Sens.* **35**, 140–146.

Aydin, K., and Tang, C. (1997b). Relationships between IWC and polarimetric radar measurands at 94 and 220 GHz for hexagonal columns and plates. *J. Atmos. Oceanic Technol.* **14**, 1055–1063.

Aydin, K., and Walsh, T. M. (1999). Millimeter wave scattering from spatial and planar bullet rosettes. *IEEE Trans. Geosci. Remote Sens.* **37**, 1138–1150.

Azzam, R. M. A., and Bashara, N. M. (1977). *Ellipsometry and Polarized Light* (North-Holland, Amsterdam).

Babenko, V. A., Astafyeva, L. G., and Kuzmin, V. N. (2003). *Electromagnetic Scattering in Disperse Media. Inhomogeneous and Anisotropic Particles* (Praxis, Chichester, UK).

Bacon, N. J., and Swanson, B. D. (2000). Laboratory measurements of light scattering by single levitated ice crystals. *J. Atmos. Sci.* **57**, 2094–2104.

Bantges, R. J., Russell, J. E., and Haigh, J. D. (1999). Cirrus cloud top-of-atmosphere radiance spectra in the thermal infrared. *J. Quant. Spectrosc. Radiat. Transfer* **63**, 487–498.

Barabanenkov, Yu. N., Kravtsov, Yu. A., Ozrin, V. D., and Saichev, A. I. (1991). Enhanced backscattering in optics. In *Progress in Optics XXIX*, ed. E. Wolf, pp. 65–197 (Elsevier, Amsterdam).

Baran, A. J., Foot, J. S., and Mitchell, D. L. (1998). Ice-crystal absorption: a comparison between theory and implications for remote sensing. *Appl. Opt.* **37**, 2207–2215.

Baran, A. J., Francis, P. N., Havemann, S., and Yang, P. (2001a). A study of the absorption and extinction properties of hexagonal ice columns and plates in random and preferred orientations, using exact *T*-matrix theory and aircraft observations of cirrus. *J. Quant. Spectrosc. Radiat. Transfer* **70**, 505–518.

Baran, A. J., Yang, P., and Havemann, S. (2001b). Calculation of the single-scattering properties of randomly oriented hexagonal ice columns: a comparison of the *T*-matrix and the finite-difference time-domain methods. *Appl. Opt.* **40**, 4376–4386.

Barber, P. W. (1977). Resonance electromagnetic absorption by nonspherical dielectric objects. *IEEE Trans. Microwave Theory Tech.* **25**, 373–381.

Barber, P. W., and Hill, S. C. (1990). *Light Scattering by Particles: Computational Methods* (World Scientific, Singapore).

Barber, P. W., and Wang, D.-S. (1978). Rayleigh–Gans–Debye applicability to scattering by nonspherical particles. *Appl. Opt.* **17**, 797–803.

Barber, P., and Yeh, C. (1975). Scattering of electromagnetic waves by arbitrarily shaped dielectric bodies. *Appl. Opt.* **14**, 2864–2872.

Barber, P. W., Miller, E. K., and Sarkar, T. K., eds. (1994). Feature issue on Scattering by Three-Dimensional Objects. *J. Opt. Soc. Am. A* **11**, 1379–1545.

Barkey, B., and Liou, K. N. (2001). Polar nephelometer for light-scattering measurements of ice crystals. *Opt. Lett.* **26**, 232–234.

Barthel, H., Sachweh, B., and Ebert, F. (1998). Measurement of airborne mineral fibers using a new differential light scattering device. *Meas. Sci. Technol.* **9**, 210–220.

Barton, J. P., and Alexander, D. R. (1991). Electromagnetic fields for an irregularly shaped, near-spherical particle illuminated by a focused laser beam. *J. Appl. Phys.* **69**, 7973–7986.

Barton, J. P., Alexander, D. R., and Schaub, S. A. (1989). Theoretical determination of net radiation force and torque for a spherical particle illuminated by a focused laser beam. *J. Appl. Phys.* **66**, 4594–4602.

Bates, R. H. T. (1975). Analytic constraints on electromagnetic field computations. *IEEE Trans. Microwave Theory Tech.* **23**, 605–623.

Bates, R. H. T., and Wall, D. J. N. (1977). Null field approach to scalar diffraction. *Phil. Trans. Roy. Soc. London A* **287**, 45–114.

Battaglia, A., Muinonen, K., Nousiainen, T., and Peltoniemi, J. I. (1999). Light scattering by Gaussian particles: Rayleigh-ellipsoid approximation. *J. Quant. Spectrosc. Radiat. Transfer* **63**, 277–303.

Battaglia, A., Prodi, F., and Sturniolo, O. (2001). Radar and scattering parameters through falling hydrometeors with axisymmetric shapes. *Appl. Opt.* **40**, 3092–3100.

Battan, L. J. (1973). *Radar Observation of the Atmosphere* (University of Chicago Press, Chicago).

Bayvel, L. P., and Jones, A. R. (1981). *Electromagnetic Scattering and its Applications* (Applied Science Publishers, London).

Beardsley, G. F. (1968). Mueller scattering matrix of sea water. *J. Opt. Soc. Am.* **58**, 52–57.

Berenger, J.-P. (1996). Three-dimensional perfectly matched layer for the absorption of electromagnetic waves. *J. Comput. Phys.* **127**, 363–379.

Berne, B. J., and Pecora, R. (2000). *Dynamic Light Scattering with Applications to Chemistry, Biology, and Physics* (Dover Publications, Mineola, NY).

Beyerle, G., Deckelmann, H., Neuber, R., *et al.* (2001). Occurrence of solid particles in the winter polar stratosphere above the nitric acid trihydrate coexistence temperature inferred from ground-based polarization lidar observations at Ny-Ålesund, Spitsbergen. *J. Geophys. Res.* **106**, 2979–2992.

Bhandari, R. (1985). Scattering coefficients for a multilayered sphere: analytic expressions and algorithms. *Appl. Opt.* **24**, 1960–1967.

Bickel, W. S., Davidson, J. F., Huffman, D. R., and Kilkson, R. (1976). Application of polarization effects in light scattering: a new biophysical tool. *Proc. Nat. Acad. Sci. USA* **73**, 486–490.

Biedenharn, L. C., and Louck, J. D. (1981). *Angular Momentum in Quantum Physics: Theory and Application* (Addison-Wesley, Reading, MA).

Bohren, C. F. (1974). Light scattering by an optically active sphere. *Chem. Phys. Lett.* **29**, 458–462.

Bohren, C. F., and Gilra, D. P. (1979). Extinction by a spherical particle in an absorbing medium. *J. Colloid Interface Sci.* **72**, 215–221.

Bohren, C. F., and Huffman, D. R. (1983). *Absorption and Scattering of Light by Small Particles* (John Wiley & Sons, New York).

Bohren, C. F., and Singham, S. B. (1991). Backscattering by nonspherical particles: a review of methods and suggested new approaches. *J. Geophys. Res.* **96**, 5269–5277.

Borghese, F., Denti, P., Toscano, G., and Sindoni, O. I. (1979). Electromagnetic scattering by a cluster of spheres. *Appl. Opt.* **18**, 116–120.

Borghese, F., Denti, P., Saija, R., *et al*. (1984). Multiple electromagnetic scattering from a cluster of spheres. I. Theory. *Aerosol Sci. Technol.* **3**, 227–235.

Borghese, F., Denti, P., Saija, R., and Sindoni, O. I. (1992). Optical properties of spheres containing a spherical eccentric inclusion. *J. Opt. Soc. Am. A* **9**, 1327–1335.

Borghese, F., Denti, P., and Saija, R. (1994). Optical properties of spheres containing several spherical inclusions. *Appl. Opt.* **33**, 484–493. (Errata: **34**, 5556 (1995).)

Borghese, F., Denti, P., Saija, R., *et al*. (2001). Optical properties of a dispersion of anisotropic particles with non-randomly distributed orientations. The case of atmospheric ice crystals. *J. Quant. Spectrosc. Radiat. Transfer* **70**, 237–251.

Borghese, F., Denti, P., and Saija, R. (2003). *Scattering from Model Nonspherical Particles. Theory and Applications to Environmental Physics* (Springer Verlag, Berlin).

Born, M., and Wolf, E. (1999). *Principles of Optics* (Cambridge University Press, Cambridge).

Borovoi, A., Naats, E., Oppel, U., and Grishin, I. (2000). Shape characterization of a large nonspherical particle by use of its Fraunhofer diffraction pattern. *Appl. Opt.* **39**, 1989–1997.

Borrmann, S., Solomon, S., Dye, J. E., and Luo, B. (1996). The potential of cirrus clouds for heterogeneous chlorine activation. *Geophys. Res. Lett.* **23**, 2133–2136.

Borrmann, S., Luo, B., and Mishchenko, M. (2000). Application of the T-matrix method to the measurement of aspherical (ellipsoidal) particles with forward scattering optical particle counters. *J. Aerosol Sci.* **31**, 789–799.

Boström, A., Kristensson, G., and Ström, S. (1991). Transformation properties of plane, spherical and cylindrical scalar and vector wave functions. In *Field Representations and Introduction to Scattering*, eds. V. V. Varadan, A. Lakhtakia, and V. K. Varadan, pp. 165–210 (North-Holland, Amsterdam).

Botet, R., Rannou, P., and Cabane, M. (1997). Mean-field approximation of Mie scattering by fractal aggregates of identical spheres. *Appl. Opt.* **36**, 8791–8797.

Bottiger, J. R., Fry, E. S., and Thompson, R. C. (1980). Phase matrix measurements for electromagnetic scattering by sphere aggregates. In *Light Scattering by Irregularly Shaped Particles*, ed. D. W. Schuerman, pp. 283–290 (Plenum Press, New York).

Boucher, O., Schwartz, S. E., Ackerman, T. P., *et al*. (1998). Intercomparison of models representing direct shortwave radiative forcing by sulfate aerosols. *J. Geophys. Res.* **103**, 16 979–16 998.

Bourrely, C., Chiappetta, P., and Lemaire, T. (1989). Electromagnetic scattering by large rotating particles in the eikonal formalism. *Opt. Commun.* **70**, 173–176.

Braak, C. J., de Haan, J. F., van der Mee, C. V. M., *et al.* (2001). Parameterized scattering matrices for small particles in planetary atmospheres. *J. Quant. Spectrosc. Radiat. Transfer* **69**, 585–604.

Braun, C., and Krieger, U. K. (2001). Two-dimensional angular light-scattering in aqueous NaCl single aerosol particles during deliquescence and efflorescence. *Opt. Express* **8**, 314–321.

Bringi, V. N., and Chandrasekar, V. (2001). *Polarimetric Doppler Weather Radar: Principles and Applications* (Cambridge University Press, Cambridge).

Bringi, V. N., and Seliga, T. A. (1977). Scattering from axisymmetric dielectrics or perfect conductors imbedded in an axisymmetric dielectric. *IEEE Trans. Antennas Propag.* **25**, 575–580.

Brink, D. M., and Satchler, G. R. (1993). *Angular Momentum* (Oxford University Press, Oxford).

Brogniez, C., Santer, R., Diallo, B. S., *et al*. (1992). Comparative observations of stratospheric aerosols by ground-based lidar, balloon-borne polarimeter, and satellite solar occultation. *J. Geophys. Res.* **97**, 20 805–20 823.

Brosseau, C. (1998). *Polarized Light: A Statistical Optics Approach* (John Wiley & Sons, New York).

Browell, E. V., Butler, C. F., Ismail, S., *et al.* (1990). Airborne lidar observations in the wintertime Arctic stratosphere: polar stratospheric clouds. *Geophys. Res. Lett*. **17**, 385–388.

Brown, W., ed. (1993). *Dynamic Light Scattering: The Method and Some Applications* (Oxford University Press, Oxford).

Bruning, J. H., and Lo, Y. T. (1971a). Multiple scattering of EM waves by spheres. I. Multipole expansion and ray-optical solutions. *IEEE Trans. Antennas Propag.* **19**, 378–390.

Bruning, J. H., and Lo, Y. T. (1971b). Multiple scattering of EM waves by spheres. II. Numerical and experimental results. *IEEE Trans. Antennas Propag.* **19**, 391–400.

Bruscaglioni, P., Ismaelli, A., and Zaccanti, G. (1993). A note on the definition of scattering cross sections and phase functions for spheres immersed in an absorbing medium. *Waves in Random Media* **3**, 147–156.

Bugaenko, O. I. (1976). Generalized spherical functions in the Mie problem. *Izv. Acad. Sci. USSR, Atmos. Oceanic Phys.* (Engl. transl.) **12**, 366–370.

Buriez, J. C., Vanbauce, C., Parol, F., *et al*. (1997). Cloud detection and derivation of cloud properties from POLDER. *Int. J. Remote Sens.* **18**, 2785–2813.

Burrows, M. L. (1969). Equivalence of the Rayleigh solution and the extended-boundary-condition solution for scattering problems. *Electron. Lett.* **5**, 277–278.

Cai, Q., and Liou, K.-N. (1982). Polarized light scattering by hexagonal ice crystals: theory. *Appl. Opt.* **21**, 3569–3580.

Card, J. B. A., and Jones, A. R. (1999a). An investigation of the use of a CCD camera for the characterisation of irregular particles. *Part. Part. Syst. Charact.* **16**, 135–142.

Card, J. B. A., and Jones, A. R. (1999b). An investigation of the potential of polarized light scattering for the characterization of irregular particles. *J. Phys. D: Appl. Phys.* **32**, 2467–2474.

Carey, L. D., Rutledge, S. A., Ahijevych, D. A., and Keenan, T. D. (2000). Correcting propagation effects in C-band polarimetric radar observations of tropical convection using differential propagation phase. *J. Appl. Meteorol.* **39**, 1405–1433.

Carslaw, K. S., Wirth, M., Tsias, A., *et al.* (1998). Particle microphysics and chemistry in remotely observed mountain polar stratospheric clouds. *J. Geophys. Res.* **103**, 5785–5796.

Chamaillard, K., and Lafon, J.-P. J. (2000). Statistical study of polarization of light scattered by rough grains. I. The method with preliminary results. *Astron. Astrophys.* **363**, 1155–1165.

Chamaillard, K., and Lafon, J.-P. J. (2001). Statistical approach of the effects of roughness on the polarization of light scattered by dust grains. *J. Quant. Spectrosc. Radiat. Transfer* **70**, 519–528.

Chandrasekhar, S. (1960). *Radiative Transfer* (Dover Publications, New York).

Chepfer, H., Brogniez, G., Goloub, P., *et al*. (1999). Observations of horizontally oriented ice crystals in cirrus clouds with POLDER-1/ADEOS-1. *J. Quant. Spectrosc. Radiat. Transfer* **63**, 521–543.

Chew, W. C. (1995). *Waves and Fields in Inhomogeneous Media* (IEEE Press, New York).

Chiappetta, P. (1980). Multiple scattering approach to light scattering by arbitrarily shaped particles. *J. Phys. A: Math. Gen.* **13**, 2101–2108.

Cho, S. K. (1990). *Electromagnetic Scattering* (Springer-Verlag, New York).

Chowdhary, J., Cairns, B., Mishchenko, M., and Travis, L. (2001). Retrieval of aerosol properties over the ocean using multispectral and multiangle photopolarimetric measurements from the Research Scanning Polarimeter. *Geophys. Res. Lett*. **28**, 243–246.

Chuang, C., and Beard, K. V. (1990). A numerical model for the equilibrium shape of electrified raindrops. *J. Atmos. Sci.* **47**, 1374–1389.

Chýlek, P. (1973). Large-sphere limits of the Mie-scattering functions. *J. Opt. Soc. Am.* **63**, 699–706.

Chýlek, P. (1976). Partial-wave resonances and the ripple structure in the Mie normalized extinction cross section. *J. Opt. Soc. Am.* **66**, 285–287.

Chýlek, P. (1977). Light scattering by small particles in an absorbing medium. *J. Opt. Soc. Am.* **67**, 561–563.

Chýlek, P., and Klett, J. D. (1991a). Extinction cross sections of nonspherical particles in the anomalous diffraction approximation. *J. Opt. Soc. Am. A* **8**, 274–281.

Chýlek, P., and Klett, J. D. (1991b). Absorption and scattering of electromagnetic radiation by prismatic columns: anomalous diffraction approximation. *J. Opt. Soc. Am. A* **8**, 1713–1720.

Chýlek, P., and Videen, G. (1994). Longwave radiative properties of polydispersed hexagonal ice crystals. *J. Atmos. Sci.* **51**, 175–190.

Chýlek, P., and Zhan, J. (1989). Interference structure of the Mie extinction cross section. *J. Opt. Soc. Am. A* **6**, 1846–1851.

Chýlek, P., Kiehl, J. T., and Ko, M. K. W. (1978). Narrow resonance structure in the Mie scattering characteristics. *Appl. Opt.* **17**, 3019–3021.

Chýlek, P., Ramaswamy, V., Ashkin, A., and Dziedzic, J. M. (1983). Simultaneous determination of refractive index and size of spherical dielectric particles from light scattering data. *Appl. Opt.* **22**, 2302–2307.

Chýlek, P., Ngo, D., and Pinnick, R. G. (1992). Resonance structure of composite and slightly absorbing spheres. *J. Opt. Soc. Am. A* **9**, 775–780.

Chýlek, P., Videen, G., Ngo, D., *et al*. (1995). Effect of black carbon on the optical properties and climate forcing of sulfate aerosols. *J. Geophys. Res.* **100**, 16 325–16 332.

Chýlek, P., Lesins, G. B., Videen, G., *et al.* (1996). Black carbon and absorption of solar radiation by clouds. *J. Geophys. Res.* **101**, 23 365–23 371.

Chýlek, P., Videen, G., Geldart, D. J. W., *et al*. (2000). Effective medium approximations for heterogeneous particles. In *Light Scattering by Nonspherical Particles: Theory, Measurements, and Applications*, eds. M. I. Mishchenko, J. W. Hovenier, and L. D. Travis, pp. 273–308 (Academic Press, San Diego).

Ciric, I. R., and Cooray, F. R. (1999). Benchmark solutions for electromagnetic scattering by systems of randomly oriented spheroids. *J. Quant. Spectrosc. Radiat. Transfer* **63**, 131–148.

Ciric, I. R., and Cooray, F. R. (2000). Separation of variables for electromagnetic scattering by spheroidal particles. In *Light Scattering by Nonspherical Particles: Theory, Measurements, and Applications*, eds. M. I. Mishchenko, J. W. Hovenier, and L. D. Travis, pp. 89–130 (Academic Press, San Diego).

C.-Labonnote, L., Brogniez, G., Doutriaux-Boucher, M., *et al.* (2000). Modeling of light scattering in cirrus clouds with inhomogeneous hexagonal monocrystals. Comparison with in-situ and ADEOS-POLDER measurements. *Geophys. Res. Lett.* **27**, 113–116.

C.-Labonnote, L., Brogniez, G., Buriez, J.-C., *et al.* (2001). Polarized light scattering by inhomogeneous hexagonal monocrystals. Validation with ADEOS-POLDER measurements. *J. Geophys. Res.* **106**, 12 139–12 153.

Clarke, D., and Grainger, J. F. (1971). *Polarized Light and Optical Measurement* (Pergamon Press, Oxford).

Cloude, S. R., and Pottier, E. (1996). A review of target decomposition theorems in radar polarimetry. *IEEE Trans. Geosci. Remote Sens.* **34**, 498–518.

Coakley, J. A., Jr., and Chýlek, P. (1975). The two-stream approximation in radiative transfer: including the angle of the incident radiation. *J. Atmos. Sci.* **32**, 409–418.

Coletti, A. (1984). Light scattering by nonspherical particles: a laboratory study. *Aerosol Sci. Technol.* **3**, 39–52.

Collett, E. (1992). *Polarized Light: Fundamentals and Applications* (Marcel Dekker, New York).

Colton, D., and Kress, R. (1983). *Integral Equation Methods in Scattering Theory* (John Wiley & Sons, New York).

Colton, D., and Kress, R. (1998). *Inverse Acoustic and Electromagnetic Scattering Theory* (Springer, Berlin).

Comberg, U., and Wriedt, T. (1999). Comparison of scattering calculations for aggregated particles based on different models. *J. Quant. Spectrosc. Radiat. Transfer* **63**, 149–162.

Conwell, P. R., Barber, P. W., and Rushforth, C. K. (1984). Resonant spectra of dielectric spheres. *J. Opt. Soc. Am. A* **1**, 62–67.

Cooper, J., Hombach, V., and Schiavoni, A. (1996). Comparison of computational electromagnetic codes applied to a sphere canonical problem. *IEE Proc. Microwave Antennas Propag.* **143**, 309–316.

Cooray, M. F. R., and Ciric, I. R. (1989). Rotational–translational addition theorems for vector spheroidal wave functions. *COMPEL* **8**, 151–166.

Cooray, M. F. R., and Ciric, I. R. (1991). Scattering by systems of spheroids in arbitrary configurations. *Comput. Phys. Commun.* **68**, 279–305.

Cooray, M. F. R., and Ciric, I. R. (1992). Scattering of electromagnetic waves by a coated dielectric spheroid. *J. Electromagn. Waves Appl.* **6**, 1491–1507.

Cooray, M. F. R., and Ciric, I. R. (1993). Wave scattering by a chiral spheroid. *J. Opt. Soc. Am. A* **10**, 1197–1203.

Crosta, G. F., Camatini, M. C., Zomer, S., *et al*. (2001). Optical scattering (TAOS) by tire debris particles: preliminary results. *Opt. Express* **8**, 302–307.

Cummins, H. Z., and Pike, E. R., eds. (1974). *Photon Correlation and Light Beating Spectroscopy* (Plenum Press, New York).

Cummins, H. Z., and Pike, E. R., eds. (1977). *Photon Correlation Spectroscopy and Velocimetry* (Plenum Press, New York).

Czekala, H., Havemann, S., Schmidt, K., *et al*. (1999). Comparison of microwave radiative transfer calculations obtained with three different approximations of hydrometeor shape. *J. Quant. Spectrosc. Radiat. Transfer* **63**, 545–558.

