Enhanced backscattering of polarized light from discrete random media: calculations in exactly the backscattering direction

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Enhanced backscattering of polarized light by disordered media composed of independently scattering particles of arbitrary size and shape is studied theoretically. Rigorous relations between the cyclical and the ladder parts of the backscattering matrix in exactly the backscattering direction are derived for three commonly used representations of polarization, and the corresponding polarization backscattering enhancement factors are introduced. The ladder part of the Stokes-backscattering matrix is calculated by solving Chandrasekhar's vector radiative transfer equation [*Radiative Transfer* (Clarendon, London, 1950)]. The general properties of the enhancement factors are studied, and the results of numerical computations are reported for finite and semi-infinite homogeneous slabs composed of spherical and randomly oriented nonspherical particles. It is shown that the enhancement factors depend strongly on the direction of light incidence, the optical thickness of the medium, the true absorption, and the particle size and shape.

INTRODUCTION

In recent years, extensive experimental and theoretical studies concerning enhanced backscattering of light from discrete disordered media have been performed (for recent reviews, see, e.g., Refs. 1 and 2). This phenomenon is associated with weak localization of photons and manifests itself as a well-defined narrow peak in the angular distribution of the intensity of the scattered light at scattering angles near 180°. Although the scalar theory of the enhanced backscattering is basically well understood (see, e.g., Refs. 3–7 and references therein), the corresponding vector theory still lacks rigorous results. Until now, approximate methods were used mainly to study theoretically the enhanced backscattering of polarized light, namely, the second-order multiple-scattering theory,[8] the diffusion approximation,[9,10] and the numerical simulation of multiple-scattering processes.[6,11] Moreover, in all these studies the simplest case of Rayleigh scattering was considered. A more general case of Mie scattering was studied by Mandt *et al.*[12] Nevertheless, like Kuga *et al.*,[8] these authors used the second-order approximation of the multiple-scattering theory.

An important result of the theory of multiple scattering of scalar waves in discrete random media is that, in exactly the backscattering direction, the contribution of all the cyclical diagrams to the backscattered intensity is identical to that of all the ladder diagrams (see, e.g., Ref. 3). Therefore the backscattering coefficient γ is given by

$$\gamma = \gamma^1 + \gamma^L + \gamma^C \equiv \gamma^1 + 2\gamma^L, \tag{1}$$

where γ^1 is the contribution of the first-order scattering, γ^L is the contribution of all the ladder diagrams, and γ^C is the contribution of all the cyclical diagrams. An important consequence of Eq. (1) is the so-called factor of two:

$$\tilde{\zeta} = \frac{\gamma^1 + 2\gamma^L}{\gamma^1 + \gamma^L} \approx 2, \tag{2}$$

where $\tilde{\zeta}$ is the backscattering enhancement factor, defined as the ratio of the total backscattered intensity and the incoherent background intensity in exactly the backscattering direction. In the present paper I generalize Eq. (1) by taking into account the vector character of light. More specifically, I consider the multiple scattering of polarized light by independently scattering particles of arbitrary size and shape and derive rigorous relations between the ladder and the cyclical parts of the backscattering matrix for three commonly used representations of polarization, namely, the coherency matrix representation, the Stokes vector representation, and the circular-polarization representation. These relations are used to define the backscattering enhancement factors that correspond to different states of polarization of the incident light. The ladder part of the Stokes-backscattering matrix is calculated by solving Chandrasekhar's vector radiative transfer equation. Both the analytical and the numerical solutions of this equation are well known, enabling one to study in detail how the backscattering enhancement factors are influenced by the parameters that specify the scattering problem. The general properties of the backscattering enhancement factors are studied, and illustrative numerical results are reported for finite and semi-infinite homogeneous slabs composed of spherical particles and randomly oriented spheroids. It is shown that the enhancement factors depend strongly on the direction of light incidence, the optical thickness of the medium, the true absorption, and the particle size and shape.

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FORMULATION

Consider a plane-parallel medium composed of uncorrelated particles of arbitrary size and shape. To describe light scattering by a particle, I use a local right-handed Cartesian coordinate system that has its origin inside the particle and a fixed orientation identical to that of the laboratory reference frame attached to the medium. The direction of light propagation is specified by a unit vector $\hat{n} = (\vartheta, \varphi) = \hat{\vartheta} \times \hat{\varphi}$, where ϑ is the polar angle, φ is the azimuth angle, and $\hat{\vartheta}$ and $\hat{\varphi}$ are the corresponding unit vectors. Assume that the concentration of the particles is low and that they may be considered independent scatterers. Thus each particle can be specified by a (2×2) amplitude scattering matrix $\mathbf{F}(\hat{n}', \hat{n})$, which describes how the θ and the φ components of a plane wave $\mathbf{E}(\hat{n})$, incident upon the particle in the direction $\hat{n}$, are transformed into the θ and the φ components of the wave $\mathbf{E}'(\hat{n}')$ scattered by the particle in the far-field zone in the direction $\hat{n}'$[12,13]:

$$\begin{bmatrix} E_\vartheta' \\ E_\varphi' \end{bmatrix} \propto \mathbf{F} \begin{bmatrix} E_\vartheta \\ E_\varphi \end{bmatrix}. \tag{3}$$

A fundamental property of the amplitude scattering matrix is the reciprocity relation[14]

$$\mathbf{F}(-\hat{n}, -\hat{n}') = \mathbf{Q}\mathbf{F}^T(\hat{n}', \hat{n})\mathbf{Q}, \tag{4}$$

where $\mathbf{Q} = \mathrm{diag}(1, -1)$ and T denotes the matrix transpose.