Czekala, H., Crewell, S., Simmer, C., and Thiele, A. (2001). Discrimination of cloud and rain liquid water path by groundbased polarized microwave radiometry. *Geophys. Res. Lett.* **28**, 267–270.

D'Almeida, G. A., Koepke, P., and Shettle, E. P. (1991). *Atmospheric Aerosols:Global Climatology and Radiative Characteristics* (A. Deepak Publishing, Hampton, VA).

Debye, P. (1909). Der Lichtdruk auf Kugeln von beliebigem Material. *Ann. Physik* **30**, 57–136.

de Boer, J. F., Srinivas, S. M., Park, B. H., *et al*. (1999). Polarization effects in optical coherence tomography of various biological tissues. *IEEE J. Select. Topics Quant. Electron.* **5**, 1200–1204.

de Haan, J. F. (1987). Effects of aerosols on the brightness and polarization of cloudless planetary atmospheres. Ph. D. dissertation, Free University, Amsterdam.

de Haan, J. F., Bosma, P. B., and Hovenier, J. W. (1987). The adding method for multiple scattering calculations of polarized light. *Astron. Astrophys.* **183**, 371–391.

de Hoop, A. T. (1995). *Handbook of Radiation and Scattering of Waves* (Academic Press, London).

Deirmendjian, D. (1969). *Electromagnetic Scattering on Spherical Polydispersions* (Elsevier, New York).

Denti, P., Borghese, F., Saija, R., *et al.* (1999a). Optical properties of aggregated spheres in the vicinity of a plane surface. *J. Opt. Soc. Am. A* **16**, 167–175.

Denti, P., Borghese, F., Saija, R., *et al*. (1999b). Optical properties of a dispersion of randomly oriented identical aggregates of spheres deposited on a plane surface. *Appl. Opt.* **38**, 6421–6430.

de Rooij, W. A. (1985). Reflection and transmission of polarized light by planetary atmospheres. Ph. D. dissertation, Free University, Amsterdam.

de Rooij, W. A., and van der Stap, C. C. A. H. (1984). Expansion of Mie scattering matrices in generalized spherical functions. *Astron. Astrophys.* **131**, 237–248.

Deuzé, J. L., Goloub, P., Herman, M., *et al*. (2000). Estimate of the aerosol properties over the ocean with POLDER. *J. Geophys. Res.* **105**, 15 329–15 346.

Dick, W. D., Zeimann, P. J., Huang, P.-F., and McMurry, P. H. (1998). Optical shape fraction measurements of submicrometre laboratory and atmospheric aerosols. *Meas. Sci. Technol.* **9**, 183–196.

Ding, J., and Xu, L. (1999). Convergence of the T-matrix approach for randomly oriented, nonabsorbing, nonspherical Chebyshev particles. *J. Quant. Spectrosc. Radiat. Transfer* **63**, 163–174.

Doicu, A. (1999). Null-field method with discrete sources. In *Generalized Multipole Techniques for Electromagnetic and Light Scattering*, ed. T. Wriedt, pp. 229–253 (Elsevier, Amsterdam).

Doicu, A., and Wriedt, T. (1997a). Multiple multipole extended boundary condition method. *Optik* **105**, 57–60.

Doicu, A., and Wriedt, T. (1997b). Extended boundary condition method with multipole sources located in the complex plane. *Opt. Commun.* **139**, 85–91.

Doicu, A., and Wriedt, T. (1999). Calculation of the T matrix in the null-field method with discrete sources. *J. Opt. Soc. Am. A* **16**, 2539–2544.

Doicu, A., Wriedt, T., and Bauckhage, K. (1997). Light scattering by homogeneous axisymmetric particles for PDA calculations to measure both axes of spheroidal particles. *Part. Part. Syst. Charact.* **14**, 3–11.

Doicu, A., Köser, J., Wriedt, T., and Bauckhage, K. (1998). Light scattering simulation and measurement of monodisperse spheroids using a phase Doppler anemometer. *Part. Part. Syst. Charact.* **15**, 257–262.

Doicu, A., Eremin, Yu. A., and Wriedt, T. (1999). Convergence of the T-matrix method for light scattering from a particle on or near a surface. *Opt. Commun.* **159**, 266–277.

Doicu, A., Eremin, Yu. A., and Wriedt, T. (2000). *Acoustic and Electromagnetic Scattering Analysis Using Discrete Sources* (Academic Press, San Diego).

Dolginov, A. Z., Gnedin, Yu. N., and Silant'ev, N. A. (1995). *Propagation and Polarization of Radiation in Cosmic Media* (Gordon and Breach, Basel). (Original Russian edi-

tion: Nauka, Moscow, 1979.)

Domke, H. (1974). The expansion of scattering matrices for an isotropic medium in generalized spherical functions. *Astrophys. Space Sci.* **29**, 379–386.

Domke, H. (1975). Fourier expansion of the phase matrix for Mie scattering. *Z. Meteorol.* **25**, 357–361.

Do-Nhat, T., and MacPhie, R. H. (1997). Accurate values of prolate spheroidal radial functions of the second kind. *Can. J. Phys.* **75**, 671–675.

Draine, B. T. (1988). The discrete-dipole approximation and its application to interstellar graphite grains. *Astrophys. J.* **333**, 848–872.

Draine, B. T. (2000). The discrete dipole approximation for light scattering by irregular targets. In *Light Scattering by Nonspherical Particles: Theory, Measurements, and Applications*, eds. M. I. Mishchenko, J. W. Hovenier, and L. D. Travis, pp. 131–145 (Academic Press, San Diego).

Draine, B. T., and Flatau, P. J. (1994). Discrete-dipole approximation for scattering calculations. *J. Opt. Soc. Am. A* **11**, 1491–1499.

Draine, B. T., and Flatau, P. J. (1997). User guide for the discrete dipole approximation code DDSCAT (Version 5a). Princeton Observatory Preprint POPe-695 (http://www.astro. princeton.edu/~draine/UserGuide/UserGuide.html).

Draine, B. T., and Goodman, J. (1993). Beyond Clausius–Mossotti: wave propagation on a polarizable point lattice and the discrete dipole approximation. *Astrophys. J.* **405**, 685–697.

Draine, B. T., and Weingartner, J. C. (1996). Radiative torques on interstellar grains. I. Superthermal spin-up. *Astrophys. J.* **470**, 551–565.

Drezek, R., Dunn, A., and Richards-Kortum, R. (1999). Light scattering from cells: finite-difference time-domain simulations and goniometric measurements. *Appl. Opt.* **38**, 3651–3661.

Drossart, P. (1990). A statistical model for the scattering by irregular particles. *Astrophys. J. Lett.* **361**, 29–32.

Dubovik, O., and King, M. D. (2000). A flexible inversion algorithm for retrieval of aerosol optical properties from Sun and sky radiance measurements. *J. Geophys. Res.* **105**, 20 673–20 696.

Dungey, C. E., and Bohren, C. F. (1991). Light scattering by nonspherical particles: a refinement to the coupled-dipole method. *J. Opt. Soc. Am. A* **8**, 81–87.

Eberhard, W. L. (1992). Ice-cloud depolarization of backscatter for $CO_2$ and other infrared lidars. *Appl. Opt.* **31**, 6485–6490.

Edmonds, A. R. (1996). *Angular Momentum in Quantum Mechanics* (Princeton University Press, Princeton, NJ).

Eide, H., Stamnes, J. J., Stamnes, K., and Schulz, F. M. (1999). New method for computing expansion coefficients for spheroidal functions. *J. Quant. Spectrosc. Radiat. Transfer* **63**, 191–203.

Eide, H. A., Stamnes, K., Hu, Y., and Stamnes, J. J. (2000). Determination of ice particle characteristics from depolarization lidar backscatter returns. In *Light Scattering by Nonspherical Particles: Halifax Contributions*, eds. G. Videen, Q. Fu, and P. Chýlek, pp. 173–176 (Army Research Laboratory, Adelphi, MD).

Eremin, Yu., and Orlov, N. (1998). Modeling of light scattering by non-spherical particles

based on discrete sources method. *J. Quant. Spectrosc. Radiat. Transfer* **60**, 451–62.

Erma, V. A. (1969). Exact solution for the scattering of electromagnetic waves from bodies of arbitrary shape. III. Obstacles with arbitrary electromagnetic properties. *Phys. Rev.* **179**, 1238–1246.

Evans, B. T. N., and Fournier, G. R. (1994). Analytic approximation to randomly oriented spheroid extinction. *Appl. Opt.* **33**, 5796–5804.

Evans, B. G., and Holt, A. R. (1977). Scattering amplitudes and cross-polarisation of ice particles. *Electron. Lett.* **13**, 342–344.

Evans, K. F., and Stephens, G. L. (1995). Microwave radiative transfer through clouds composed of realistically shaped ice crystals. I. Single scattering properties. *J. Atmos. Sci.* **52**, 2041–2057.

Farafonov, V. G. (1983). The scattering of a plane electromagnetic wave by a dielectric spheroid. *Differential Equations (Sov.)* **19**, 1765–1777.

Farafonov, V. G. (2000). Light scattering by multilayer ellipsoids in the Rayleigh approximation. *Opt. Spectrosc.* **88**, 441–443.

Farafonov, V. G., Voshchinnikov, N. V., and Somsikov, V. V. (1996). Light scattering by a core-mantle spheroidal particle. *Appl. Opt.* **35**, 5412–5426.

Farafonov, V. G., Il'in, V. B., and Henning, T. (1999). A new solution of the light scattering problem for axisymmetric particles. *J. Quant. Spectrosc. Radiat. Transfer* **63**, 205–215.

Fikioris, J. G., and Uzunoglu, N. K. (1979). Scattering from an eccentrically stratified dielectric sphere. *J. Opt. Soc. Am.* **69**, 1359–1366.

Flammer, C. (1957). *Spheroidal Wave Functions* (Stanford University Press, Stanford, CA).

Flatau, P. J. (1997). Improvements in the discrete-dipole approximation method of computing scattering and absorption. *Opt. Lett.* **22**, 1205–1207.

Flatau, P. J. (2000). SCATTERLIB: Light Scattering Codes Library. URL: atol.ucsd.edu/~pflatau/scatlib/.

Flatau, P. J., Fuller, K. A., and Mackowski, D. W. (1993). Scattering by two spheres in contact: comparisons between the discrete dipole approximation and modal analysis. *Appl. Opt.* **32**, 3302–3305.

Forsythe, G. E., Malcolm, M. A., and Moler, C. B. (1977). *Computer Methods for Mathematical Computations* (Prentice-Hall, Englewood Cliffs, NJ).

Francis, P. N. (1995). Some aircraft observations of the scattering properties of ice crystals. *J. Atmos. Sci.* **52**, 1142–1154.

Francis, P. N., Hignett, P., and Macke, A. (1998). The retrieval of cirrus cloud properties from aircraft multi-spectral reflectance measurements during EUCREX'93. *Q. J. R. Meteorol. Soc.* **124**, 1273–1291.

Francis, P. N., Foot, J. S., and Baran, A. J. (1999). Aircraft measurements of the solar and infrared radiative properties of cirrus and their dependence on ice crystal shape. *J. Geophys. Res.* **104**, 31 685–31 695.

Freudenthaler, V., Homburg, F., and Jäger, H. (1996). Optical parameters of contrails from lidar measurements: linear depolarization. *Geophys. Res. Lett.* **23**, 3715–3718.

Fu, Q., and Sun, W. (2001). Mie theory for light scattering by a spherical particle in an absorbing medium. *Appl. Opt.* **40**, 1354–1361.

Fu, Q., Sun, W. B., and Yang, P. (1999). Modeling of scattering and absorption by nonspheri-

cal cirrus ice particles at thermal infrared wavelengths. *J. Atmos. Sci.* **56**, 2937–2947.

Fucile, E., Borghese, F., Denti, P., and Saija, R. (1993). Theoretical description of dynamic light scattering from an assembly of large axially symmetric particles. *J. Opt. Soc. Am. A* **10**, 2611–2617.

Fucile, E., Borghese, F., Denti, P., and Saija, R. (1995). Effect of an electrostatic field on the optical properties of a cloud of dielectric particles. *Appl. Opt.* **34**, 4552–4562.

Fuller, K. A. (1991). Optical resonances and two-sphere systems. *Appl. Opt.* **30**, 4716–4731.

Fuller, K. A. (1993). Scattering of light by coated spheres. *Opt. Lett.* **18**, 257–259.

Fuller, K. A. (1994). Scattering and absorption cross sections of compounded spheres. I. Theory for external aggregation. *J. Opt. Soc. Am. A* **11**, 3251–3260.

Fuller, K. A. (1995a). Scattering and absorption cross sections of compounded spheres. II. Calculations for external aggregation. *J. Opt. Soc. Am. A* **12**, 881–892.

Fuller, K. A. (1995b). Scattering and absorption cross sections of compounded spheres. III. Spheres containing arbitrarily located spherical inhomogeneities. *J. Opt. Soc. Am. A* **12**, 893–904.

Fuller, K. A., and Mackowski, D. W. (2000). Electromagnetic scattering by compounded spherical particles. In *Light Scattering by Nonspherical Particles: Theory, Measurements, and Applications*, eds. M. I. Mishchenko, J. W. Hovenier, and L. D. Travis, pp. 225–272 (Academic Press, San Diego).

Fung, A. K. (1994). *Microwave Scattering and Emission Models and Their Applications* (Artech House, Boston, MA).

Gan, H., and Chew, W. C. (1995). A discrete BCG-FFT algorithm for solving 3D inhomogeneous scatterer problems. *J. Electromagn. Waves Appl.* **9**, 1339–1357.

Garrett, T. J., Hobbs, P. V., and Gerber, H. (2001). Shortwave, single-scattering properties of arctic ice clouds. *J. Geophys. Res.* **106**, 15 155–15 172.

Gayet, J.-F., Auriol, F., Oshchepkov, S., *et al*. (1998). In situ measurements of the scattering phase function of stratocumulus, contrails and cirrus. *Geophys. Res. Lett.* **25**, 971–974.

Gehrels, T., ed. (1974). *Planets, Stars and Nebulae Studied with Photopolarimetry* (University of Arizona Press, Tucson, AZ).

Gelfand, I. M., Minlos, R. A., and Shapiro, Z. Ya. (1963). *Representations of the Rotation and Lorentz Groups and their Applications* (Pergamon Press, New York). (Original Russian edition: Fizmatgiz, Moscow, 1958.)

Gerber, H., Takano, Y., Garrett, T. J., and Hobbs, P. V. (2000). Nephelometer measurements of the asymmetry parameter, volume extinction coefficient, and backscatter ratio in Arctic clouds. *J. Atmos. Sci.* **57**, 3021–3034.

Gledhill, T. M., and McCall, A. (2000). Circular polarization by scattering from spheroidal dust grains. *Mon. Not. R. Astron. Soc.* **314**, 123–137.

Gobbi, G. P. (1998). Polarization lidar returns from aerosols and thin clouds: a framework for the analysis. *Appl. Opt*. **37**, 5505–5508.

Göbel, G., Lippek, A., Wriedt, T., and Bauckhage, K. (1998). Monte Carlo simulation of light scattering by inhomogeneous spheres. In *Radiative Transfer II*, ed. M. P. Mengüc, pp. 367–376 (Begel House, New York).

Goedecke, G. H., and O'Brien, S. G. (1988). Scattering by irregular inhomogeneous particles via the digitized Green's function algorithm. *Appl. Opt.* **27**, 2431–2438.

Goncharenko, A. V., Semenov, Yu. G., and Venger, E. F. (1999). Effective scattering cross section of an assembly of small ellipsoidal particles. *J. Opt. Soc. Am. A* **16**, 517–522.

Goodman, J. J., Draine, B. T., and Flatau, P. J. (1991). Application of fast-Fourier-transform techniques to the discrete-dipole approximation. *Opt. Lett.* **16**, 1198–1200.

Gouesbet, G., and Gréhan, G., eds. (1988). *Optical Particle Sizing: Theory and Practice* (Plenum Press, New York).

Gouesbet, G., Maheu, B., and Gréhan, G. (1988). Light scattering from a sphere arbitrarily located in a Gaussian beam, using a Bromwich formulation. *J. Opt. Soc. Am. A* **5**, 1427–1443.

Gouesbet, G., Gréhan, G., and Maheu, B. (1991). Generalized Lorenz–Mie theory and applications to optical sizing. In *Combustion Measurements*, ed. N. Chigier, pp. 339–384 (Hemisphere Publishing Corporation, New York).

Grandy, W. T., Jr. (2000). *Scattering of Waves from Large Spheres* (Cambridge University Press, Cambridge).

Greenberg, J. M., Pedersen, N. E., and Pedersen, J. C. (1961). Microwave analog to the scattering of light by nonspherical particles. *J. Appl. Phys.* **32**, 233–242.

Greenler, R. (1980). *Rainbows, Halos, and Glories* (Cambridge University Press, Cambridge).

Grenfell, T. C., and Warren, S. G. (1999). Representation of a nonspherical ice particle by a collection of independent spheres for scattering and absorption of radiation. *J. Geophys. Res.* **104**, 31 697–31 709.

Grote, M. J., and Keller, J. B. (1998). Nonreflecting boundary conditions for Maxwell's equations. *J. Comput. Phys.* **139**, 327–342.

Grundy, W. M., Douté, S., and Schmitt, B. (2000). A Monte Carlo ray-tracing model for scattering and polarization by large particles with complex shapes. *J. Geophys. Res.* **105**, 29 291–29 314.

Guggenheimer, H. W. (1977). *Differential Geometry* (Dover Publications, New York).

Gurwich, I., Kleiman, M., Shiloah, N., and Cohen, A. (2000). Scattering of electromagnetic radiation by multilayered spheroidal particles: recursive procedure. *Appl. Opt.* **39**, 470–477.

Gustafson, B. Å. S. (1996). Microwave analog to light scattering measurements: a modern implementation of a proven method to achieve precise control. *J. Quant. Spectrosc. Radiat. Transfer* **55**, 663–672.

Gustafson, B. Å. S. (2000). Microwave analog to light-scattering measurements. In *Light Scattering by Nonspherical Particles: Theory, Measurements, and Applications*, eds. M. I. Mishchenko, J. W. Hovenier, and L. D. Travis, pp. 367–390 (Academic Press, San Diego).

Gustafson, B. Å. S., and Kolokolova, L. (1999). A systematic study of light scattering by aggregate particles using the microwave analog technique: angular and wavelength dependence of intensity and polarization. *J. Geophys. Res.* **104**, 31 711–31 720.

Haferman, J. L. (2000). Microwave scattering by precipitation. In *Light Scattering by Nonspherical Particles: Theory, Measurements, and Applications*, eds. M. I. Mishchenko, J. W. Hovenier, and L. D. Travis, pp. 481–524 (Academic Press, San Diego).

Hafner, C. (1990). *The Generalized Multipole Technique for Computational Electromagnetics* (Artech House, Boston, MA).

Hage, J. I., Greenberg, J. M., and Wang, R. T. (1991). Scattering from arbitrarily shaped particles: theory and experiment. *Appl. Opt.* **30**, 1141–1152.

Hamid, A.-K. (1996). Modeling the scattering from a dielectric spheroid by a system of dielectric spheres. *J. Electromagn. Waves Appl.* **10**, 723–729.

Hamid, A.-K., Ciric, I. R., and Hamid, M. (1990). Electromagnetic scattering by an arbitrary configuration of dielectric spheres. *Can. J. Phys.* **68**, 1419–1428.

Hamid, A.-K., Ciric, I. R., and Hamid, M. (1991). Iterative solution of the scattering by an arbitrary configuration of conducting or dielectric spheres. *IEE Proc. H* **138**, 565–572.

Hamid, A.-K., Ciric, I. R., and Hamid, M. (1992). Analytic solutions of the scattering by two multilayered dielectric spheres. *Can. J. Phys.* **70**, 696–705.

Han, Q., Rossow, W. B., Chou, J., *et al*. (1999). The effects of aspect ratio and surface roughness on satellite retrievals of ice-cloud properties. *J. Quant. Spectrosc. Radiat. Transfer* **63**, 559–583.

Hansen, J. E. (1971). Multiple scattering of polarized light in planetary atmospheres. II. Sunlight reflected by terrestrial water clouds. *J. Atmos. Sci.* **28**, 1400–1426.

Hansen, J. E., and Hovenier, J. W. (1974). Interpretation of the polarization of Venus. *J. Atmos. Sci.* **27**, 265–281.

Hansen, J. E., and Travis, L. D. (1974). Light scattering in planetary atmospheres. *Space Sci. Rev.* **16**, 527–610.

Haracz, R. D., Cohen, L. D., and Cohen, A. (1984). Perturbation theory for scattering from dielectric spheroids and short cylinders. *Appl. Opt.* **23**, 436–441.

Haracz, R. D., Cohen, L. D., and Cohen, A. (1985). Scattering of linearly polarized light from randomly oriented cylinders and spheroids. *J. Appl. Phys.* **58**, 3322–3327.

Haracz, R. D., Cohen, L. D., Cohen, A., and Acquista, C. (1986). Light scattering from dielectric targets composed of a continuous assembly of circular disks. *Appl. Opt.* **25**, 4386–4395.

Havemann, S., and Baran, A. J. (2001). Extension of *T*-matrix to scattering of electromagnetic plane waves by non-axisymmetric dielectric particles: application to hexagonal ice cylinders. *J. Quant. Spectrosc. Radiat. Transfer* **70**, 139–158.

He, S., and Cao, J. (1998). Scattering from a bi-isotropic object of arbitrary shape. *J. Electromagn. Waves Appl.* **12**, 1547–1574.

Heintzenberg, J., and Charlson, R. J. (1996). Design and application of the integrating nephelometer: a review. *J. Atmos. Oceanic Technol.* **13**, 987–1000.

Hess, M., Koelemeijer, R. B. A., and Stammes, P. (1998). Scattering matrices of imperfect hexagonal ice crystals. *J. Quant. Spectrosc. Radiat. Transfer* **60**, 301–308.

Heymsfield, A. J., and Iaquinta, J. (2000). Cirrus crystal terminal velocities. *J. Atmos. Sci.* **57**, 916–938.

Hill, S. C., and Benner, R. E. (1988). Morphology-dependent resonances. In *Optical Effects Associated with Small Particles*, eds. P. W. Barber and R. K. Chang, pp. 3–61 (World Scientific, Singapore).

Hill, S. C., Hill, A. C., and Barber, P. W. (1984). Light scattering by size/shape distributions of soil particles and spheroids. *Appl. Opt.* **23**, 1025–1031.

Hillier, J. K., and Buratti, B. J. (2001). Monte Carlo simulations of light scattering by composite particles in a planetary surface. *Icarus* **149**, 251–261.

Hirleman, E. D., and Bohren, C. F., eds. (1991). Special feature on optical particle sizing. *Appl. Opt.* **30**, 4685–4986.

Hirst, E., and Kaye, P. H. (1996). Experimental and theoretical light scattering profiles from spherical and nonspherical particles. *J. Geophys. Res.* **101**, 19 231–19 235.

Hirst, E., Kaye, P. H., and Guppy, J. R. (1994). Light scattering from nonspherical airborne particles: experimental and theoretical comparisons. *Appl. Opt.* **33**, 7180–7186.

Hirst, E., Kaye, P. H., Greenaway, R. S., *et al*. (2001). Discrimination of micrometre-sized ice and super-cooled droplets in mixed-phase cloud. *Atmos. Environ.* **35**, 33–47.

Hodkinson, J. R. (1963). Light scattering and extinction by irregular particles larger than the wavelength. In *ICES Electromagnetic Scattering*, ed. M. Kerker, pp. 87–100 (Pergamon Press, New York).

Hoekstra, A. G., and Sloot, P. M. A. (2000). Biophysical and biomedical applications of nonspherical scattering. In *Light Scattering by Nonspherical Particles: Theory, Measurements, and Applications*, eds. M. I. Mishchenko, J. W. Hovenier, and L. D. Travis, pp. 585–602 (Academic Press, San Diego).

Hoekstra, A. G., Grimminck, M., and Sloot, P. M. A. (1998). Large scale simulations of elastic light scattering by a fast discrete dipole approximation. *Int. J. Mod. Phys. C* **9**, 87–102.

Hoekstra, A., Frijlink, M., and Sloot, P. (2000). Calculation of the radiation forces on each dipole in the discrete dipole approximation. In *Light Scattering by Nonspherical Particles: Halifax Contributions*, eds. G. Videen, Q. Fu, and P. Chýlek, pp. 237–240 (Army Research Laboratory, Adelphi, MD).

Hogan, R. J., Illingworth, A. J., and Sauvageot, H. (2000). Measuring crystal size in cirrus using 35- and 84-GHz radars. *J. Atmos. Oceanic Technol.* **17**, 27–37.

Holler, S., Pan, Y., Chang, R. K., *et al*. (1998). Two-dimensional angular optical scattering for the characterization of airborne microparticles. *Opt. Lett.* **23**, 1489–1491.

Holler, S., Surbek, M., Chang, R. K., and Pan, Y.-L. (1999). Two-dimensional angular optical scattering patterns as droplets evolve into clusters. *Opt. Lett.* **24**, 1185–1187.

Holler, S., Auger, J.-C., Stout, B., *et al*. (2000). Observations and calculations of light scattering from clusters of spheres. *Appl. Opt.* **39**, 6873–6887.

Holt, A. R. (1982). The scattering of electromagnetic waves by single hydrometeors. *Radio Sci.* **17**, 929–945.

Holt, A. R., and Shepherd, J. W. (1979). Electromagnetic scattering by dielectric spheroids in the forward and backward directions. *J. Phys. A: Math. Gen.* **12**, 159–166.

Holt, A. R., Uzunoglu, N. K., and Evans, B. G. (1978). An integral equation solution to the scattering of electromagnetic radiation by dielectric spheroids and ellipsoids. *IEEE Trans. Antennas Propag.* **26**, 706–712.

Hovenier, J. W. (1969). Symmetry relationships for scattering of polarized light in a slab of randomly oriented particles. *J. Atmos. Sci.* **26**, 488–499.

Hovenier, J. W. (1971). Multiple scattering of polarized light in planetary atmospheres. *Astron. Astrophys.* **13**, 7–29.

Hovenier, J. W., ed. (1996). Special issue on light scattering by non-spherical particles. *J. Quant. Spectrosc. Radiat. Transfer* **55**, 535–694.

Hovenier, J. W. (2000). Measuring scattering matrices of small particles at optical wavelengths. In *Light Scattering by Nonspherical Particles: Theory, Measurements, and Appli-*

*cations*, eds. M. I. Mishchenko, J. W. Hovenier, and L. D. Travis, pp. 355–365 (Academic Press, San Diego).

Hovenier, J. W., and Mackowski, D. W. (1998). Symmetry relations for forward and backward scattering by randomly oriented particles. *J. Quant. Spectrosc. Radiat. Transfer* **60**, 483–492.

Hovenier, J. W., and van der Mee, C. V. M. (1983). Fundamental relationships relevant to the transfer of polarized light in a scattering atmosphere. *Astron. Astrophys.* **128**, 1–16.

Hovenier, J. W., and van der Mee, C. V. M. (1996). Testing scattering matrices: a compendium of recipes. *J. Quant. Spectrosc. Radiat. Transfer* **55**, 649–661.

Hovenier, J. W., and van der Mee, C. V. M. (2000). Basic relationships for matrices describing scattering by small particles. In *Light Scattering by Nonspherical Particles: Theory, Measurements, and Applications*, eds. M. I. Mishchenko, J. W. Hovenier, and L. D. Travis, pp. 61–85 (Academic Press, San Diego).

Hovenier, J. W., van de Hulst, H. C., and van der Mee, C. V. M. (1986). Conditions for the elements of the scattering matrix. *Astron. Astrophys.* **157**, 301–310.

Hovenier, J. W., Lumme, K., Mishchenko, M. I., *et al.* (1996). Computations of scattering matrices of four types of non-spherical particles using diverse methods. *J. Quant. Spectrosc. Radiat. Transfer* **55**, 695–705.

Hovenier, J. W., van der Mee, C. V. M., and Domke, H. (2005). *Transfer of Polarized Light in Planetary Atmospheres* (Kluwer Academic Publishers, Dordrecht, The Netherlands).

Huang, X., and Jin, Y.-Q. (1998). Numerical simulations of polarized scattering from random clusters of spatially-oriented, nonspherical scatterers. *J. Electron.* **15**, 267–273.

Huckaby, J. L., Ray, A. K., and Das, B. (1994). Determination of size, refractive index, and dispersion of single droplets from wavelength-dependent scattering spectra. *Appl. Opt.* **33**, 7112–7125.

Huffman, D. R. (1988). The applicability of bulk optical constants to small particles. In *Optical Effects Associated with Small Particles*, eds. P. W. Barber and R. K. Chang, pp. 279–324 (World Scientific, Singapore).

Hunt, A. J., and Huffman, D. R. (1973). A new polarization-modulated light scattering instrument. *Rev. Sci. Instrum.* **44**, 1753–1762.

Hunter, B. A., Box, M. A., and Maier, B. (1988). Resonance structure in weakly absorbing spheres. *J. Opt. Soc. Am. A* **5**, 1281–1286.

Iaquinta, J., Isaka, H., and Personne, P. (1995). Scattering phase function of bullet rosette ice crystals. *J. Atmos. Sci.* **52**, 1401–1413.

Il'in, V. B., and Voshchinnikov, N. V. (1998). Radiation pressure on non-spherical dust grains in envelopes of late-type giants. *Astron. Astrophys. Suppl.* **128**, 187–196.

Irvine, W. M. (1963). The asymmetry of the scattering diagram of a spherical particle. *Bull. Astron. Inst. Neth.* **17**, 176–184.

Ishimaru, A. (1997). *Wave Propagation and Scattering in Random Media* (IEEE Press, New York).

Iskander, M. F., Lakhtakia, A., and Durney, C. H. (1983). A new procedure for improving the solution stability and extending the frequency range of EBCM. *IEEE Trans. Antennas Propag.* **31**, 317–324.

Iskander, M. F., Chen, H. Y., and Penner, J. E. (1989a). Optical scattering and absorption by

branched chains of aerosols. *Appl. Opt.* **28**, 3083–3091.

Iskander, M. F., Chen, H. Y., and Duong, T. V. (1989b). A new sectioning procedure for calculating scattering and absorption by elongated dielectric objects. *IEEE Trans. Electromagn. Compat.* **31**, 157–163.

Ismagilov, F. M., and Kravtsov, Yu. A. (1993). Backscattering enhancement polarization effects on a system of two small randomly oriented scatterers. *Waves in Random Media* **3**, 17–24.

Iwai, T., Furukawa, H., and Asakura, T. (1995). Numerical analysis of enhanced backscattering of light based on Rayleigh–Debye scattering theory. *Opt. Rev.* **2**, 413–419.

Jackson, J. D. (1998). *Classical Electrodynamics* (John Wiley & Sons, New York).

Jaggard, D. L., Hill, C., Shorthill, R. W., *et al*. (1981). Light scattering from particles of regular and irregular shape. *Atmos. Environ.* **15**, 2511–2519.

Jakeman, E. (2000). Polarization fluctuations in light scattered by small particles. In *Light Scattering from Microstructures*, eds. F. Moreno and F. González, pp. 179–189 (Springer, Berlin).

Jin, Y.-Q., and Huang, X. (1996a). Numerical simulations of *T*-matrix solution for polarized bistatic scattering from a cluster of scatterers. *Opt. Commun.* **124**, 27–32.

Jin, Y.-Q., and Huang, X. (1996b). Numerical *T*-matrix solution for polarized scattering from a cluster of spatially oriented, nonspherical scatterers. *Microwave Opt. Technol. Lett.* **12**, 154–158.

Jones, A. R. (1979). Electromagnetic wave scattering by assemblies of particles in the Rayleigh approximation. *Proc. Roy. Soc. London A* **366**, 111–127. (Errata: **375**, 453–454 (1981).)

Jones, A. R. (1999). Light scattering for particle characterization. *Progr. Energy Combust. Sci.* **25**, 1–53.

Joo, K., and Iskander, M. F. (1990). A new procedure of point-matching method for calculating the absorption and scattering of lossy dielectric objects. *IEEE Trans. Antennas Propag.* **38**, 1483–1490.

Kahnert, F. M. (2003). Numerical methods in electromagnetic scattering theory. *J. Quant. Spectrosc. Radiat. Transfer* **79–80**, 775–824.

Kahnert, F. M., Stamnes, J. J., and Stamnes, K. (2001a). Application of the extended boundary condition method to homogeneous particles with point-group symmetries. *Appl. Opt.* **40**, 3110–3123.

Kahnert, F. M., Stamnes, J. J., and Stamnes, K. (2001b). Application of the extended boundary condition method to particles with sharp edges: a comparison of two surface integration approaches. *Appl. Opt.* **40**, 3101–3109.

Kaiser, T., and Schweiger, G. (1993). Stable algorithm for the computation of Mie coefficients for scattered and transmitted fields of a coated sphere. *Computers in Physics* **7**, 682–686.

Kang, X. K., Li, L. W., Leong, M. S., and Kooi, P. S. (2000). Specific absorption rate distributions in a multilayered spheroidal human head model exposed to mobile dipoles. *Radio Sci.* **35**, 247–256.

Kaplan, B., Compain, E., Drevillon, B., and Chaput, F. (1999). Phase modulated Mueller ellipsometry characterization of the scattering by particle suspensions. In *4th Conference on*

*Electromagnetic and Light Scattering by Nonspherical Particles:Theory and Applications*, eds. F. Obelleiro, J. L. Rodríguez, and T. Wriedt, pp. 13–20 (University of Vigo, Vigo, Spain).

Kaplan, B., Compain, E., and Drevillon, B. (2000). Phase-modulated Mueller ellipsometry characterization of scattering by latex sphere suspensions. *Appl. Opt.* **39**, 629–636.

Kapustin, V. N., Lyubovtseva, Yu. S., and Rozenberg, G. V. (1975). An experiment in electro-optical investigation of aerosols. *Izv. Acad. Sci. USSR, Atmos. Oceanic Phys.* (Engl. transl.) **11**, 636–639.

Kapustin, V. N., Rozenberg, G. V., Ahlquist, N. C., *et al*. (1980). Characterization of nonspherical atmospheric aerosol particles with electrooptical nephelometry. *Appl. Opt.* **19**, 1345–1348.

Karam, M. A., and Fung, A. K. (1988). Electromagnetic scattering from a layer of finite length, randomly oriented, dielectric, circular cylinders over a rough interface with application to vegetation. *Int. J. Remote Sens.* **9**, 1109–1134.

Karonis, G. J., Kaklamani, D. I., and Uzunoglu, N. K. (1999). Scattering of electromagnetic waves by a chiral object of ellipsoidal shape. *J. Electromagn. Waves Appl.* **13**, 817–845.

Kattawar, G. W., Hu, C.-R., Parkin, M. E., and Herb, P. (1987). Mueller matrix calculations for dielectric cubes: Comparison with experiments. *Appl. Opt.* **26**, 4174–4180.

Kaye, P. H. (1998). Spatial light-scattering analysis as a means of characterizing and classifying non-spherical particles. *Meas. Sci. Technol.* **9**, 141–149.

Kaye, P. H., Alexander-Buckley, K., Hirst, E., *et al*. (1996). A real-time monitoring system for airborne particle shape and size analysis. *J. Geophys. Res.* **101**, 19 215–19 221.

Kaye, P., Hirst, E., and Wang-Thomas, Z. (1997). Neural-network-based spatial light-scattering instrument for hazardous airborne fiber detection. *Appl. Opt.* **36**, 6149–6156.

Keenan, T. D., Carey, L. D., Zrnić, D. S., and May, P. T. (2001). Sensitivity of 5-cm wavelength polarimetric radar variables to raindrop axial ratio and drop size distribution. *J. Appl. Meteorol.* **40**, 526–545.

Kerker, M. (1969). *The Scattering of Light and Other Electromagnetic Radiation* (Academic Press, San Diego).

Kerker, M., ed. (1988). *Selected Papers on Light Scattering* (SPIE Press, Bellingham, WA).

Kerker, M., Scheiner, P., and Cooke, D. D. (1978). The range of validity of the Rayleigh and Thomson limits for Lorenz–Mie scattering. *J. Opt. Soc. Am.* **68**, 135–137.

Kerola, D. X., and Larson, S. M. (2001). Analysis of coma dust optical properties in comet C/1995 O1 (Hale-Bopp). II. Effects of polarization. *Icarus* **149**, 351–356.

Khlebtsov, N. G. (1979). Analysis and numerical calculation of the scattering matrix for soft spheroids comparable in size with the wavelength of light. *Opt. Spectrosc.* **46**, 292–295.

Khlebtsov, N. G. (1984). Integral equation for problems of light scattering by particles of the medium. *Opt. Spectrosc.* **57**, 399–401.

Khlebtsov, N. G. (1992). Orientational averaging of light-scattering observables in the *T*-matrix approach. *Appl. Opt.* **31**, 5359–5365.

Khlebtsov, N. G. (1993). Optics of fractal clusters in the anomalous diffraction approximation. *J. Mod. Opt.* **40**, 2221–2235.

Khlebtsov, N. G. (2001). Orientational averaging of integrated cross sections in the discrete dipole method. *Opt. Spectrosc.* **90**, 408–415.

Khlebtsov, N. G., and Melnikov, A. G. (1991). Integral equation for light scattering problems: application to the orientationally induced birefringence of colloidal dispersions. *J. Colloid Interface Sci.* **142**, 396–408.

Khlebtsov, N. G., Melnikov, A. G., and Bogatyrev, V. A. (1991). The linear dichroism and birefringence of colloidal dispersions: approximate and exact approaches. *J. Colloid Interface Sci.* **146**, 463–478.

Khlebtsov, N. G., Melnikov, A. G., Bogatyrev, V. A., and Sirota, A. I. (1999). Electrooptics effects in dilute suspensions of bacterial cells and fractal aggregates. *J. Quant. Spectrosc. Radiat. Transfer* **63**, 469–478.

Kiehl, J. T., Ko, M. W., Mugnai, A., and Chýlek, P. (1980). Perturbation approach to light scattering by nonspherical particles. In *Light Scattering by Irregularly Shaped Particles*, ed. D. Schuerman, pp. 135–140 (Plenum Press, New York).

Kim, S.-H., and Martin, P. G. (1995). The size distribution of interstellar dust particles as determined from polarization: Spheroids. *Astrophys. J.* **444**, 293–305.

Kim, C. S., and Yeh, C. (1991). Scattering of an obliquely incident wave by a multilayered elliptical lossy dielectric cylinder. *Radio Sci.* **26**, 1165–1176.

Kimura, H. (2001). Light-scattering properties of fractal aggregates: numerical calculations by a superposition technique and the discrete-dipole approximation. *J. Quant. Spectrosc. Radiat. Transfer* **70**, 581–594.

Kimura, H., and Mann, I. (1998). Radiation pressure cross section for fluffy aggregates. *J. Quant. Spectrosc. Radiat. Transfer* **60**, 425–438.

King, M. D., Kaufman, Y. J., Tanré, D., and Nakajima, T. (1999). Remote sensing of tropospheric aerosols from space: Past, present, and future. *Bull. Am. Meteorol. Soc.* **80**, 2229–2259.

Kleinman, R. E., and Senior, T. B. A. (1986). Rayleigh scattering. In *Low and High Frequency Asymptotics*, eds. V. K. Varadan and V. V. Varadan, pp. 1–70 (Elsevier, Amsterdam).

Klett, J. D., and Sutherland, R. A. (1992). Approximate methods for modeling the scattering properties of non-spherical particles: evaluation of the Wentzel–Kramers–Brillouin method. *Appl. Opt.* **31**, 373–386.

Kliger, D. S., Lewis, J. W., and Randall, C. E. (1990). *Polarized Light in Optics and Spectroscopy* (Academic Press, San Diego).

Kokhanovsky, A. (2001). *Optics of Light Scattering Media: Problems and Solutions* (Praxis, Chichester).

Kokhanovsky, A. A., and Nakajima, T. Y. (1998). The dependence of phase functions of large transparent particles on their refractive index and shape. *J. Phys. D: Appl. Phys.* **31**, 1329–1335.

Kolokolova, L., and Gustafson, B. Å. S. (2001). Scattering by inhomogeneous particles: microwave analog experiments and comparison to effective medium theories. *J. Quant. Spectrosc. Radiat. Transfer* **70**, 611–625.

Kong, J. A. (1990). *Electromagnetic Wave Theory* (John Wiley & Sons, New York).

Korolev, A. V., Isaac, G. A., and Hallett, J. (1999). Ice particle habits in Arctic clouds. *Geophys. Res. Lett.* **26**, 1299–1302.

Korolev, A. V., Isaac, G. A., and Hallett, J. (2000). Ice particle habits in stratiform clouds.

*Q. J. R. Meteorol. Soc.* **126**, 2873–2902.

Kouzoubov, A., Brennan, M. J., Thomas, J. C., and Abbot, R. H. (1999). Monte Carlo simulations of the influence of particle nonsphericity on remote sensing of ocean water. *J. Geophys. Res.* **104**, 31 731–31 737.

Kravtsov, Yu. A., and Orlov, Yu. I. (1990). *Geometrical Optics of Inhomogeneous Media* (Springer, Berlin). (Original Russian edition: Nauka, Moscow, 1980).

Kravtsov, Yu. A., and Orlov, Yu. I. (1999). *Caustics, Catastrophes and Wave Fields* (Springer, Berlin).

Kristensson, G. (1980). Electromagnetic scattering from buried inhomogeneities – a general three-dimensional approach. *J. Appl. Phys.* **51**, 3486–3500.

Krotkov, N. A., Flittner, D. E., Krueger, A. J., *et al*. (1999). Effect of particle nonsphericity on satellite monitoring of drifting volcanic ash clouds. *J. Quant. Spectrosc. Radiat. Transfer* **63**, 613–630.

Krylov, V. I. (1962). *Approximate Calculation of Integrals* (Macmillan, New York).

Ku, J. C. (1993). Comparisons of coupled-dipole solutions and dipole refractive indices for light scattering and absorption by arbitrarily shaped or agglomerated particles. *J. Opt. Soc. Am. A* **10**, 336–342.

Ku, J. C., and Felske, J. D. (1984). The range of validity of the Rayleigh limit for computing Mie scattering and extinction efficiencies. *J. Quant. Spectrosc. Radiat. Transfer* **31**, 569–574.

Kuga, Y., and Ishimaru, A. (1984). Retroreflectance from a dense distribution of spherical particles. *J. Opt. Soc. Am. A* **1**, 831–835.

Kuik, F. (1992). Single scattering of light by ensembles of particles with various shapes. Ph. D. dissertation, Free University, Amsterdam.

Kuik, F., Stammes, P., and Hovenier, J. W. (1991). Experimental determination of scattering matrices of water droplets and quartz particles. *Appl. Opt.* **30**, 4872–4881.

Kuik, F., de Haan, J. F., and Hovenier, J. W. (1992). Benchmark results for single scattering by spheroids. *J. Quant. Spectrosc. Radiat. Transfer* **47**, 477–489.

Kuik, F., de Haan, J. F., and Hovenier, J. W. (1994). Single scattering of light by circular cylinders. *Appl. Opt.* **33**, 4906–4918.

Kunz, K. S., and Luebbers, R. J. (1993). *Finite Difference Time Domain Method for Electromagnetics* (CRC Press, Boca Raton, FL).

Kurtz, V., and Salib, S. (1993). Scattering and absorption of electromagnetic radiation by spheroidally shaped particles: computation of the scattering properties. *J. Imaging Sci. Technol.* **37**, 43–60.

Kuščer, I., and Ribarič, M. (1959). Matrix formalism in the theory of diffusion of light. *Opt. Acta* **6**, 42–51 (1959).

Kuzmin, V. N., and Babenko, V. A. (1981). Light scattering by a weakly anisotropic spherical particle. *Opt. Spectrosc.* **50**, 269–273.

Labeyrie, G., Müller, C. A., Wiersma, D. S., *et al*. (2000). Observation of coherent backscattering of light by cold atoms. *J. Opt. B: Quantum Semiclass. Opt.* **2**, 672–685.

Lacis, A. A., Carlson, B. E., and Hansen, J. E. (2000). Retrieval of atmospheric $NO_2$, $O_3$, aerosol optical depth, effective radius and variance information from SAGE II multispectral extinction measurements. *Appl. Math. Comput.* **116**, 133–151.

Lacoste, D., and van Tiggelen, B. A. (1999). Stokes parameters for light scattering from a Faraday-active sphere. *J. Quant. Spectrosc. Radiat. Transfer* **63**, 305–319.

Lacoste, D., van Tiggelen, B. A., Rikken, G. L. J. A., and Sparenberg, A. (1998). Optics of a Faraday-active Mie sphere. *J. Opt. Soc. Am. A* **15**, 1636–1642.

Lagendijk, A., and van Tiggelen, B. A. (1996). Resonant multiple scattering of light. *Phys. Rep.* **270**, 143–215.

Laitinen, H., and Lumme, K. (1998). *T*-matrix method for general star-shaped particles: first results. *J. Quant. Spectrosc. Radiat. Transfer* **60**, 325–334.

Lakhtakia, A. (1991). The extended boundary condition method for scattering by a chiral scatterer in a chiral medium: formulation and analysis. *Optik* **86**, 155–161.

Lakhtakia, A. (1992). General theory of the Purcell–Pennypacker scattering approach and its extension to bianisotropic scatterers. *Astrophys. J.* **394**, 494–499.

Lakhtakia, A., and Mulholland, G. W. (1993). On two numerical techniques for light scattering by dielectric agglomerated structures. *J. Res. Natl. Inst. Stand. Technol.* **98**, 699–716.

Lakhtakia, A., Varadan, V. K., and Varadan, V. V. (1984). Scattering by highly aspherical targets: EBCM coupled with reinforced orthogonalization. *Appl. Opt.* **23**, 3502–3504.

Lakhtakia, A., Varadan, V. V., and Varadan, V. K. (1985a). Scattering of ultrasonic waves by oblate spheroidal voids of high aspect ratio. *J. Appl. Phys.* **58**, 4525–4530.

Lakhtakia, A., Varadan, V. K., and Varadan, V. V. (1985b). Scattering and absorption characteristics of lossy dielectric, chiral, nonspherical objects. *Appl. Opt.* **24**, 4146–4154.

Landgraf, M., Augustsson, K., Grün, E., and Gustafson, B. Å. S. (1999). Deflection of the local interstellar dust flow by solar radiation pressure. *Science* **286**, 2319–2322.

Langley, D. S., and Marston, P. L. (1998). Generalized tertiary rainbow of slightly oblate drops: observations with laser illumination. *Appl. Opt.* **37**, 1520–1526.

Latimer, P. (1975). Light scattering by ellipsoids. *J. Colloid. Interface Sci.* **53**, 102–109.

Latimer, P., and Barber, P. (1978). Scattering by ellipsoids of revolution: a comparison of theoretical methods. *J. Colloid Interface Sci.* **63**, 310–316.

Lawson, R. P., Heymsfield, A. J., Aulenbach, S. M., and Jensen, T. L. (1998). Shapes, sizes and light scattering properties of ice crystals in cirrus and a persistent contrail during SUCCESS. *Geophys. Res. Lett.* **25**, 1331–1334.

Lebedev, A. N., Gartz, M., Kreibig, U., and Stenzel, O. (1999). Optical extinction by spherical particles in an absorbing medium: application to composite absorbing films. *Eur. Phys. J. D* **6**, 365–373.

Lemaire, T. (1997). Coupled-dipole formulation for the treatment of electromagnetic scattering by a small dielectric particle of arbitrary shape. *J. Opt. Soc. Am. A* **14**, 470–474.

Lemke, H. M., and Quante, M. (1999). Backscatter characteristics of nonspherical ice crystals: assessing the potential of polarimetric radar measurements. *J. Geophys. Res.* **104**, 31 739–31 751.

Lemke, H., Okamoto, H., and Quante, M. (1998). Comment on error analysis of backscatter from discrete dipole approximation for different ice particle shapes. *Atmos. Res.* **49**, 189–197.

Lenke, R., and Maret, G. (2000). Multiple scattering of light: coherent backscattering and transmission. In *Scattering in Polymeric and Colloidal Systems*, eds. W. Brown and K. Mortensen, pp. 1–73 (Gordon and Breach Science Publishers, Amsterdam).

Lenoble, J., ed. (1985). *Radiative Transfer in Scattering and Absorbing Atmospheres: Standard Computational Procedures* (A. Deepak Publishing, Hampton, VA).

Le Vine, D. M., Schneider, A., Lang, R. H., and Carter, H. G. (1985). Scattering from thin dielectric disks. *IEEE Trans. Antennas Propag.* **33**, 1410–1413.

Lewin, L. (1970). On the restricted validity of point-matching techniques. *IEEE Trans. Microwave Theory Tech.* **18**, 1041–1047.

Li, L.-W., Leong, M.-S., Yeo, T.-S., *et al*. (1998a). Computations of spheroidal harmonics with complex arguments: a review with an algorithm. *Phys. Rev. E* **58**, 6792–6806.

Li, L. W., Yeo, T. S., Kooi, P. S., and Leong, M. S. (1998b). Microwave specific attenuation by oblate spheroidal raindrops: an exact analysis of TCS's in terms of spheroidal wave functions. *Progr. Electromagn. Res. (PIER)* **18**, 127–150.

Li, L.-W., Leong, M.-S., Kooi, P.-S., and Yeo, T.-S. (2001). Spheroidal vector wave eigenfunction expansion of dyadic Green's functions for a dielectric spheroid. *IEEE Trans. Antennas Propag.* **49**, 645–659.

Lin, Y.-C., and Sarabandi, K. (1995). Electromagnetic scattering model for a tree trunk above a tilted ground plane. *IEEE Trans. Geosci. Remote Sens.* **33**, 1063–1070.

Lind, A. C., Wang, R. T., and Greenberg, J. M. (1965). Microwave scattering by nonspherical particles. *Appl. Opt.* **4**, 1555–1561.

Liou, K.-N. (1980). *An Introduction to Atmospheric Radiation* (Academic Press, New York).

Liou, K. N. (1992). *Radiation and Cloud Processes in the Atmosphere: Theory, Observation, and Modeling* (Oxford University Press, New York).

Liou, K.-N., and Hansen, J. E. (1971). Intensity and polarization for single scattering by polydisperse spheres: a comparison of ray optics and Mie theory. *J. Atmos. Sci.* **28**, 995–1004.

Liou, K.-N., and Lahore, H. (1974). Laser sensing of cloud composition: a backscattered depolarization technique. *J. Appl. Meteorol.* **13**, 257–263.

Liou, K. N., Cai, Q., Pollack, J. B., and Cuzzi, J. N. (1983). Light scattering by randomly oriented cubes and parallelepipeds. *Appl. Opt.* **22**, 3001–3008.

Liou, K. N., Yang, P., Takano, Y., *et al*. (1998). On the radiative properties of contrail cirrus. *Geophys. Res. Lett.* **25**, 1161–1164.

Liou, K. N., Takano, Y., and Yang, P. (2000). Light scattering and radiative transfer in ice crystal clouds: aplications to climate research. In *Light Scattering by Nonspherical Particles: Theory, Measurements, and Applications*, eds. M. I. Mishchenko, J. W. Hovenier, and L. D. Travis, pp. 3–27 (Academic Press, San Diego).

Lippmann, B. A., and Schwinger, J. (1950). Variational principles for scattering processes. *Phys. Rev.* **79**, 469–480.

Liu, H., and Chandrasekar, V. (2000). Classification of hydrometeors based on polarimetric radar measurements: development of fuzzy logic and neuro-fuzzy systems, and in situ verification. *J. Atmos. Oceanic Technol.* **17**, 140–164.

Liu, L., and Mishchenko, M. I. (2001). Constraints on PSC particle microphysics derived from lidar observations. *J. Quant. Spectrosc. Radiat. Transfer* **70**, 817–831.

Liu, C., Jonas, P. R., and Saunders, C. P. R. (1996). Pyramidal ice crystal scattering phase functions and concentric halos. *Ann. Geophys.* **14**, 1192–1197.

Liu, Y., Arnott, W. P., and Hallett, J. (1998). Anomalous diffraction theory for arbitrarily oriented finite circular cylinders and comparison with exact *T*-matrix results. *Appl. Opt.* **37**,

5019–5030.

Liu, Y., Arnott, W. P., and Hallett, J. (1999). Particle size distribution retrieval from multispectral optical depth: influences of particle nonsphericity and refractive index. *J. Geophys. Res.* **104**, 31 753–31 762.

Lock, J. A. (1995). Improved Gaussian beam-scattering algorithm. *Appl. Opt.* **34**, 559–570.

Lock, J. A., and Yang, L. (1991). Interference between diffraction and transmission in the Mie extinction efficiency. *J. Opt. Soc. Am. A* **8**, 1132–1134.

Lopatin, V. N., and Sid'ko, F. Ya. (1988). *Introduction to Optics of Cell Suspensions* (Nauka, Novosibirsk) (in Russian).

Love, A. E. H. (1899). The scattering of electric waves by a dielectric sphere. *Proc. London Math. Soc.* **30**, 308–321.

Ludwig, A. C. (1991). The generalized multipole technique. *Comput. Phys. Commun.* **68**, 306–314.

Lumme, K., ed. (1998). Special issue on light scattering by non-spherical particles. *J. Quant. Spectrosc. Radiat. Transfer* **60**, 301–500.

Lumme, K. (2000). Scattering properties of interplanetary dust particles. In *Light Scattering by Nonspherical Particles: Theory, Measurements, and Applications*, eds. M. I. Mishchenko, J. W. Hovenier, and L. D. Travis, pp. 555–583 (Academic Press, San Diego).

Lumme, K., and Rahola, J. (1994). Light scattering by porous dust particles in the discrete-dipole approximation. *Astrophys. J.* **425**, 653–667.

Lumme, K., and Rahola, J. (1998). Comparison of light scattering by stochastically rough spheres, best-fit spheroids and spheres. *J. Quant. Spectrosc. Radiat. Transfer* **60**, 439–450.

Lumme, K., Rahola, J., and Hovenier, J. W. (1997). Light scattering by dense clusters of spheres. *Icarus* **126**, 455–469.

Lynch, D. K., and Livingston, W. (1995). *Color and Light in Nature* (Cambridge University Press, Cambridge).

Lyot, B. (1929). Recherches sur la polarisation de la lumiere des planetes et de quelques substances terrestres. *Ann. Observ. Paris* **8**, No. 1. (English translation: Research on the polarization of light from planets and from some terrestrial substances, NASA Tech. Transl. NASA TT F-187, Washington, DC, 1964.)

Macke, A. (1993). Scattering of light by polyhedral ice crystals. *Appl. Opt.* **32**, 2780–2788.

Macke, A. (2000). Monte Carlo calculations of light scattering by large particles with multiple internal inclusions. In *Light Scattering by Nonspherical Particles: Theory, Measurements, and Applications*, eds. M. I. Mishchenko, J. W. Hovenier, and L. D. Travis, pp. 309–322 (Academic Press, San Diego).

Macke, A., and Großklaus, M. (1998). Light scattering by nonspherical raindrops: implications for lidar remote sensing of rainrates. *J. Quant. Spectrosc. Radiat. Transfer* **60**, 355–363.

Macke, A., and Mishchenko, M. I. (1996). Applicability of regular particle shapes in light scattering calculations for atmospheric ice particles. *Appl. Opt.* **35**, 4291–4296.

Macke, A., Mishchenko, M. I., Muinonen, K., and Carlson, B. E. (1995). Scattering of light by large nonspherical particles: ray tracing approximation versus *T*-matrix method. *Opt. Lett.* **20**, 1934–1936.

Macke, A., Mishchenko, M. I., and Cairns, B. (1996a). The influence of inclusions on light

scattering by large ice particles. *J. Geophys. Res.* **101**, 23 311–23 316.

Macke, A., Mueller, J., and Raschke, E. (1996b). Scattering properties of atmospheric ice crystals. *J. Atmos. Sci.* **53**, 2813–2825.

Mackowski, D. W. (1991). Analysis of radiative scattering for multiple sphere configurations. *Proc. Roy. Soc. London A* **433**, 599–614.

Mackowski, D. W. (1994). Calculation of total cross sections of multiple-sphere clusters. *J. Opt. Soc. Am. A* **11**, 2851–2861.

Mackowski, D. W. (1995). Electrostatic analysis of radiative absorption by sphere clusters in the Rayleigh limit: application to soot. *Appl. Opt.* **34**, 3535–3545.

Mackowski, D. W. (2001). An effective medium method for calculation of the *T* matrix of aggregated spheres. *J. Quant. Spectrosc. Radiat. Transfer* **70**, 441–464.

Mackowski, D. W., and Jones, P. D. (1995). Theoretical investigation of particles having directionally dependent absorption cross section. *J. Thermophys. Heat Transfer* **9**, 193–201.

Mackowski, D. W., and Mishchenko, M. I. (1996). Calculation of the *T* matrix and the scattering matrix for ensembles of spheres. *J. Opt. Soc. Am. A* **13**, 2266–2278.

Mackowski, D. W., Altenkirch, R. A., Menguc, M. P. (1990). Internal absorption cross sections in a stratified sphere. *Appl. Opt.* **29**, 1551–1559.

Maltsev, V. P., Chernyshev, A. V., Sem'yanov, K. A., and Soini, E. (1997). Absolute real-time determination of size and refractive index of individual microspheres. *Meas. Sci. Technol.* **8**, 1023–1027.

Mandel, L., and Wolf, E. (1995). *Optical Coherence and Quantum Optics* (Cambridge University Press, Cambridge).

Markel, V. A., and George, T. F., eds. (2001). *Optics of Nanostructured Materials* (John Wiley & Sons, New York).

Markel, V. A., Shalaev, V. M., and George, T. F. (2000). Some theoretical and numerical approaches to the optics of fractal smoke. In *Optics of Nanostructured Materials*, eds. V. A. Markel and T. F. George, pp. 355–411 (John Wiley & Sons, New York).

Marshall, S. F., Covert. D. S., and Charlson, R. J. (1995). Relationship between asymmetry parameter and hemispheric backscatter ratio: implications for climate forcing by aerosols. *Appl. Opt.* **34**, 6306–6311.

Marston, P. L. (1992). Geometrical and catastrophe optics methods in scattering. In *High Frequency and Pulse Scattering*, eds. A. D. Pierce and R. N. Thurston, pp. 1–234 (Academic Press, San Diego).

Marston, P. L., ed. (1994). *Geometrical Aspects of Scattering* (SPIE Press, Bellingham, WA).

Marston, P. L. (1999). Catastrophe optics of spheroidal drops and generalized rainbows. *J. Quant. Spectrosc. Radiat. Transfer* **63**, 341–351.

Marston, P. L., and Crichton, J. H. (1984). Radiation torque on a sphere caused by a circularly-polarized electromagnetic wave. *Phys. Rev. A* **30**, 2508–2516.

Martin, P. G. (1978). *Cosmic Dust* (Oxford University Press, Oxford).

Martin, R. J. (1993). Mie scattering formulae for non-spherical particles. *J. Mod. Opt.* **40**, 2467–2494.

Martin, T. (1998). An improved near- to far-zone transformation for the finite-difference time-domain method. *IEEE Trans. Antennas Propag.* **46**, 1263–1271.

Martinez, A. S., and Maynard, R. (1994). Faraday effect and multiple scattering of light.

*Phys. Rev. B* **50**, 3714–3732.

Masłowska, A., Flatau, P. J., and Stephens, G. L. (1994). On the validity of the anomalous diffraction theory to light scattering by cubes. *Opt. Commun.* **107**, 35–40.

Masuda, K., and Takashima, T. (1992). Feasibility study of derivation of cirrus information using polarimetric measurements from satellite. *Remote Sens. Environ.* **39**, 45–59.

Masuda, K., Takashima, T., Kawata, Y., *et al*. (2000). Retrieval of aerosol optical properties over the ocean using multispectral polarization measurements from space. *Appl. Math. Comput.* **116**, 103–114.

Matsumura, M., and Seki, M. (1991). Light scattering calculations by the Fredholm integral equation method. *Astrophys. Space Sci.* **176**, 283–295.

Matsumura, M., and Seki, M. (1996). Extinction and polarization by ellipsoidal particles in the infrared. *Astrophys. J.* **456**, 557–565.

Mazeron, P., and Muller, S. (1996). Light scattering by ellipsoids in a physical optics approximation. *Appl. Opt.* **35**, 3726–3735.

Mazin, I. P., and Khrgian, A. Kh., eds. (1989). *Handbook of Clouds and Cloudy Atmosphere* (Gidrometeoizdat, Leningrad) (in Russian).

McClain, W. M., and Ghoul, W. A. (1986). Elastic light scattering by randomly oriented macromolecules: computation of the complete set of observables. *J. Chem. Phys.* **84**, 6609–6622.

McGuire, A. F., and Hapke, B. W. (1995). An experimental study of light scattering by large, irregular particles. *Icarus* **113**, 134–155.

Medgyesi-Mitschang, L. N., Putnam, J. M., and Gedera, M. B. (1994). Generalized method of moments for three-dimensional penetrable scatterers. *J. Opt. Soc. Am. A* **11**, 1383–1398.

Meeten, G. H. (1982). An anomalous diffraction theory of linear birefringence and dichroism in colloidal dispersions. *J. Colloid. Interface Sci.* **87**, 407–415.

Mei, K. K. (1974). Unimoment method of solving antenna and scattering problems. *IEEE Trans. Antennas Propag.* **22**, 760–766.

Meyer, W. V., Smart, A. E., Brown, R. G. W., and Anisimov, M. A., eds. (1997). Feature issue on photon correlation and scattering. *Appl. Opt.* **36**, 7477–7677.

Meyer, W. V., Smart, A. E., and Brown, R. G. W., eds. (2001). Feature issue on photon correlation and scattering. *Appl. Opt.* **40**, 3965–4242.

Mie, G. (1908). Beiträge zur Optik trüber Medien, speziell kolloidaler Metallösungen. *Ann. Physik* **25**, 377–445.

Mikulski, J. J., and Murphy, E. L. (1963). The computation of electromagnetic scattering from concentric spherical structures. *IEEE Trans. Antennas Propag.* **11**, 169–177.

Millar, R. F. (1969). Rayleigh hypothesis in scattering problems. *Electron. Lett.* **5**, 416–417.

Miller, E. K., Medgyesi-Mitschang, L. N., and Newman, E. H. (1991). *Computational Electromagnetics: Frequency Domain Method of Moments* ( IEEE Press, New York).

Mishchenko, M. I. (1990a). Multiple scattering of polarized light in anisotropic plane-parallel media. *Transp. Theory Statist. Phys.* **19**, 293–316.

Mishchenko, M. I. (1990b). Extinction of light by randomly-oriented non-spherical grains. *Astrophys. Space Sci.* **164**, 1–13.

Mishchenko, M. I. (1990c). Expansion of scattering matrix in generalized spherical functions for radially inhomogeneous spherical particles. *Kinem. Phys. Celest. Bodies* **6**(1), 93–95.

Mishchenko, M. I. (1991a). Light scattering by randomly oriented axially symmetric particles. *J. Opt. Soc. Am. A* **8**, 871–882. (Errata: **9**, 497 (1992).)

Mishchenko, M. I. (1991b). Extinction and polarization of transmitted light by partially aligned nonspherical grains. *Astrophys. J.* **367**, 561–574.

Mishchenko, M. I. (1991c). Scattering cross section for randomly oriented particles of arbitrary shape. *Kinem. Phys. Celest. Bodies* **7**(5), 83–86.

Mishchenko, M. I. (1992a). Coherent propagation of polarized millimeter waves through falling hydrometeors. *J. Electromagn. Waves Appl.* **6**, 1341–1351.

Mishchenko, M. I. (1992b). Enhanced backscattering of polarized light from discrete random media: calculations in exactly the backscattering direction. *J. Opt. Soc. Am. A* **9**, 978–982.

Mishchenko, M. I. (1993). Light scattering by size-shape distributions of randomly oriented axially symmetric particles of a size comparable to a wavelength. *Appl. Opt.* **32**, 4652–4666.

Mishchenko, M. I. (1996a). Coherent backscattering by two-sphere clusters. *Opt. Lett.* **21**, 623–625.

Mishchenko, M. I. (1996b). Diffuse and coherent backscattering by discrete random media. I. Radar reflectivity, polarization ratios, and enhancement factors for a half-space of polydisperse, nonabsorbing and absorbing spherical particles. *J. Quant. Spectrosc. Radiat. Transfer* **56**, 673–702.

Mishchenko, M. I. (2000). Calculation of the amplitude matrix for a nonspherical particle in a fixed orientation. *Appl. Opt.* **39**, 1026–1031.

Mishchenko, M. I. (2001). Radiation force caused by scattering, absorption, and emission of light by nonspherical particles. *J. Quant. Spectrosc. Radiat. Transfer* **70**, 811–816.

Mishchenko, M. I. (2002). Vector radiative transfer equation for arbitrarily shaped and arbitrarily oriented particles: a microphysical derivation from statistical electromagnetics. *Appl. Opt.* **41**, 7114–7134.

Mishchenko, M. I. (2003). Microphysical approach to polarized radiative transfer: extension to the case of an external observation point. *Appl. Opt.* **42**, 4963–4967.

Mishchenko, M. I. (2005). Scale invariance rule in electromagnetic scattering. *J. Quant. Spectrosc. Radiat. Transfer* (in press).

Mishchenko, M. I., and Hovenier, J. W. (1995). Depolarization of light backscattered by randomly oriented nonspherical particles. *Opt. Lett.* **20**, 1356–1358.

Mishchenko, M. I., and Lacis, A. A. (2000). Manifestations of morphology-dependent resonances in Mie scattering matrices. *Appl. Math. Comput*. **116**, 167–179.

Mishchenko, M. I., and Macke, A. (1997). Asymmetry parameters of the phase function for isolated and densely packed spherical particles with multiple internal inclusions in the geometric optics limit. *J. Quant. Spectrosc. Radiat. Transfer* **57**, 767–794.

Mishchenko, M. I., and Macke, A. (1998). Incorporation of physical optics effects and computation of the Legendre expansion for ray-tracing phase functions involving δ-function transmission. *J. Geophys. Res.* **103**, 1799–1805.

Mishchenko, M. I., and Macke, A. (1999). How big should hexagonal ice crystals be to produce halos? *Appl. Opt.* **38**, 1626–1629.

Mishchenko, M. I., and Mackowski, D. W. (1994). Light scattering by randomly oriented bispheres. *Opt. Lett.* **19**, 1604–1606.

Mishchenko, M. I., and Mackowski, D. W. (1996). Electromagnetic scattering by randomly oriented bispheres: comparison of theory and experiment and benchmark calculations. *J. Quant. Spectrosc. Radiat. Transfer* **55**, 683–694.

Mishchenko, M. I., and Sassen, K. (1998). Depolarization of lidar returns by small ice crystals: an application to contrails. *Geophys. Res. Lett.* **25**, 309–312.

Mishchenko, M. I., and Travis, L. D. (1994a). T-matrix computations of light scattering by large spheroidal particles. *Opt. Commun.* **109**, 16–21.

Mishchenko, M. I., and Travis, L. D. (1994b). Light scattering by polydisperse, rotationally symmetric nonspherical particles: linear polarization. *J. Quant. Spectrosc. Radiat. Transfer* **51**, 759–778.

Mishchenko, M. I., and Travis, L. D. (1994c). Light scattering by polydispersions of randomly oriented spheroids with sizes comparable to wavelengths of observation. *Appl. Opt.* **33**, 7206–7225.

Mishchenko, M. I., and Travis, D. L. (1997). Satellite retrieval of aerosol properties over the ocean using polarization as well as intensity of reflected sunlight. *J. Geophys. Res.* **102**, 16 989–17 013.

Mishchenko, M. I., and Travis, L. D. (1998). Capabilities and limitations of a current FORTRAN implementation of the *T*-matrix method for randomly oriented, rotationally symmetric scatterers. *J. Quant. Spectrosc. Radiat. Transfer* **60**, 309–324.

Mishchenko, M. I., and Videen, G. (1999). Single-expansion EBCM computations for osculating spheres. *J. Quant. Spectrosc. Radiat. Transfer* **63**, 231–236.

Mishchenko, M. I., Lacis, A. A., and Travis, L. D. (1994). Errors introduced by the neglect of polarization in radiance calculations for Rayleigh-scattering atmospheres. *J. Quant. Spectrosc. Radiat. Transfer* **51**, 491–510.

Mishchenko, M. I., Mackowski, D. W., and Travis, L. D. (1995). Scattering of light by bispheres with touching and separated components. *Appl. Opt.* **34**, 4589–4599.

Mishchenko, M. I., Travis, L. D., and Macke, A. (1996a). Scattering of light by polydisperse, randomly oriented, finite circular cylinders. *Appl. Opt.* **35**, 4927–4940.

Mishchenko, M. I., Travis, L. D., and Mackowski, D. W. (1996b). *T*-matrix computations of light scattering by nonspherical particles: a review. *J. Quant. Spectrosc. Radiat. Transfer* **55**, 535–575.

Mishchenko, M. I., Rossow, W. B., Macke, A., and Lacis, A. A. (1996c). Sensitivity of cirrus cloud albedo, bidirectional reflectance and optical thickness retrieval accuracy to ice particle shape. *J. Geophys. Res.* **101**, 16 973–16 985.

Mishchenko, M. I., Travis, L. D., Kahn, R. A., and West, R. A. (1997a). Modeling phase functions for dustlike tropospheric aerosols using a shape mixture of randomly oriented polydisperse spheroids. *J. Geophys. Res.* **102**, 16 831–16 847.

Mishchenko, M. I., Wielaard, D. J., and Carlson, B. E. (1997b). *T*-matrix computations of zenith-enhanced lidar backscatter from horizontally oriented ice plates. *Geophys. Res. Lett.* **24**, 771–774.

Mishchenko, M. I., Hovenier, J. W., and Travis, L. D., eds. (1999a). Special issue on light scattering by nonspherical particles'98. *J. Quant. Spectrosc. Radiat. Transfer* **63**, 127–738.

Mishchenko, M. I., Travis, L. D., and Hovenier, J. W., coordinators (1999b). Special sec-

tion on electromagnetic scattering by nonspherical particles. *J. Geophys. Res.* **104**, 31 671–31 808.

Mishchenko, M. I., Hovenier, J. W., and Travis, L. D., eds. (2000a). *Light Scattering by Nonspherical Particles: Theory, Measurements, and Applications* (Academic Press, San Diego).

Mishchenko, M. I., Hovenier, J. W., and Travis, L. D. (2000b). Concepts, terms, notation. In *Light Scattering by Nonspherical Particles: Theory, Measurements, and Applications*, eds. M. I. Mishchenko, J. W. Hovenier, and L. D. Travis, pp. 3–27 (Academic Press, San Diego).

Mishchenko, M. I., Hovenier, J. W., Wiscombe, W. J., and Travis, L. D. (2000c). Overview of scattering by nonspherical particles. In *Light Scattering by Nonspherical Particles: Theory, Measurements, and Applications*, eds. M. I. Mishchenko, J. W. Hovenier, and L. D. Travis, pp. 29–60 (Academic Press, San Diego).

Mishchenko, M. I., Travis, L. D., and Macke, A. (2000d). *T*-matrix method and its applications. In *Light Scattering by Nonspherical Particles: Theory, Measurements, and Applications*, eds. M. I. Mishchenko, J. W. Hovenier, and L. D. Travis, pp. 147–172 (Academic Press, San Diego).

Mishchenko, M. I., Luck, J.-M., and Nieuwenhuizen, Th. M. (2000e). Full angular profile of the coherent polarization opposition effect. *J. Opt. Soc. Am. A* **17**, 888–891.

Mishchenko, M. I., Hovenier, J. W., and Mackowski, D. W. (2004a). Singe scattering by a small volume element. *J. Opt. Soc. Am. A* **21**, 71–87.

Mishchenko, M. I., Videen, G., Babenko, V. A., *et al*. (2004b). *T*-matrix theory of electromagnetic scattering by particles and its applications: a comprehensive reference database. *J. Quant. Spectrosc. Radiat. Transfer* **88**, 357–406.

Mittra, R., and Ramahi, O. (1990). Absorbing boundary conditions for the direct solution of partial differential equations arising in electromagnetic scattering problems. In *Finite Element and Finite Difference Methods in Electromagnetic Scattering*, ed. M. A. Morgan, pp. 133–173 (Elsevier, New York).

Moreno, F., and González, F., eds. (2000). *Light Scattering from Microstructures* (Springer, Berlin).

Morgan, M. A. (1980). Finite element computation of microwave scattering by raindrops. *Radio Sci.* **15**, 1109–1119.

Morgan, M. A., ed. (1990). *Finite Element and Finite Difference Methods in Electromagnetic Scattering* (Elsevier, New York).

Morgan, M. A., and Mei, K. K. (1979). Finite-element computation of scattering by inhomogeneous penetrable bodies of revolution. *IEEE Trans. Antennas Propag.* **27**, 202–214.

Morita, N. (1979). Another method of extending the boundary condition for the problem of scattering by dielectric cylinders. *IEEE Trans. Antennas Propag.* **27**, 97–99.

Morrison, J. A., and Cross, M.-J. (1974). Scattering of a plane electromagnetic wave by axisymmetric raindrops. *Bell Syst. Tech. J.* **53**, 955–1019.

Morse, P. M., and Feshbach, H. (1953). *Methods of Theoretical Physics* (McGraw-Hill, New York).

Mugnai, A., and Wiscombe, W. J. (1980). Scattering of radiation by moderately nonspherical particles. *J. Atmos. Sci.* **37**, 1291–1307.

Mugnai, A., and Wiscombe, W. J. (1986). Scattering from nonspherical Chebyshev particles. 1. Cross sections, single-scattering albedo, asymmetry factor, and backscattered fraction. *Appl. Opt.* **25**, 1235–1244.

Mugnai, A., and Wiscombe, W. J. (1989). Scattering from nonspherical Chebyshev particles. 3. Variability in angular scattering patterns. *Appl. Opt.* **28**, 3061–3073.

Muinonen, K. (1989). Scattering of light by crystals: a modified Kirchhoff approximation. *Appl. Opt.* **28**, 3044–3050.

Muinonen, K. (1993). Coherent backscattering by solar system dust particles. In *Asteroids, Comets, Meteors*, eds. A. Milani *et al*., pp. 271–296 (Kluwer Academic Publishers, Dordrecht, The Netherlands).

Muinonen, K. (1996). Light scattering by Gaussian random particles: Rayleigh and Rayleigh–Gans approximations. *J. Quant. Spectrosc. Radiat. Transfer* **55**, 603–613.

Muinonen, K. (2000). Light scattering by stochastically shaped particles. In *Light Scattering by Nonspherical Particles: Theory, Measurements, and Applications*, eds. M. I. Mishchenko, J. W. Hovenier, and L. D. Travis, pp. 323–352 (Academic Press, San Diego).

Muinonen, K., Lumme, K., Peltoniemi, J., and Irvine, W. M. (1989). Light scattering by randomly oriented crystals. *Appl. Opt.* **28**, 3051–3060.

Muinonen, K., Nousiainen, T., Fast, P., *et al*. (1996). Light scattering by Gaussian random particles: ray optics approximation. *J. Quant. Spectrosc. Radiat. Transfer* **55**, 577–601.

Mulholland, G. W., Bohren, C. F., and Fuller, K. A. (1994). Light scattering by agglomerates: coupled electric and magnetic dipole method. *Langmuir* **10**, 2533–2546.

Muñoz, O., Volten, H., de Haan, J. F., *et al*. (2000a). Experimental determination of scattering matrices of olivine and Allende meteorite particles. *Astron. Astrophys.* **360**, 777–788.

Muñoz, O., Volten, H., Vermeulen, K., *et al*. (2000b). Scattering matrices of fly-ash and green clay particles. In *Light Scattering by Nonspherical Particles: Halifax Contributions*, eds. G. Videen, Q. Fu, and P. Chýlek, pp. 45–48 (Army Research Laboratory, Adelphi, MD).

Muñoz, O., Volten, H., de Haan, J. F., *et al*. (2001). Experimental determination of scattering matrices of randomly oriented fly ash and clay particles at 442 and 633 nm. *J. Geophys. Res.* **106**, 22 833–22 844.

Murayama, T., Okamoto, H., Kaneyasu, N., *et al*. (1999). Application of lidar depolarization measurement in the atmospheric boundary layer: effects of dust and sea-salt particles. *J. Geophys. Res.* **104**, 31 781–31 792.

Nag, S., and Sinha, B. P. (1995). Electromagnetic plane wave scattering by a system of two uniformly lossy dielectric prolate spheroids in arbitrary orientation. *IEEE Trans. Antennas Propag.* **43**, 322–327.

Nakajima, T., and King, M. D. (1990). Determination of optical thickness and effective particle radius of clouds from reflected solar radiation measurements. I. Theory. *J. Atmos. Sci.* **47**, 1878–1893.

Nakajima, T., Tanaka, M., Yamano, M., *et al*. (1989). Aerosol optical characteristics in the yellow sand events observed in May, 1982 at Nagasaki. II. Models. *J. Meteorol. Soc. Japan* **67**, 279–291.

Napper, D. H., and Ottewill, R. H. (1963). Light scattering studies on monodisperse silver bromide sols. In *ICES Electromagnetic Scattering*, ed. M. Kerker, pp. 377–386 (Pergamon Press, New York).

Nevitt, T. J., and Bohren, C. F. (1984). Infrared backscattering by irregularly shaped particles: a statistical approach. *J. Climate Appl. Meteorol.* **23**, 1342–1349.

Newton, R. G. (1966). *Scattering Theory of Waves and Particles* (McGraw-Hill, New York).

Ngo, D., Videen, G., and Dalling, R. (1997). Chaotic light scattering from a system of osculating, conducting spheres. *Phys. Lett. A* **227**, 197–202.

Nieminen, T. A., Rubinsztein-Dunlop, H., and Heckenberg, N. R. (2001). Calculation and optical measurement of laser trapping forces on non-spherical particles. *J. Quant. Spectrosc. Radiat. Transfer* **70**, 627–637.

Nousiainen, T. (2000). Scattering of light by raindrops with single-mode oscillations. *J. Atmos. Sci.* **57**, 789–802.

Nousiainen, T., and Muinonen, K. (1999). Light scattering by Gaussian, randomly oriented raindrops. *J. Quant. Spectrosc. Radiat. Transfer* **63**, 643–666.

Nousiainen, T., Muinonen, K., Avelin, J., and Sihvola, A. (2001). Microwave backscattering by nonspherical ice particles at 5.6 GHz using second-order perturbation series. *J. Quant. Spectrosc. Radiat. Transfer* **70**, 639–661.

Nussenzveig, H. M. (1992). *Diffraction Effects in Semiclassical Scattering* (Cambridge University Press, Cambridge).

Obelleiro, F., Rodríguez, J. L., and Wriedt, T., eds. (1999). *4th Conference on Electromagnetic and Light Scattering by Nonspherical Particles: Theory and Applications* (University of Vigo, Vigo, Spain).

Oguchi, T. (1960). Attenuation of electromagnetic waves due to rain with distorted raindrops. *J. Radio Res. Lab. Japan* **7**, 467–485.

Oguchi, T. (1973). Scattering properties of oblate raindrops and cross polarization of radio waves due to rain: calculations at 19.3 and 34.8 GHz. *J. Radio Res. Lab. Japan* **20**, 79–118.

Oguchi, T. (1981). Scattering from hydrometeors: a survey. *Radio Sci.* **16**, 691–730.

Oguchi, T. (1983). Electromagnetic wave propagation and scattering in rain and other hydrometeors. *Proc. IEEE* **71**, 1029–1078.

Oguchi, T., and Hosoya, Y. (1974). Scattering properties of oblate raindrops and cross polarization of radio waves due to rain. II. Calculations at microwave and millimeter wave regions. *J. Radio Res. Lab. Japan* **21**, 191–259.

Ogura, H., and Takahashi, N. (1990). Scattering of waves from a random spherical surface – Mie scattering. *J. Math. Phys.* **31**, 61–75.

Okamoto, H. (1995). Light scattering by clusters: the $a_1$-term method. *Opt. Rev.* **2**, 407–412.

Okamoto, H., Macke, A., Quante, M., and Raschke, E. (1995). Modeling of backscattering by nonspherical ice particles for the interpretation of cloud radar signals at 94 GHz. An error analysis. *Beitr. Phys. Atmos.* **68**, 319–334.

Onaka, T. (1980). Light scattering by spheroidal grains. *Ann. Tokyo Astron. Observ.* **18**, 1–54.

O'Neill, E. L. (1992). *Introduction to Statistical Optics* (Dover Publications, New York).

Ozrin, V. D. (1992). Exact solution for coherent backscattering of polarized light from a random medium of Rayleigh scatterers. *Waves in Random Media* **2**, 141–164.

Palik, E. D., and Ghosh, G. (1997). *Handbook of Optical Constants of Solids* (Academic Press, San Diego).

Papadakis, S. N., Uzunoglu, N. K., and Capsalis, C. N. (1990). Scattering of a plane wave by

a general anisotropic dielectric ellipsoid. *J. Opt. Soc. Am. A* **7**, 991–997.

Paramonov, L. E. (1995). *T*-matrix approach and the angular momentum theory in light scattering problems by ensembles of arbitrarily shaped particles. *J. Opt. Soc. Am. A* **12**, 2698–2707.

Paulick, T. C. (1990). Applicability of the Rayleigh hypothesis to real materials. *Phys. Rev. B* **42**, 2801–2824.

Pecora, R., ed. (1985). *Dynamic Light Scattering: Applications of Photon Correlation Spectroscopy* (Plenum Press, New York).

Peltoniemi, J. I. (1996). Variational volume integral equation method for electromagnetic scattering by irregular grains. *J. Quant. Spectrosc. Radiat. Transfer* **55**, 637–647.

Peltoniemi, J. I., Lumme, K., Muinonen, K., and Irvine, W. M. (1989). Scattering of light by stochastically rough particles. *Appl. Opt.* **28**, 4088–4095.

Perelman, A. Y. (1996). Scattering by particles with radially variable refractive index. *Appl. Opt.* **35**, 5452–5460.

Perrin, J.-M., and Lamy, P. L. (1986). Light scattering by large particles. II. A vectorial description in the eikonal picture. *Opt. Acta* **33**, 1001–1022.

Perry, R. J., Hunt, A. J., and Huffman, D. R. (1978). Experimental determination of Mueller scattering matrices for nonspherical particles. *Appl. Opt.* **17**, 2700–2710.

Peterson, B., and Ström, S. (1973). *T* matrix for electromagnetic scattering from an arbitrary number of scatterers and representations of E(3)*. *Phys. Rev. D* **8**, 3661–3678.

Peterson, B., and Ström, S. (1974). *T*-matrix formulation of electromagnetic scattering from multilayered scatterers. *Phys. Rev. D* **10**, 2670–2684.

Peterson, A. F., Ray, S. L., and Mittra, R. (1998). *Computational Methods for Electromagnetics* (IEEE Press, New York).

Petrova, E. V. (1999). Mars aerosol optical thickness retrieved from measurements of the polarization inversion angle and the shape of dust particles. *J. Quant. Spectrosc. Radiat. Transfer* **63**, 667–676.

Petrova, E. V., Jockers, K., and Kiselev, N. N. (2000). Light scattering by aggregates with sizes comparable to the wavelength: an application to cometary dust. *Icarus* **148**, 526–536.

Petrushin, A. G. (1994). Light scattering by hexagonal ice prisms. I. Small scattering angles. *Izv., Atmos. Oceanic Phys.* (Engl. transl.) **30**, 291–299.

Pike, E. R., and Abbiss, J. B., eds. (1997). *Light Scattering and Photon Correlation Spectroscopy* (Kluwer Academic Publishers, Dordrecht, The Netherlands).

Piller, N. B. (1999). Coupled-dipole approximation for high permittivity materials. *Opt. Commun.* **160**, 10–14.

Piller, N. B., and Martin, O. J. F. (1998a). Extension of the generalized multipole technique to three-dimensional anisotropic scatterers. *Opt. Lett.* **23**, 579–581.

Piller, N. B., and Martin, O. J. F. (1998b). Increasing the performance of the coupled-dipole approximation: a spectral approach. *IEEE Trans. Antennas Propag.* **46**, 1126–1137.

Pitter, M. C., Hopcraft, K. I., Jakeman, E., and Walker, J. G. (1999). Structure of polarization fluctuations and their relation to particle shape. *J. Quant. Spectrosc. Radiat. Transfer* **63**, 433–444.

Platt, C. M. R. (1978). Lidar backscattering from horizontally oriented ice crystal plates. *J.*

*Appl. Meteorol.* **17**, 482–488.

POAN Research Group, ed. (1998). *New Aspects of Electromagnetic and Acoustic Wave Diffusion* (Springer, Berlin).

Pocock, M. D., Bluck, M. J., and Walker, S. P. (1998). Electromagnetic scattering from 3-D curved dielectric bodies using time-domain integral equations. *IEEE Trans. Antennas Propag.* **46**, 1212–1219.

Poggio, A. J., and Miller, E. K. (1973). Integral equation sol utions of three-dimensional scattering problems. In *Computer Techniques for Electromagnetics*, ed. R. Mittra, pp. 159–264 (Pergamon, Oxford).

Pollack, J. B., and Cuzzi, J. N. (1980). Scattering by nonspherical particles of size comparable to a wavelength: a new semi-empirical theory and its application to tropospheric aerosols. *J. Atmos. Sci.* **37**, 868–881.

Porstendorfer, J., Berg, K.-J., and Berg, G. (1999). Calculation of extinction and scattering spectra of large spheroidal gold particles embedded in a glass matrix. *J. Quant. Spectrosc. Radiat. Transfer* **63**, 479–486.

Prabhu, D. R., Davies, M., and Videen, G. (2001). Light scattering calculations from oleic-acid droplets with water inclusions. *Opt. Express* **8**, 308–313.

Prigent, C., Pardo, J. R., Mishchenko, M. I., and Rossow, W. B. (2001). Microwave polarized signatures generated within cloud systems: Special Sensor Microwave Imager (SSM/I) observations interpreted with radiative transfer simulations. *J. Geophys. Res.* **106**, 28 243–28 258.

Prishivalko, A. P., Babenko, V. A., and Kuzmin, V. N. (1984). *Scattering and Absorption of Light by Inhomogeneous and Anisotropic Spherical Particles* (Nauka i Tekhnika, Minsk, USSR) (in Russian).

Pritchard, B. S., and Elliott, W. G. (1960). Two instruments for atmospheric optics measurements. *J. Opt. Soc. Am.* **50**, 191–202.

Probert-Jones, J. R. (1984). Resonance component of backscattering by large dielectric spheres. *J. Opt. Soc. Am. A* **1**, 822–830.

Prodi, F., Sturniolo, O., Battaglia, A., and Medini, R. (1999). Radar parameters simulation for populations of spherical and non-spherical hydrometeors: dependence on size distributions, shapes and composition. *J. Quant. Spectrosc. Radiat. Transfer* **63**, 677–699.

Purcell, E. M., and Pennypacker, C. R. (1973). Scattering and absorption of light by nonspherical dielectric grains. *Astrophys. J.* **186**, 705–714.

Pye, D. (2001). *Polarised Light in Science and Nature* (Institute of Physics Publishing, Bristol, UK).

Qingan, W., Zixiang, O., Liping, L., and Zengming, C. (1998). A comparative study on the backscattering ability of raindrops and ice particles (hail). *Contr. Atmos. Phys.* **71**, 377–386.

Qingan, W., Kang, C., and Xiang, O. Y. Z. (1999). Discussion of key algorithms for computing scattering cross sections using the separation of variables method for spheroids. *J. Quant. Spectrosc. Radiat. Transfer* **63**, 251–261.

Quinby-Hunt, M. S., Hull, P. G., and Hunt, A. (2000). Polarized light scattering in the marine environment. In *Light Scattering by Nonspherical Particles: Theory, Measurements, and Applications*, eds. M. I. Mishchenko, J. W. Hovenier, and L. D. Travis, pp. 525–554 (Aca-

demic Press, San Diego).

Quinten, M. (1999). Optical effects associated with aggregates of clusters. *J. Cluster Sci.* **10**, 319–358.

Quinten, M., and Kreibig, U. (1993). Absorption and elastic scattering of light by particle aggregates. *Appl. Opt.* **32**, 6173–6182.

Quinten, M., and Rostalski, J. (1996). Lorenz–Mie theory for spheres immersed in an absorbing host medium. *Part. Part. Syst. Charact.* **13**, 89–96.

Quirantes, A. (1999). Light scattering properties of spheroidal coated particles in random orientation. *J. Quant. Spectrosc. Radiat. Transfer* **63**, 263–275.

Quirantes, A., and Delgado, A. V. (1995). Particle size determinations in colloidal suspensions of randomly oriented ellipsoids. *Progr. Colloid Polymer Sci.* **98**, 145–150.

Quirantes, A., and Delgado, A. V. (2001). Scattering cross sections of randomly orientated coated spheroids. *J. Quant. Spectrosc. Radiat. Transfer* **70**, 261–272.

Ravey, J.-C., and Mazeron, P. (1982). Light scattering in the physical optics approximation: application to large spheroids. *J. Opt. (Paris)* **13**, 273–282.

Ravey, J.-C., and Mazeron, P. (1983). Light scattering by large spheroids in the physical optics approximation: numerical comparison with other approximate and exact results. *J. Opt. (Paris)* **14**, 29–41.

Ray, A. K., and Nandakumar, R. (1995). Simultaneous determination of size and wavelength-dependent refractive indices of thin-layered droplets from optical resonances. *Appl. Opt.* **34**, 7759–7770.

Rayleigh, Lord (1897). On the incidence of aerial and electric waves upon small obstacles in the form of ellipsoids or elliptic cylinders, and on the passage of electric waves through a circular aperture in a conducting screen. *Phil. Mag.* **44**, 28–52.

Reagan, J. A., McCormick, M. P., and Spinhirne, J. D. (1989). Lidar sensing of aerosols and clouds in the troposphere and stratosphere. *Proc. IEEE* **77**, 433–448.

Reichardt, J., Tsias, A., and Behrendt, A. (2000a). Optical properties of PSC Ia-enhanced at UV and visible wavelengths: Model and observations. *Geophys. Res. Lett.* **27**, 201–204.

Reichardt, J., Hess, M., and Macke, A. (2000b). Lidar inelastic multiple-scattering parameters of cirrus particle ensembles determined with geometrical-optics crystal phase functions. *Appl. Opt.* **39**, 1895–1910.

Rockwitz, K.-D. (1989). Scattering properties of horizontally oriented ice crystal columns in cirrus clouds. *Appl. Opt.* **28**, 4103–4110.

Rogers, C., and Martin, P. G. (1979). On the shape of interstellar grains. *Astrophys. J.* **228**, 450–464.

Rojas, R. G. (1992). Integral equations for the scattering by a three dimensional inhomogeneous chiral body. *J. Electromagn. Waves Appl.* **6**, 733–750.

Roll, G., and Schweiger, G. (2000). Geometrical optics model of Mie resonances. *J. Opt. Soc. Am. A* **17**, 1301–1311.

Rose, M. E. (1995). *Elementary Theory of Angular Momentum* (Dover Publications, New York).

Rosen, J. M., Pinnick, R. G., and Garvey, D. M. (1997). Nephelometer optical response model for the interpretation of atmospheric aerosol measurements. *Appl. Opt.* **36**, 2642–2649.

Rossow, W. B., and Schiffer, R. A. (1999). Advances in understanding clouds from ISCCP.

*Bull. Am. Meteorol. Soc.* **80**, 2261–2287.

Rother, T. (1998). Generalization of the separation of variables method for non-spherical scattering of dielectric objects. *J. Quant. Spectrosc. Radiat. Transfer* **60**, 335–353.

Rother, T., and Schmidt, K. (1996). The discretized Mie-formalism for plane wave scattering on dielectric objects with non-separable geometries. *J. Quant. Spectrosc. Radiat. Transfer* **55**, 615–625.

Rother, T., Havemann, S., and Schmidt, K. (1999). Scattering of plane waves on finite cylinders with non-circular cross-sections. *Progr. Electromagn. Res. (PIER)* **23**, 79–105.

Rozenberg, G. V. (1977). The light ray (contribution to the theory of the light field). *Sov. Phys. Usp.* **20**, 55–79.

Rozenberg, G. V., Lyubovtseva, Yu. S., Kadyshevich, Ye. A., and Amnuil, N. R. (1970). Measurement of the light scattering matrix of a hydrosol. *Izv. Acad. Sci. USSR, Atmos. Oceanic Phys.* (Engl. transl.) **6**, 747–750.

Ruppin, R. (1999). Effects of high-order multipoles on the extinction spectra of dispersive bispheres. *Opt. Commun.* **168**, 35–38.

Sachweh, B. A., Dick, W. D., and McMurry, P. H. (1995). Distinguishing between spherical and nonspherical particles by measuring the variability in azimuthal light scattering. *Aerosol Sci. Technol.* **23**, 373–391.

Sachweh, B., Barthel, H., Polke, R., *et al*. (1999). Particle shape and structure analysis from the spatial intensity pattern of scattered light using different measuring devices. *J. Aerosol Sci.* **30**, 1257–1270.

Saija, R., Iatì, M. A., Denti, P., *et al*. (2001). Backscattered intensity from model atmospheric ice crystals in the millimeter-wave range. *Appl. Opt.* **40**, 5337–5342.

Sano, I., and Mukai, S. (2000). Algorithm description of system flow for global aerosol distribution. *Appl. Math. Comput.* **116**, 79–91.

Sarkar, D., and Halas, N. J. (1997). General vector basis function solution of Maxwell's equations. *Phys. Rev. E* **56**, 1102–1112.

Sassen, K. (1991). The polarization lidar technique for cloud research: a review and current assessment. *Bull. Am. Meteorol. Soc.* **72**, 1848–1866.

Sassen, K. (2000). Lidar backscatter depolarization technique for cloud and aerosol research. In *Light Scattering by Nonspherical Particles: Theory, Measurements, and Applications*, eds. M. I. Mishchenko, J. W. Hovenier, and L. D. Travis, pp. 393–416 (Academic Press, San Diego).

Sassen, K., and Arnott, W. P. (1998). Light and color in the open air: introduction to the feature issue. *Appl. Opt.* **37**, 1425–1426.

Sassen, K., and Hsueh, C.-y. (1998). Contrail properties derived from high-resolution polarization lidar studies during SUCCESS. *Geophys. Res. Lett.* **25**, 1165–1168.

Sassen, K., and Liou, K.-N. (1979). Scattering of polarized laser light by water droplet, mixed-phase and ice crystal clouds. I. Angular scattering patterns. *J. Atmos. Sci.* **36**, 838–851.

Sassen, K., Knight, N. C., Takano, Y., and Heymsfield, A. J. (1994). Effects of ice-crystal structure on halo formation: cirrus cloud experimental and ray-tracing modeling studies. *Appl. Opt.* **33**, 4590–4601.

Sassen, K., Comstock, J. M., Wang, Zh., and Mace, G. G. (2001). Cloud and aerosol research

capabilities at FARS: the Facility for Atmospheric Remote Sensing. *Bull. Am. Meteorol. Soc.* **82**, 1119–1138.

Sato, M., Travis, L. D., and Kawabata, K. (1996). Photopolarimetry analysis of the Venus atmosphere in polar regions. *Icarus* **124**, 569–585.

Saxon, D. S. (1955a). Tensor scattering matrix for the electromagnetic field. *Phys. Rev.* **100**, 1771–1775.

Saxon, D. S. (1955b). Lectures on the scattering of light (scientific report No. 9, Department of Meteorology, University of California at Los Angeles).

Schaefer, R. W. (1980). Calculations of the light scattered by randomly oriented ensembles of spheroids of size comparable to the wavelength. Ph. D. dissertation, State University of New York at Albany.

Schiffer, R. (1989). Light scattering by perfectly conducting statistically irregular particles. *J. Opt. Soc. Am. A* **6**, 385–402.

Schiffer, R. (1990). Perturbation approach for light scattering by an ensemble of irregular particles of arbitrary material. *Appl. Opt.* **29**, 1536–1550.

Schiffer, R., and Thielheim, K. O. (1979). Light scattering by dielectric needles and disks. *J. Appl. Phys.* **50**, 2476–2483.

Schmidt, K., Rother, T., and Wauer, J. (1998). The equivalence of applying the Extended Boundary Condition and the continuity conditions for solving electromagnetic scattering problems. *Opt. Commun.* **150**, 1–4.

Schmitt, J. M., and Xiang, S. H. (1998). Cross-polarized backscatter in optical coherence tomography of biological tissue. *Opt. Lett.* **23**, 1060–1062.

Schneider, J. B., and Peden, I. C. (1988). Differential cross section of a dielectric ellipsoid by the *T*-matrix extended boundary condition method. *IEEE Trans. Antennas Propag.* **36**, 1317–1321.

Schneider, J., Brew, J., and Peden, I. C. (1991). Electromagnetic detection of buried dielectric targets. *IEEE Trans. Geosci. Remote Sens.* **29**, 555–562.

Schuerman, D. W., ed. (1980). *Light Scattering by Irregularly Shaped Particles* (Plenum Press, New York).

Schuh, R., and Wriedt, T. (2001). Computer programs for light scattering by particles with inclusions. *J. Quant. Spectrosc. Radiat. Transfer* **70**, 715–723.

Schulten, K., and Gordon, R. G. (1975). Exact recursive evaluation of 3*j*- and 6*j*-coefficients for quantum-mechanical coupling of angular momenta. *J. Math. Phys.* **16**, 1961–1970.

Schulz, F. M., Stamnes, K., and Stamnes, J. J. (1998a). Scattering of electromagnetic waves by spheroidal particles: a novel approach exploiting the *T* matrix computed in spheroidal coordinates. *Appl. Opt.* **37**, 7875–7896.

Schulz, F. M., Stamnes, K., and Stamnes, J. J. (1998b). Modeling the radiative transfer properties of media containing particles of moderately and highly elongated shape. *Geophys. Res. Lett.* **25**, 4481–4484.

Schulz, F. M., Stamnes, K., and Stamnes, J. J. (1999a). Point-group symmetries in electromagnetic scattering. *J. Opt. Soc. Am. A* **16**, 853–865.

Schulz, F. M., Stamnes, K., and Stamnes, J. J. (1999b). Shape dependence of the optical properties in size-shape distributions of randomly oriented prolate spheroids, including highly

elongated shapes. *J. Geophys. Res.* **104**, 9413–9421.

Sebak, A. R., and Sinha, B. P. (1992). Scattering by a conducting spheroidal object with dielectric coating at axial incidence. *IEEE Trans. Antennas Propag.* **40**, 268–274.

Secker, D. R., Kaye, P. H., Greenaway, R. S., *et al*. (2000). Light scattering from deformed droplets and droplets with inclusions. I. Experimental results. *Appl. Opt.* **39**, 5023–5030.

Secker, D. R., Kaye, P. H., and Hirst, E. (2001). Real-time observation of the change in light scattering from droplets with increasing deformity. *Opt. Express* **8**, 290–295.

Seker, S. S., and Schneider, A. (1988). Electromagnetic scattering from a dielectric cylinder of finite length. *IEEE Trans. Antennas Propag.* **36**, 303–307.

Senior, T. B. A., and Weil, H. (1977). Electromagnetic scattering and absorption by thin walled dielectric cylinders with application to ice crystals. *Appl. Opt.* **16**, 2979–2985.

Shafai, L., ed. (1991). Thematic issue on computational electromagnetics. *Comput. Phys. Commun.* **68**, 1–498.

Sharma, S. K., and Somerford, D. J. (1999). Scattering of light in the eikonal approximation. *Progr. Opt.* **39**, 213–290.

Sheng, P. (1995). *Introduction to Wave Scattering, Localization, and Mesoscopic Phenomena* (Academic Press, San Diego).

Sheng, X.-Q., Jin, J.-M., Song, J., *et al*. (1998). On the formulation of hybrid finite-element and boundary-integral methods for 3-D scattering. *IEEE Trans. Antennas Propag.* **46**, 303–311.

Shepelevich, N. V., Prostakova, I. V., and Lopatin, V. N. (1999). Extrema in the light-scattering indicatrix of a homogeneous spheroid. *J. Quant. Spectrosc. Radiat. Transfer* **63**, 353–367.

Shepherd, J. W., and Holt, A. R. (1983). The scattering of electromagnetic radiation from finite dielectric circular cylinders. *J. Phys. A: Math. Gen.* **16**, 651–662.

Shifrin, K. S. (1968). *Scattering of Light in a Turbid Medium* (NASA Technical Translation NASA TT F-477). (Original Russian edition: Gostehteorizdat, Moscow, 1951).

Shifrin, K. S., and Mikulinsky, I. A. (1987). Ensemble approach to the problem of light scattering by a system of tenuous particles. *Appl. Opt.* **26**, 3012–3017.

Shkuratov, Yu. G. (1994). Light backscattering by the solid surfaces of celestial bodies: theoretical models of the opposition effect. *Solar Syst. Res.* **28**, 418–431.

Shurcliff, W. A. (1962). *Polarized Light: Production and Use* (Harvard University Press, Cambridge, MA).

Shvalov, A. N., Soini, J. T., Surovtsev, I. V., *et al*. (2000). Individual *Escherichia coli* cells studied from light scattering with the scanning flow cytometer. *Cytometry* **41**, 41–45.

Siewert, C. E. (1981). On the equation of transfer relevant to the scattering of polarized light. *Astrophys. J.* **245**, 1080–1086.

Sihvola, A. (1999). *Electromagnetic Mixing Formulas and Applications* (IEE Press, London).

Silvester, P. P., and Ferrari, R. L. (1996). *Finite Elements for Electrical Engineers* (Cambridge University Press, New York).

Singham, S. B. (1989). Theoretical factors in modeling polarized light scattering by arbitrary particles. *Appl. Opt.* **28**, 5058–5064.

Singham, S. B., and Bohren, C. F. (1987). Light scattering by an arbitrary particle: a physical

reformulation of the coupled dipole method. *Opt. Lett.* **12**, 10–12.

Singham, S. B., and Bohren, C. F. (1988). Light scattering by an arbitrary particle: the scattering-order formulation of the coupled-dipole method. *J. Opt. Soc. Am. A* **5**, 1867–1872.

Singham, M. K., Singham, S. B., and Salzman, G. C. (1986). The scattering matrix for randomly oriented particles. *J. Chem. Phys.* **85**, 3807–3815.

Sinha, B. P., and MacPhie, R. H. (1977). Electromagnetic scattering by prolate spheroids for plane waves with arbitrary polarization and angle of incidence. *Radio Sci.* **12**, 171–184.

Siqueira, P. R., and Sarabandi, K. (2000). T-matrix determination of effective permittivity for three-dimensional dense random media. *IEEE Trans. Antennas Propag.* **48**, 317–327.

Skaropoulos, N. C., Ioannidou, M. P., and Chrissoulidis, D. P. (1994). Indirect mode-matching solution to scattering from a dielectric sphere with an eccentric inclusion. *J. Opt. Soc. Am. A* **11**, 1859–1866.

Sobolev, V. V. (1974). *Light Scattering in Planetary Atmospheres* (Pergamon Press, Oxford).

Somsikov, V. V. (1996). Optical properties of two-layered spheroidal dust grains. *Astron. Lett.* **22**, 696–703.

Sorensen, C. M. (2001). Light scattering by fractal aggregates: a review. *Aerosol Sci. Technol.* **35**, 648–687.

Stamatakos, G. S., and Uzunoglu, N. K. (1997). An integral equation solution to the scattering of electromagnetic radiation by a linear chain of interacting triaxial dielectric ellipsoids. The case of a red blood cell rouleau. *J. Electromagn. Waves Appl.* **11**, 949–980.

Stamatakos, G. S., Yova, D., and Uzunoglu, N. K. (1997). Integral equation model of light scattering by an oriented monodisperse system of triaxial dielectric ellipsoids: application in ectacytometry. *Appl. Opt.* **36**, 6503–6512.

Stammes, P. (1989). Light scattering properties of aerosols and the radiation inside a planetary atmosphere. Ph. D. dissertation, Free University, Amsterdam.

Stephen, M. J., and Cwilich, G. (1986). Rayleigh scattering and weak localization: effect of polarization. *Phys. Rev. B* **34**, 7564–7572.

Stephens, G. L. (1994). *Remote Sensing of the Lower Atmosphere* (Oxford University Press, New York).

Stephens, G. L., Tsay, S.-C., Stackhouse, P. W., Jr., and Flatau, P. J. (1990). The relevance of the microphysical and radiative properties of cirrus clouds to climate and climatic feedback. *J. Atmos. Sci.* **47**, 1742–1753.

Stevenson, A. F. (1953). Solution of electromagnetic scattering problems as power series in the ratio (dimension of scatterer/wavelength). *J. Appl. Phys.* **24**, 1134–1142.

Stevenson, A. F. (1968). Light scattering by spheroidal particles oriented by streaming. *J. Chem. Phys.* **49**, 4545–4550.

Straka, J. M., Zrnić, D. S., and Ryzhkov, A. V. (2000). Bulk hydrometeor classification and quantification using polarimetric radar data: synthesis of relations. *J. Appl. Meteorol.* **39**, 1341–1372.

Stratton, J. A. (1941). *Electromagnetic Theory* (McGraw Hill, New York).

Streekstra, G. J., Hoekstra, A. G., and Heethaar, R. M. (1994). Anomalous diffraction by arbitrarily oriented ellipsoids: applications in ektacytometry. *Appl. Opt.* **33**, 7288–7296.

Ström, S. (1975). On the integral equations for electromagnetic scattering. *Am. J. Phys.* **43**, 1060–1069.

Ström, S., and Zheng, W. (1987). Basic features of the null field method for dielectric scatterers. *Radio Sci.* **22**, 1273–1281.

Ström, S., and Zheng, W. (1988). The null field approach to electromagnetic scattering from composite objects. *IEEE Trans. Antennas Propag.* **36**, 376–382.

Stubenrauch, C. J., Holz, R., Chédin, A., *et al.* (1999). Retrieval of cirrus ice crystal sizes from 8.3 and 11.1 μm emissivities determined by the improved initialization inversion of TIROS-N Operational Vertical Sounder observations. *J. Geophys. Res.* **104**, 31 793–31 808.

Su, C.-C. (1989). Electromagnetic scattering by a dielectric body with arbitrary inhomogeneity and anisotropy. *IEEE Trans. Antennas Propag.* **37**, 384–389.

Sudiarta, I. W., and Chýlek, P. (2001). Mie-scattering formalism for spherical particles embedded in an absorbing medium. *J. Opt. Soc. Am. A* **18**, 1275–1278.

Sun, W., and Fu, Q. (1999). Anomalous diffraction theory for arbitrarily oriented hexagonal crystals. *J. Quant. Spectrosc. Radiat. Transfer* **63**, 727–737.

Sun, W., and Fu, Q. (2000). Finite-difference time-domain solution of light scattering by dielectric particles with large complex refractive indices. *Appl. Opt.* **39**, 5569–5578.

Sun, W., Fu, Q., and Chen, Z. (1999). Finite-difference time-domain solution of light scattering by dielectric particles with a perfectly matched layer absorbing boundary condition. *Appl. Opt.* **38**, 3141–3151.

Swatek, D. R., and Ciric, I. R. (2000a). Reduction of multiply-nested dielectric bodies for wave scattering analysis by single source surface integral equations. *J. Electromagn. Waves Appl.* **14**, 405–422.

Swatek, D. R., and Ciric, I. R. (2000b). A recursive single-source surface integral equation analysis for wave scattering by heterogeneous dielectric bodies. *IEEE Trans. Antennas Propag.* **48**, 1175–1185.

Szegő, G. (1959). *Orthogonal Polynomials* (American Mathematical Society, New York).

Taflove, A. (1995). *Computational Electrodynamics: The Finite-difference Time-domain Method* (Artech House, Boston, MA).

Taflove, A., ed. (1998). *Advances in Computational Electrodynamics: The Finite-difference Time-domain Method* (Artech House, Boston, MA).

Tai, C.-T. (1993). *Dyadic Green Functions in Electromagnetic Theory* (IEEE Press, New York).

Takano, Y., and Asano, S. (1983). Fraunhofer diffraction by ice crystals suspended in the atmosphere. *J. Meteorol. Soc. Japan* **61**, 289–300.

Takano, Y., and Jayaweera, K. (1985). Scattering phase matrix for hexagonal ice crystals computed from ray optics. *Appl. Opt.* **24**, 3254–3263.

Takano, Y., and Liou, K. N. (1989a). Solar radiative transfer in cirrus clouds. I. Single-scattering and optical properties of hexagonal ice crystals. *J. Atmos. Sci.* **46**, 3–19.

Takano, Y., and Liou, K.N. (1989b). Radiative transfer in cirrus clouds. II. Theory and computation of multiple scattering in an anisotropic medium. *J. Atmos. Sci.* **46**, 20–36.

Takano, Y., and Liou, K. N. (1995). Solar radiative transfer in cirrus clouds. III. Light scattering by irregular ice crystals. *J. Atmos. Sci.* **52**, 818–837.

Tang, C., and Aydin, K. (1995). Scattering from ice crystals at 94 and 220 GHz millimeter wave frequencies. *IEEE Trans. Geosci. Remote Sens.* **33**, 93–99.

Tavker, S., and Kumar, P. P. (2000). Laser depolarization studies of simulated crystallized $H_2O/H_2SO_4$ clouds. *Atmos. Res.* **53**, 219–230.

Tazaki, T., Tabuchi, H., Ikeda, K., *et al*. (2000). Laboratory measurements of polarimetric radar signatures of randomly distributed spherical and spheroidal scatterers at 30 GHz. *IEE Proc. Microwave Antennas Propag.* **147**, 8–12.

Thomas, G. E., and Stamnes, K. (1999). *Radiative Transfer in the Atmosphere and Ocean* (Cambridge University Press, New York).

Thompson, R. C. (1978). An electro-optic light scattering photometric polarimeter. Ph. D. dissertation, Texas A&M University, College Station, Texas.

Thompson, R. C., Bottiger, J. R,. and Fry, E. S. (1980). Measurement of polarized light interactions via the Mueller matrix. *Appl. Opt.* **19**, 1323–1332. (Errata: **19**, 2657 (1980).)

Tishkovets, V. P. (1994). Light scattering by clusters of spherical particles. Cooperative effects under chaotic orientation. *Kinem. Phys. Celest. Bodies* **10**(2), 50–54.

Tomasko, M. G., West, R. A., and Castillo, N. D. (1978). Photometry and polarimetry of Jupiter at large phase angles. I. Analysis of imaging data of a prominent belt and a zone from Pioneer 10. *Icarus* **33**, 558–592.

Toon, O. B., and Ackerman, T. P. (1981). Algorithms for the calculation of scattering by stratified spheres. *Appl. Opt.* **20**, 3657–3660.

Toon, O. B., Tabazadeh, A., Browell, E. V., and Jordan, J. (2000). Analysis of lidar observations of Arctic polar stratospheric clouds during January 1989. *J. Geophys. Res*. **105**, 20 589–20 615.

Tozer, W. F., and Beeson, D. E. (1974). Optical model of noctilucent clouds based on polarimetric measurements from two sounding rocket campaigns. *J. Geophys. Res.* **79**, 5607–5612.

Tsang, L., Kong, J. A., and Shin, R. T. (1985). *Theory of Microwave Remote Sensing* (John Wiley & Sons, New York).

Tsang, L., Mandt, C. E., and Ding, K. H. (1992). Monte Carlo simulations of the extinction rate of dense media with randomly distributed dielectric spheres based on solution of Maxwell's equations. *Opt. Lett.* **17**, 314–316.

Tsang, L., Kong, J. A., and Ding, K.-H. (2000). *Scattering of Electromagnetic Waves: Theories and Applications* (John Wiley & Sons, New York).

Tsias, A., Wirth, M., Carslaw, K. S., *et al*. (1999). Aircraft observations of an enhanced type Ia polar stratospheric cloud during APE-POLECAT. *J. Geophys. Res.* **104**, 23 961–23 969.

Tu, H., and Ray, A. K. (2001). Analysis of time-dependent scattering spectra for studying processes associated with microdroplets. *Appl. Opt.* **40**, 2522–2534.

Ulaby, F. T., and Elachi, C., eds. (1990). *Radar Polarimetry for Geoscience Applications* (Artech House, Norwood, MA).

Ulaby, F. T., Moore, R. K., and Fung, A. K. (1981). *Microwave Remote Sensing: Active and Passive*, Vol. 1 (Artech House, Boston, MA).

Umashankar, K., Taflove, A., and Rao, S. M. (1986). Electromagnetic scattering by arbitrarily shaped three-dimensional homogeneous lossy dielectric objects. *IEEE Trans. Antennas Propag.* **34**, 758–766.

Uzunoglu, N. K., Alexopoulos, N. G., and Fikioris, J. G. (1978). Scattering from thin and

finite dielectric fibers. *J. Opt. Soc. Am.* **68**, 194–197.

van Albada, M. P., and Lagendijk, A. (1985). Observation of weak localization of light in a random medium. *Phys. Rev. Lett.* **55**, 2692–2695.

van Albada, M. P., and Lagendijk, A. (1987). Vector character of light in weak localization: spatial anisotropy in coherent backscattering from a random medium. *Phys. Rev. B* **36**, 2353–2356.

van Albada, M. P., van der Mark, M. B., and Lagendijk, A. (1987). Observation of weak localization of light in a finite slab: anisotropy effects and light-path classification. *Phys. Rev. Lett.* **58**, 361–364.

Van Bladel, J. (1964). *Electromagnetic Fields* (McGraw Hill, New York).

van de Hulst, H. C. (1957). *Light Scattering by Small Particles* (John Wiley & Sons, New York).

van de Hulst, H. C. (1980). *Multiple Light Scattering. Tables, Formulas, and Applications* (Academic Press, San Diego).

van der Mee, C. V. M., and Hovenier, J. W. (1990). Expansion coefficients in polarized light transfer. *Astron. Astrophys.* **228**, 559–568.

van Tiggelen, B. A., and Maynard, R. (1997). Reciprocity and coherent backscattering of light. In *Wave Propagation in Complex Media*, ed. G. Papanicolaou, pp. 247–271 (Springer, Berlin).

Varadan, V. K., and Varadan, V. V., eds. (1980). *Acoustic, Electromagnetic and Elastic Wave Scattering – Focus on the T-matrix Approach* (Pergamon Press, New York).

Varadan, V. V., Lakhtakia, A., and Varadan, V. K. (1989). Scattering by three-dimensional anisotropic scatterers. *IEEE Trans. Antennas Propag.* **37**, 800–802.

Varadan, V. V., Lakhtakia, A., and Varadan, V. K. (1991). *Field Representations and Introduction to Scattering* (North-Holland, Amsterdam).

Vargas, W. E., and Niklasson, G. A. (2001). Reflectance of pigmented polymer coatings: comparisons between measurements and radiative transfer calculations. *Appl. Opt.* **40**, 85–94.

Varshalovich, D. A., Moskalev, A. N., and Khersonskii, V. K. (1988). *Quantum Theory of Angular Momentum* (World Scientific, Singapore). (Original Russian edition: Nauka, Leningrad, 1975.)

Vechinski, D. A., Rao, S. M., and Sarkar, T. K. (1994). Transient scattering from three-dimensional arbitrarily shaped dielectric bodies. *J. Opt. Soc. Am. A* **11**, 1458–1470.

Vermeulen, K. (1999). Calculated scattering matrices of randomly oriented spheroids distributed over size and shape: a comparison with laboratory measurements for feldspar. Graduation report, Free University, Amsterdam.

Videen, G. (1996). Light scattering from an irregular particle behind a plane interface. *Opt. Commun.* **128**, 81–90.

Videen, G., Ngo, D., Chýlek, P., and Pinnick, R. G. (1995). Light scattering from a sphere with an irregular inclusion. *J. Opt. Soc. Am. A* **12**, 922–928.

Videen, G., Ngo, D., and Hart, M. B. (1996). Light scattering from a pair of conducting, osculating spheres. *Opt. Commun.* **125**, 275–287.

Videen, G., Pinnick, R. G., Ngo, D., *et al.* (1998a). Asymmetry parameter and aggregate particles. *Appl. Opt.* **37**, 1104–1109.

Videen, G., Sun, W., and Fu, Q. (1998b). Light scattering from irregular tetrahedral aggregates. *Opt. Commun.* **156**, 5–9.

Videen, G., Sun, W., Fu, Q., *et al.* (2000a). Light scattering from deformed droplets and droplets with inclusions. II. Theoretical treatment. *Appl. Opt.* **39**, 5031–5039.

Videen, G., Fu, Q., and Chýlek, P., eds. (2000b). *Light Scattering by Nonspherical Particles: Halifax Contributions* (Army Research Laboratory, Adelphi, MD).

Videen, G., Fu, Q., and Chýlek, P., eds. (2001). Special issue on light scattering by nonspherical particles. *J. Quant. Spectrosc. Radiat. Transfer* **70**, 373–831.

Vilenkin, N. Ja. (1968). *Special Functions and the Theory of Group Representations* (American Mathematical Society, New York). (Original Russian edition: Nauka, Moscow, 1965.)

Volakis, J. L., Chatterjee, A., and Kempel, L. C. (1998). *Finite Element Method for Electromagnetics* (IEEE Press, New York).

Volkovitsky, O. A., Pavlova, L. N., and Petrushin, A. G. (1984). *Optical Properties of Ice Clouds* (Gidrometeoizdat, Leningrad) (in Russian).

Volten, H. (2001). Light scattering by small planetary particles: an experimental study. Ph. D. dissertation, Free University, Amsterdam.

Volten, H., de Haan, J. F., Hovenier, J. W., *et al*. (1998). Laboratory measurements of angular distributions of light scattered by phytoplancton and slit. *Limnol. Oceanogr.* **43**, 1180–1197.

Volten, H., Jalava, J.-P., Lumme, K., *et al*. (1999). Laboratory measurements and T-matrix calculations of the scattering matrix of rutile particles in water. *Appl. Opt.* **38**, 5232–5240.

Volten, H., Muñoz, O., Rol, E., *et al*. (2001). Scattering matrices of mineral aerosol particles at 441.6 nm and 632.8 nm. *J. Geophys. Res.* **106**, 17 375–17 402.

von Ross, O. (1971). Method for the solution of electromagnetic scattering problems for inhomogeneous dielectrics as a power series in the ratio (dimension of scatterer)/wavelength. *J. Appl. Phys.* **42**, 4197–4201.

Voshchinnikov, N. V. (1996). Electromagnetic scattering by homogeneous and coated spheroids: calculations using the separation of variables method. *J. Quant. Spectrosc. Radiat. Transfer* **55**, 627–636.

Voshchinnikov, N. V., and Farafonov, V. G. (1993). Optical properties of spheroidal particles. *Astrophys. Space Sci.* **204**, 19–86.

Voshchinnikov, N. G., and Farafonov, V. G. (1994). Peculiarities of scattering of radiation by the nonspherical particles of atmospheric aerosol. *Izv., Atmos. Oceanic Phys.* (Engl. transl.) **30**, 43–49.

Voshchinnikov, N. V., and Farafonov, V. G. (2000). Applicability of quasi-static and Rayleigh approximations for spheroidal particles. *Opt. Spectrosc.* **88**, 71–75.

Voshchinnikov, N. V., Il'in, V. B., Henning, Th., *et al*. (2000). Extinction and polarization of radiation by absorbing spheroids: shape/size effects and benchmark results. *J. Quant. Spectrosc. Radiat. Transfer* **65**, 877–893.

Vouk, V. (1948). Projected area of convex bodies. *Nature (London)* **162**, 330–331.

Wait, J. R. (1955). Scattering of a plane wave from a circular dielectric cylinder at oblique incidence. *Can. J. Phys.* **33**, 189–195.

Wait, J. R. (1963). Electromagnetic scattering from a radially inhomogeneous sphere. *Appl.*

*Sci. Res. Sect. B* **10**, 441–450.

Waldemarsson, K. W. T., and Gustafson, B. Å. S. (2000). Light scattering by thin flakes. In *Light Scattering by Nonspherical Particles: Halifax Contributions*, eds. G. Videen, Q. Fu, and P. Chýlek, pp. 34–37 (Army Research Laboratory, Adelphi, MD).

Wang, J. J. H. (1991). *Generalized Moment Methods in Electromagnetics: Formulation and Computer Solution of Integral Equations* (John Wiley & Sons, New York).

Wang, D.-S., and Barber, P. W. (1979). Scattering by inhomogeneous nonspherical objects. *Appl. Opt.* **18**, 1190–1197.

Wang, Y. M., and Chew, W. C. (1993). A recursive T-matrix approach for the solution of electromagnetic scattering by many spheres. *IEEE Trans. Antennas Propag.* **41**, 1633–1639.

Warren, S. G. (1984). Optical constants of ice from the ultraviolet to the microwave. *Appl. Opt.* **23**, 1206–1225.

Waterman, P. C. (1965). Matrix formulation of electromagnetic scattering. *Proc. IEEE* **53**, 805–812.

Waterman, P. C. (1971). Symmetry, unitarity, and geometry in electromagnetic scattering. *Phys. Rev. D* **3**, 825–839.

Waterman, P. C. (1973). Numerical solution of electromagnetic scattering problems. In *Computer Techniques for Electromagnetics*, ed. R. Mittra, pp. 97–157 (Pergamon, Oxford).

Waterman, P. C. (1979). Matrix methods in potential theory and electromagnetic scattering. *J. Appl. Phys.* **50**, 4550–4566.

Watson, K. M. (1969). Multiple scattering of electromagnetic waves in an underdense plasma. *J. Math. Phys.* **10**, 688–702.

Weil, H., and Chu, C. M. (1976). Scattering and absorption of electromagnetic radiation by thin dielectric disks. *Appl. Opt*. **15**, 1832–1836.

Weil, H., and Chu, C. M. (1980). Scattering and absorption by thin flat aerosols. *Appl. Opt.* **19**, 2066–2071.

Wendling, P., Wendling, R., and Weickmann, H. K. (1979). Scattering of solar radiation by hexagonal ice crystals. *Appl. Opt.* **18**, 2663–2671.

West, R. A. (1991). Optical properties of aggregate particles whose outer diameter is comparable to the wavelength. *Appl. Opt.* **30**, 5316–5324.

West, R. A., and Smith, P. H. (1991). Evidence for aggregate particles in the atmospheres of Titan and Jupiter. *Icarus* **90**, 330–333.

West, R. A., Lane, A. L., Hart, H., *et al*. (1983). Voyager 2 photopolarimeter observations of Titan. *J. Geophys. Res*. **88**, 8699–8707.

Wielaard, D. J., Mishchenko, M. I., Macke, A., and Carlson, B. E. (1997). Improved **T**-matrix computations for large, nonabsorbing and weakly absorbing nonspherical particles and comparison with geometrical-optics approximation. *Appl. Opt.* **36**, 4305–4313.

Wigner, E. P. (1959). *Group Theory and its Application to the Quantum Mechanics of Atomic Spectra* (Academic Press, New York).

Wirth, M., Tsias, A., Dörnbrack, A., *et al*. (1999). Model-guided Lagrangian observation and simulation of mountain polar stratospheric clouds. *J. Geophys. Res.* **104**, 23 971–23 981.

Wiscombe, W. J. (1980). Improved Mie scattering algorithms. *Appl. Opt.* **19**, 1505–1509.

Wiscombe, W. J., and Grams, G. W. (1976). The backscattered fraction in two-stream ap-

proximations. *J. Atmos. Sci.* **33**, 2440–2451.

Wiscombe, W. J., and Mugnai, A. (1986). Single scattering from nonspherical Chebyshev particles: a compendium of calculations (NASA Ref. Publ. NASA RP-1157).

Wiscombe, W. J., and Mugnai, A. (1988). Scattering from nonspherical Chebyshev particles. 2. Means of angular scattering patterns. *Appl. Opt.* **27**, 2405–2421.

Witt, G., Dye, J. E., and Wilhelm, N. (1976). Rocket-borne measurements of scattered sunlight in the mesosphere. *J. Atmos. Terrestrial Phys*. **38**, 223–238.

Wolf, P. E., Maret, G., Akkermans, E., and Maynard, R. (1988). Optical coherent backscattering by random media: an experimental study. *J. Phys. (Paris)* **49**, 63–75.

Woodard, R., Collins, R. L., Disselkamp, R. S., *et al.* (1998). Circular depolarization lidar measurements of cirrus clouds. In *Proc. 19th Int. Laser Radar Conf.*, NASA Conf. Publ. NASA/CP-1998-207671/PT1, pp. 47–50.

Worms, J.-C., Renard, J.-B., Hadamcik, E., *et al*. (2000). Light scattering by dust particles with the PROGRA$^2$ instrument – comparative measurements between clouds under microgravity and layers on the ground. *Planet. Space Sci.* **48**, 493–505.

Wriedt, T., ed. (1999). *Generalized Multipole Techniques for Electromagnetic and Light Scattering* (Elsevier, Amsterdam).

Wriedt, T. (2000). Electromagnetic scattering programs. URL: www.t-matrix.de.

Wriedt, T., and Comberg, U. (1998). Comparison of computational scattering methods. *J. Quant. Spectrosc. Radiat. Transfer* **60**, 411–423.

Wriedt, T., and Eremin, Yu., eds. (1998). *Electromagnetic and Light Scattering: Theory and Applications III* (University of Bremen, Bremen).

Wriedt, T., and Doicu, A. (1998a). Formulations of the extended boundary condition method for three-dimensional scattering using the method of discrete sources. *J. Mod. Opt.* **45**, 199–213.

Wriedt, T., and Doicu, A. (1998b). Light scattering from a particle on or near a surface. *Opt. Commun.* **152**, 376–384.

Wriedt, T., Quinten, M., and Bauckhage, K., eds. (1996). *Electromagnetic and Light Scattering: Theory and Applications* (University of Bremen, Bremen).

Wurm, G., Heim, L.-O., and Göhzold, T. (2000). Light scattering experiments on CCA dust aggregates and applications to astrophysics. In *Light Scattering by Nonspherical Particles: Halifax Contributions*, eds. G. Videen, Q. Fu, and P. Chýlek, pp. 226–229 (Army Research Laboratory, Adelphi, MD).

Wyatt, P. J. (1962). Scattering of electromagnetic plane waves from inhomogeneous spherically symmetric objects. *Phys. Rev.* **127**, 1837–1843.

Xu, Y.-l. (1995). Electromagnetic scattering by an aggregate of spheres. *Appl. Opt.* **34**, 4573–4588.

Xu, R. (2000). *Particle Characterization: Light Scattering Methods* (Kluwer Academic Publishers, Dordrecht, The Netherlands).

Xu, L., Zhang, G., Ding, J., and Chen, H. (1997). Light scattering by polydispersions of randomly oriented hexagonal ice crystals: phase function analysis. *Optik* **106**, 103–114.

Yang, P., and Cai, Q. (1991). Light scattering phase matrices for spheroidal and cylindric large particles. *Chinese J. Atmos. Sci.* **14**, 345–358.

Yang, P., and Liou, K. N. (1995). Light scattering by hexagonal ice crystals: comparison of

finite-difference time domain and geometric optics models. *J. Opt. Soc. Am. A* **12**, 162–176.

Yang, P., and Liou, K. N. (1996a). Finite-difference time domain method for light scattering by small ice crystals in three-dimensional space. *J. Opt. Soc. Am. A* **13**, 2072–2085.

Yang, P., and Liou, K. N. (1996b). Geometric-optics–integral-equation method for light scattering by nonspherical ice crystals. *Appl. Opt.* **35**, 6568–6584.

Yang, P., and Liou, K. N. (1998a). Single-scattering properties of complex ice crystals in terrestrial atmosphere. *Contr. Atmos. Phys.* **71**, 223–248.

Yang, P., and Liou, K. N. (1998b). An efficient algorithm for truncating spatial domain in modeling light scattering by finite-difference technique. *J. Comput. Phys.* **140**, 346–369.

Yang, P., and Liou, K. N. (2000). Finite difference time domain method for light scattering by nonspherical and inhomogeneous particles. In *Light Scattering by Nonspherical Particles: Theory, Measurements, and Applications*, eds. M. I. Mishchenko, J. W. Hovenier, and L. D. Travis, pp. 173–221 (Academic Press, San Diego).

Yang, P., Liou, K. N., and Arnott, W. P. (1997). Extinction efficiency and single-scattering albedo for laboratory and natural cirrus clouds. *J. Geophys. Res.* **102**, 21 825–21 835.

Yang, P., Liou, K. N., Wyser, K., and Mitchell, D. (2000a). Parameterization of the scattering and absorption properties of individual ice crystals. *J. Geophys. Res.* **105**, 4699–4718.

Yang, P., Liou, K. N., Mishchenko, M. I., and Gao, B.-C. (2000b). Efficient finite-difference time-domain scheme for light scattering by dielectric particles: application to aerosols. *Appl. Opt.* **39**, 3727–3737.

Yang, P., Gao, B.-C., Baum, B. A., *et al*. (2001a). Asymptotic solutions for optical properties of large particles with strong absorption. *Appl. Opt.* **40**, 1532–1547.

Yang, P., Gao, B.-C., Baum, B. A., *et al.* (2001b). Sensitivity of cirrus bidirectional reflectance to vertical inhomogeneity of ice crystal habits and size distributions for two MODIS bands. *J. Geophys. Res.* **106**, 17 267–17 292.

Yanovitskij, E. G. (1997). *Light Scattering in Inhomogeneous Atmospheres* (Springer, Berlin).

Yee, K. S. (1966). Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media. *IEEE Trans. Antennas Propag.* **14**, 302–307.

Yeh, C. (1964). Perturbation approach to the diffraction of electromagnetic waves by arbitrarily shaped dielectric obstacles. *Phys. Rev. A* **135**, 1193–1201.

Zakharova, N. T., and Mishchenko, M. I. (2000). Scattering properties of needlelike and platelike ice spheroids with moderate size parameters. *Appl. Opt.* **39**, 5052–5057.

Zakharova, N. T., and Mishchenko, M. I. (2001) Scattering by randomly oriented thin ice disks with moderate equivalent-sphere size parameters. *J. Quant. Spectrosc. Radiat. Transfer* **70**, 465–471.

Zerull, R. H. (1976). Scattering measurements of dielectric and absorbing nonspherical particles. *Beitr. Phys. Atmos.* **49**, 168–188.

Zhang, J., and Xu, L. (1995). Light scattering by absorbing hexagonal ice crystals in cirrus clouds. *Appl. Opt.* **34**, 5867–5874.

Zheng, W., and Ström. S. (1989). The null field approach to electromagnetic scattering from composite objects: the case of concavo-convex constituents. *IEEE Trans. Antennas Propag.* **37**, 373–383.

Zrnić, D. S., Keenan, T. D., Carey, L. D., and May, P. (2000). Sensitivity analysis of pola-

rimetric variables at a 5-cm wavelength in rain. *J. Appl. Meteorol.* **39**, 1514–1526.

Zuffada, C., and Crisp, D. (1997). Particle scattering in the resonance regime: full-wave solution for axisymmetric particles with large aspect ratios. *J. Opt. Soc. Am. A* **14**, 459–469.

Zurk, L. M., Tsang, L., Ding, K. H., and Winebrenner, D. P. (1995). Monte Carlo simulations of the extinction rate of densely packed spheres with clustered and nonclustered geometries. *J. Opt. Soc. Am. A* **12**, 1772–1781.

Zurk, L. M., Tsang, L., and Winebrenner, D. P. (1996). Scattering properties of dense media from Monte Carlo simulations with application to active remote sensing of snow. *Radio Sci.* **31**, 803–819.

# Index


absorption, 3, 11, 58
absorption coefficient, 15
aggregated particles, *see* particles, aggregated
Alexander's dark band, 259
amplitude scattering matrix, 37–8, 118
  backscattering, 94
  backscattering theorem for, 41–2
  circular-polarization, 106
    for spherically symmetric particles, 141
  for rotationally symmetric particles, 183
  for spherically symmetric particles, 139–40
  forward-scattering, 91–3
  reciprocity relation for, 41, 84
  rotation transformation law for, 46
  symmetry properties of, 84–5
  translation transformation law for, 66–7
angle
  azimuth, 16
  polar, 16
  scattering, 60, 83, 87
  zenith, *see* angle, polar
angular frequency, 5, 10
anomalous diffraction approximation, *see* approximation, anomalous diffraction
approximation
  anomalous diffraction, 210
  Born, *see* approximation, Rayleigh–Gans
  effective-medium, 350
  eikonal, 210
  geometrical optics, 210–21, 258–60, 264–5, 320–5
  high-energy, 210
  Kirchhoff, 212, 220
  ray optics, *see* approximation, geometrical optics
  ray-tracing, *see* approximation, geometrical optics
  Rayleigh, 206–9, 264
  Rayleigh–Debye, *see* approximation, Rayleigh–Gans
  Rayleigh–Gans, 35, 209–10, 256, 264, 271
  Rayleigh–Gans–Stevenson, 207
  scalar, 109
  single-scattering, 6
  Wentzel–Kramers–Brillouin, 210
aspect ratio, 73, 192, 290
asymmetry parameter, 60, 96, 101, 105, 295, 299–300, 304, 309, 327, 340, 344, 350–1
  for spherically symmetric particles, 142, 239, 242–3, 245, 256–7
averaging
  ensemble, 72–3, 159–60, 334–7
  orientation, 72–3, 172–3, 334–7, 338, 344

analytical, 121, 127, 131, 134, 186, 199, 201
effects of, 279–82
numerical, 134, 137, 186
shape, 72–3, 334–7
size, 72–3, 159–61, 172–3, 335
effects of, 250–2, 279–82, 341
axis ratio for spheroids, 171
azimuth angle, 16

backscatter-to-extinction ratio, 267, 270, 292, 295, 303, 308
backscattered fraction, 299–300, 305
backscattering, 7, 94–5, 99, 267–71, 320–1
backscattering depolarization ratio
circular, 100, 228, 295–6, 307, 310, 339, 342, 352, 356–8
linear, 100, 228, 295–7, 307, 309, 339, 342, 352–8
backscattering theorem, 41–2, 54
benchmark results, 180–2, 189, 203–4
Bessel functions, spherical, 167–9, 360, 370
asymptotic form for, 360
bispheres, *see* clusters, two-sphere
blackbody
energy distribution, 63
Stokes column vector, 65, 98
blue moon, 250
Born approximation, *see* approximation, Rayleigh–Gans
boundary conditions, 5, 7, 9–10, 78, 113, 145, 191, 192, 196
absorbing, 194, 195
boundary-matching method, 196–7
Brewster angle, 272
bulk matter, 32
optical constants of, 32

charge density, 9
surface, 10
Chebyshev particles, 171–2, 182, 185, 319
generalized, 182, 185
cirrus clouds, *see* ice clouds
Clebsch–Gordan coefficients, 128, 131, 141, 369, 380–3
Clebsch–Gordan expansion, 128, 141, 368
clusters, 5, 80, 154
of spheres, 156, 190, 201, 337–47
of spheroids, 201
two-sphere, 74–6, 80–1, 186–9, 203–4, 337–45, 357–8
coherency matrix, 17
additivity of, 28
coherency vector, 17, 50, 54–5
coherent backscattering, 7, 42, 79–82, 220
completely polarized light, *see* fully polarized light
composite particles, 154–8
computer codes, 158–90, 193, 195, 199, 205, 214
condensation trails (contrails), 353–5
conductivity, 9
constitutive relations, 9, 117
continuity equation, 9
coordinate system
Cartesian, 15
device, 130
laboratory, 42–6, 119, 130
particle, 42–6, 119
right-handed, 15–6
spherical, 15–6
coupled dipole method, *see* discrete dipole approximation
cross section
absorption, 57–8, 227–8, 254–5, 294, 299, 302, 340, 343
average, 71, 97
backscattering, 267
differential scattering, 59
extinction, 57–9, 119, 227, 294–5, 299, 300, 339–40, 343
average, 70, 93
for randomly oriented particles, 132–3
for spherically symmetric particles, 140
geometrical, 59
radar backscattering, 267
radiation-pressure, 62
average, 71, 97
scattering, 57–8, 119, 294, 299, 301, 340, 343
average, 69, 71, 96
for macroscopically isotropic and mirror-symmetric media, 95–6
for randomly oriented particles,

138–9
for spherically symmetric particles, 140
current density, 9
surface, 10
cylinders
circular, 171–2, 185, 203–4, 210, 213, 215–20, 299–310
with extreme aspect ratios, 221–2, 315–8
hexagonal, 146, 210, 213, 320–2, 350

delta-function transmission, 218–20
density matrix, *see* coherency matrix
depolarization, 54, 100
dichroism, 3, 56, 78
differential equation methods, 191
diffraction, *see* Fraunhofer diffraction
Dirac delta function
solid-angle, 41
three-dimensional, 33
direct scattering problem, 237, 351
discrete dipole approximation, 198–9, 203, 205
discretized Mie formalism, 205, 222
dyad, 34
dyadic, 33–34
dyadic transition operator, 35
integral equation for, 35

effective-medium approximation, 350
effective radius, 161, 285–7
effective size parameter, 250, 286
effective variance, 161, 251–2, 285–7
efficiency factor
for absorption, 59, 97, 254–5
for backscattering, 267, 269, 292, 295
for extinction, 59, 97, 238–47, 253–4, 309, 312
for radiation pressure, 62, 97, 257–8, 309, 312
for scattering, 59, 97, 253–4
eikonal approximation, 210
electric displacement, 9
electric field, 9
electric permittivity, 9
electric polarization, 9
electric susceptibility, 9
electromagnetic wave, 3, 12
circular components of, 105
emission column vector, Stokes, 63–5
average, 71, 78, 98
emission, thermal, 3, 63–6, 97–8
energy conservation law, 11, 37, 122
ensemble averaging, 72–3, 159–60, 334–7
equilibrium, thermal, 63–5
Euler angles, 42, 119–20, 367
Ewald–Oseen extinction theorem, 115
extended boundary condition, 144
extended boundary condition method, 115, 142–7, 196
convergence of, 150–3, 170–1, 178–80
for rotationally symmetric particles, 148–50
iterative, 152, 205
multiple-multipole, 152
extinction, 3, 58, 78
extinction matrix
circular-polarization, 56
reciprocity relation for, 56
coherency, 55
reciprocity relation for, 56
modified Stokes, 56
reciprocity relation for, 56
Stokes, 55–6
average, 70, 78
for axially oriented particles, 127–32
for macroscopically isotropic and mirror-symmetric media, 91–3
reciprocity relation for, 56, 71
symmetry property of, 56
extinction-to-backscatter ratio, 267

far-field zone, 5, 35–8
finite-difference method, 194, 205
finite-difference time-domain method, 195–6, 205, 321
finite-element method, 193–5, 205
Fraunhofer diffraction, 212–3, 220, 227, 258–9
Fredholm integral equation method, 200, 205
fully polarized light, 27, 54

Gaussian random spheres, 328
generalized spherical functions, 103, 107, 366–7

geometrical optics approximation, 210–21, 258–60, 264–5, 320–5
glory, 260, 264
Green's function
  dyadic, free space, 33–6, 377
  scalar, 34–6
Green's vector theorem, 142

halos, 320–2, 350
Hankel functions, 169, 370
Helmholtz equation
  scalar, 371
  vector, 193, 205, 372–3
homogeneous wave, 13
Huygens' principle, 115
hydrometeors
  nonspherical, 100, 127, 352
  partially aligned, 100

ice clouds, 321, 322, 352, 354–5, 358
identity dyadic, 33, 36
incident field, 31, 33, 47, 116
independently scattering particles, 6, 74
inhomogeneous wave, 13
integral equation methods, 191, 197–200
intensity of light, 15, 18
interference of light, 7, 58, 79, 343, 344
interference structure, 240, 250–1, 279–82
interstellar dust grains, 100, 127
interstellar polarization, 99, 351
inverse scattering problem, 237, 351
irradiance, *see* intensity
irregular particles, 322–34

Jacobi polynomials, 364

Kirchhoff approximation, 212, 220
Koch fractals, 325, 327
  random, 325–8
Kronecker delta, 363
Legendre functions, associated, 140, 360, 366, 374
Legendre polynomials, 360, 365
levitation
  electrostatic, 273, 358
  optical, 60, 228, 273
lidars, 228, 352, 355
Lippmann–Schwinger equations, 35
Lorenz–Mie
  coefficients, 122, 153–4
  computer code, 158–65, 238
  identities, 100, 352
  scattering matrix, 99
  theory, 99, 114, 115, 139–42, 238

magnetic field, 9
magnetic induction, 9
magnetic permiability, 9
magnetization, 9
Maxwell equations, 5, 8–9, 32, 77, 78, 113, 191, 195
  linearity of, 37, 39, 117
  plane-wave solution of, 12–15
Maxwell stress tensor, 60
measurement techniques for scattering, 224–33
  using microwaves, 224, 230–3
  using visible and infrared light, 113, 224–30, 273–4, 329–34, 345, 351–8
    two-dimensional angular, 230, 351–2
meridional plane, 16, 23, 38, 83, 88
method of moments, 198–9, 205
microwave analog technique, 113, 147, 230–3
microwave measurement techniques, 224, 230–3
mineral particles, 329–39
monochromatic light, 3, 12
monodisperse particles, 165, 177, 238–50, 279–82, 335, 341

natural light, *see* unpolarized light
near-field effects, 80
nonsphericity, effects of, 99–100, 279–82
null-field method, 115
  with discrete sources, 152

optical equivalence principle, 18, 28
optical theorem, 49, 56, 58, 207
orientation angle of the polarization ellipse, 20
orientation averaging, *see* averaging, orientation
orientation distribution, 72–3

axially symmetric, 73, 127
random, 73
orientation of the scattering object, *see* particle orientation
outgoing wave, 34, 36, 371, 374

parallel beam of light, 3, 12
particle characterization, optical, 7, 82, 229–30, 249, 273–8, 350–8
particle collection, 5, 68
tenuous, 7
particle orientation, 42
effects of, 99–100, 279–82
particle shapes, 171–2, 182, 283, 322, 325, 330
particles
absorbing, 124, 216
aggregated, 154–8
Chebyshev, 171–2, 182, 185, 319
generalized, 182, 185
clay, 329–32
composite, 154–8
dust, 355–6
feldspar, 329–31
fly ash, 345, 348
independently scattering, 6, 74
irregular, 322–34
loess, 329–31, 335
mineral, 329–39
monodisperse, 165, 177, 238–50, 279–82, 335, 341
nonabsorbing, 122–5, 139, 152
polydisperse, 158, 165, 250–73, 282–310, 345, 354, 356
polyhedral, 146, 213, 320–3
quartz, 329–31, 333
randomly and sparsely distributed, 68–71, 75, 77
randomly oriented, 73, 87, 100, 132–9
randomly shaped, 322
rotationally symmetric, 46, 93, 102, 121, 125, 129, 133, 135, 138, 148–50, 165, 169, 171–2, 180
Sahara sand, 329–31, 337
spherical, 98–9, 102, 122, 139–42, 158, 214–6, 238–78
anisotropic, 222
Faraday-active, 221
monodisperse, 238–50
polydisperse, 250–73
stochastically shaped, *see* particles, randomly shaped
volcanic, 329–31, 334, 336
with inclusions, 156–7, 201, 213–14, 347–51
with rough surfaces, 325–6, 328
perturbation theories, 221
phase, 13, 18, 20
phase function, 59–60, 101
for irregular particles, 322–8
for monodisperse spherical particles, 248
for polydisperse spherical particles, 258–67
for randomly oriented circular cylinders, 301–2, 306, 315–6
for randomly oriented spheroids, 286–91, 297, 309
normalization condition for, 60, 101
Rayleigh, 208–9, 264
Rayleigh–Gans, 264
phase matrix, 49–54
circular-polarization, 52, 106
average, 107
normalized, 106–7
reciprocity relation for, 53
coherency, 50–1
reciprocity relation for, 53
modified Stokes, 52
reciprocity relation for, 53
normalized Stokes, 100–2
Fourier decomposition in azimuth for, 105
symmetry properties of, 102
Stokes, 51–2, 83
average, 70, 72, 78
backscattering, 54
expression in terms of the scattering matrix, 88–91
for macroscopically isotropic and mirror-symmetric media, 88–91
inequalities for, 52, 71
reciprocity relation for, 52–3, 71
symmetry relations for, 90–1
phase velocity, 13, 14
photon correlation spectroscopy, 7
physical optics approximation, 220

plane wave, 3, 12, 16–7, 37, 116
expansion in spherical waves, 360–1
expansion in vector spherical wave functions, 117, 376–7
point matching method, 196–7
generalized, 197, 204
multiple-expansion, 197, 204–5
polar angle, 16
polar stratospheric clouds, 352, 354
polarization, 54
circular, 21, 23, 54
degree of, 28
complete, *see* polarization, full
degree of, 28
elliptical, 21, 23
degree of, 28, 54
full, 27, 54
left-handed, 20
linear, 21, 23, 54
degree of, 28, 29, 271
natural, 27
right-handed, 20
polarization analyzer, 225
polarization ellipse, 20–4
ellipticity of, 20
orientation of, 20
polarization modulation technique, 226, 228
polarization modulator, 225
polarizer, 225
polydisperse particles, 158, 165, 250–73, 282–310, 345, 354, 356
polyhedral particles, 146, 213, 320–3
position vector, 10
Poynting vector, 11
complex, 12
time-averaged, 12, 14–5, 47, 123–4
probability density function, 72
normalization condition for, 72

quarter-wave plate, 225
quasi-monochromatic light, 5, 26–9, 52, 56, 58, 63, 71, 77

radars, 233, 352
radiance, *see* specific intensity
radiation condition, 116–7, 194, 199
radiation force, 60–2, 96–7
emission component of, 62, 65, 98
radiation pressure, 60
radiation torque, 62–3
emission component of, 66
radiative transfer equation, 6–7, 76, 78
for macroscopically isotropic and mirror-symmetric media, 105, 108–9
radius
effective, 161, 285–7
equivalent-sphere, 73, 172
surface-equivalent-sphere, 171–2
volume-equivalent-sphere, 172
radius vector, 10
rainbows, 258–60, 264
randomly and sparsely distributed particles, 68–71, 75, 77
randomly oriented particles, 73, 87, 100, 132–9
randomly shaped particles, 322
ray optics approximation, *see* approximation, geometrical optics
ray-tracing approximation, *see* approximation, geometrical optics
Rayleigh approximation, 206–9, 264
Rayleigh–Debye approximation, *see* approximation, Rayleigh–Gans
Rayleigh–Gans approximation, 35, 209–10, 256, 264, 271
Rayleigh–Gans scattering, 264, 271–3
Rayleigh hypothesis, 117, 146, 196, 221
Rayleigh scattering, 247, 250, 264, 271–3, 309, 310
reciprocity relation, 41, 52–3, 71, 84
reference plane, 24
rotations of, 24–5
refractive index, 5, 14
relative, 33
resonance structure, 241–50, 279–82, 338
resonances, 241–50, 273
ripple structure, 239, 241, 245, 250–1
rotation matrix
for circular-polarization representation, 25–6
for modified Stokes vector, 25
Stokes, 25
rotationally symmetric particles, 46, 93, 102, 121, 125, 129, 133, 135, 138, 148–50, 165, 169, 171–2, 180

*S* matrix, 122–5
unitarity condition for, 124–5
scale invariance rule, 147, 230
scattered field, 3, 31, 33, 36–7, 47, 116–7
scattering, 3
diffuse, 6
dynamic, 7
elastic, 3
electromagnetic, 3
far-field, 5, 35–8, 66–7
forward, 6, 49, 54, 91–4, 99
incoherent, 6, 74
independent, 6, 74, 75
condition for, 74
multiple, 6, 75, 344
single, 68–71
static, 7
scattering angle, 60, 83, 87
scattering dyadic, 37, 41, 117–8
reciprocity relation for, 41
scattering matrix, circular-polarization, normalized, 106–7
expansion in generalized spherical functions, 107–8, 136, 141
symmetry properties of, 107
scattering matrix, Stokes, 83, 85–6, 94
average, 87
effects of nonsphericity on, 99–100
for backward scattering, 94–5
for forward scattering, 93–4
for macroscopically isotropic and mirror-symmetric media, 87
for randomly oriented particles, 133–7
for rotationally symmetric particles, 93–4
for spherically symmetric particles, 98–9
inequalities for, 87–8
normalized, 100–3
expansion in generalized spherical functions, 103–5, 136–7, 141, 166, 172–3, 180–1
expansion in Wigner *d*-functions, 103–5
for clusters of spheres, 341–7
for irregular particles, 323–39
for randomly oriented circular cylinders, 215–8, 301–7, 315–8
for randomly oriented spheroids, 180, 182, 285–94, 297–8, 309–11, 313–4
for rotationally symmetric particles, 102
for spherically symmetric particles, 102, 140, 160, 167, 229–30, 245–9, 258–67, 271–6
properties of, 101–3
symmetries of, 85–6
scattering medium
macroscopically isotropic, 87
macroscopically isotropic and mirror-symmetric, 87, 100
macroscopically mirror-symmetric, 87
scattering plane, 83, 88
scattering tensor, 39
reciprocity condition for, 40
Schelkunoff equivalent current method, 115
secondary wave, 3
separation of variables method, 113–4, 179, 192–3, 202–5
multi-sphere, 156, 201
shape averaging, 72–3, 334–7
shape distribution, 72–3
single-scattering albedo, 58, 228, 255–6, 284, 293–6, 299, 303, 340, 344
average, 97
single-scattering approximation, 6
size averaging, 72–3, 159–61, 172–3, 335
effects of, 250–2, 279–82, 341
size distribution, 72–3, 160–1, 173, 177, 250–2, 283–6
gamma, 160, 252, 283–6
log normal, 160, 283–6
modified bimodal log normal, 161
modified gamma, 160, 283
modified power law, 161, 283–7
power law, 160, 285
size parameter, 74, 147, 192
effective, 250, 286
major-axis, 178
surface-equivalent-sphere, 178, 308
volume-equivalent-sphere, 308
specific intensity, 77
column vector, 77–8
speed of light, 14
spherical harmonics
scalar, 360
vector, 373–6

spherical particles, 98–9, 102, 122, 139–42, 158, 214–6, 238–78
anisotropic, 222
Faraday-active, 221
monodisperse, 238–50
polydisperse, 250–73
spherical wave, 36
spherical wave functions
scalar, 371
vector, 115, 116, 372–8
spheroids, 171–2
optical properties of, 180–2, 185, 203–4, 210, 213, 280–98
with extreme aspect ratios, 307–14
statistical approach, 334–7
stochastically shaped particles, 322
Stokes parameters, 6, 7, 15, 17–8
ellipsometric interpretation of, 19–24
for quasi-monochromatic light, 26
additivity of, 27–8
quadratic inequality for, 27
quadratic identity for, 18
rotation transformation rule for, 24–5
Stokes column vector, 17, 51
circular-polarization, 19, 23, 105–6
additivity of, 28
rotation transformation rule for, 25–6
for quasi-monochromatic light, 27
additivity of, 27, 28
modified, 18, 23
additivity of, 28
rotation transformation rule for, 25
rotation transformation rule for, 24–5
subtraction method, 226, 228
superposition method, 156, 201–2, 204
superposition principle, 37
surface integral equation methods, 200, 205
time-domain, 205
Système International, 384

*T* matrix, 115, 117, 146
for aggregated particles, 154–8, 201
for rotationally symmetric particles, 148–50, 167–71
orientation-averaged, 127–9, 131
rotation transformation rule for, 119–21
symmetry relations for, 121–2
translation transformation rule for, 125–7
unitarity property of, 122–5, 152
*T*-matrix computer code
for randomly oriented, rotationally symmetric particles, 165–80
for randomly oriented two-sphere clusters, 186–9
for a rotationally symmetric particle in an arbitrary orientation, 180–6
multi-sphere superposition, 190
*T*-matrix method, 114, 115–90, 202–5
superposition, 154–8, 201
recursive, 158
temperature, absolute, 3, 63
time, 5, 9
time factor, 5, 15
time-harmonic field, 10
total field, 31, 35, 54–5, 197
transition matrix, *see T* matrix
translation addition theorem, 126, 156, 157, 201, 378–9
transverse wave, 13, 36

unimoment method, 194
unitarity condition, 124
unpolarized light, 27

Venus clouds, 274–8
volume integral equation, 31–5, 197
volume integral equation method, 197–9

wave equation, vector, 33, 113
wave number, 14
wave vector, 12
wavelength, 15
weak photon localization, *see* coherent backscattering
Wentzel–Kramers–Brillouin approximation, 210
Wigner *d*-functions, 103–4, 362–9, 371, 377
Wigner *D*-functions, 367–8, 377
Wigner 3*j* symbols, 141, 379, 381

zenith angle, *see* angle, polar