Let a plane wave $\mathbf{E}_0 = \left[\begin{smallmatrix} E_{0\vartheta} \\ E_{0\varphi} \end{smallmatrix}\right]$ be incident upon the upper boundary of the medium in the direction $\hat{n}_0$. Denote by $\mathbf{J}^0$ the corresponding coherency (or density) matrix, which is defined by[13]

$$\mathbf{J}^0 = \mathbf{E}_0 \mathbf{E}_0^{*T} = \begin{bmatrix} E_{0\vartheta}E_{0\vartheta}^* & E_{0\vartheta}E_{0\varphi}^* \\ E_{0\varphi}E_{0\vartheta}^* & E_{0\varphi}E_{0\varphi}^* \end{bmatrix}, \tag{5}$$

where the asterisk denotes complex conjugation. Let $\mathbf{J}$ be the coherency matrix of the light scattered by the medium in the far-field region in the direction $-\hat{n}_0$. Transformation of the elements of the matrix $\mathbf{J}^0$ into those of the matrix $\mathbf{J}$ is given by

$$\mathbf{D} \propto \mathbf{G}(-\hat{n}_0, \hat{n}_0)\mathbf{D}^0, \tag{6}$$

where $\mathbf{G}(-\hat{n}_0, \hat{n}_0)$ is the (4×4) reflection matrix for the pure backscattering direction and $\mathbf{D}$ and $\mathbf{D}^0$ are the (4×1) columns defined as

$$\mathbf{D}^0 = \begin{bmatrix} J_{11}^0 \\ J_{12}^0 \\ J_{21}^0 \\ J_{22}^0 \end{bmatrix}, \qquad \mathbf{D} = \begin{bmatrix} J_{11} \\ J_{12} \\ J_{21} \\ J_{22} \end{bmatrix}. \tag{7}$$

Let us decompose the matrices $\mathbf{J}$ and $\mathbf{G}(-\hat{n}_0, \hat{n}_0)$ as

$$\mathbf{J} = \mathbf{J}^1 + \mathbf{J}^L + \mathbf{J}^C, \tag{8}$$

$$\mathbf{G} = \mathbf{G}^1 + \mathbf{G}^L + \mathbf{G}^C, \tag{9}$$

where, as above, 1 denotes the single-scattering terms, L denotes the ladder terms, and C denotes the cyclical terms. The problem is to express the elements of the matrix $\mathbf{G}^C$ in those of the matrix $\mathbf{G}^L$.

Denote by $(1, n)$ a light path formed by $n \geq 2$ arbitrary scattering centers along which a wave travels, and denote by $(n, 1)$ the time-reversed path, i.e., the path that is formed by the same scatterers but along which a wave travels in the opposite direction. The waves that are scattered by the chains $(1, n)$ and $(n, 1)$ in the pure backscattering direction $-\hat{n}_0$ have equal phases and will constructively interfere. Denote by $\mathbf{E}^{(1,n)}$ and $\mathbf{E}^{(n,1)}$ the amplitudes of the two scattered waves, and denote by $\mathbf{P}^{(1,n)}$ and $\mathbf{P}^{(n,1)}$ the corresponding (2×2) amplitude transformation matrices such that $\mathbf{E}^{(1,n)} \propto \mathbf{P}^{(1,n)}\mathbf{E}_0$ and $\mathbf{E}^{(n,1)} \propto \mathbf{P}^{(n,1)}\mathbf{E}_0$. The matrices $\mathbf{P}^{(1,n)}$ and $\mathbf{P}^{(n,1)}$ can be expressed in terms of the products of the amplitude scattering matrices of the individual particles that enter the chains $(1, n)$ and $(n, 1)$. Therefore, by using the single-scattering reciprocity relation [Eq. (4)], one easily derives the reciprocity relation for the matrices $\mathbf{P}^{(1,n)}$ and $\mathbf{P}^{(n,1)}$:

$$\mathbf{P}^{(n,1)} = \mathbf{Q}[\mathbf{P}^{(1,n)}]^T\mathbf{Q}. \tag{10}$$

Denote

$$\mathbf{P}^{(1,n)} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}. \tag{11}$$

The contributions of the chains $(1, n)$ and $(n, 1)$ to the matrix $\mathbf{J}^L$ are given by

$$\mathbf{E}^{(1,n)}[\mathbf{E}^{(1,n)}]^{*T} + \mathbf{E}^{(n,1)}[\mathbf{E}^{(n,1)}]^{*T},$$

while the contribution to the matrix $\mathbf{J}^C$ is given by

$$\mathbf{E}^{(1,n)}[\mathbf{E}^{(n,1)}]^{*T} + \mathbf{E}^{(n,1)}[\mathbf{E}^{(1,n)}]^{*T}.$$

The corresponding contributions to the matrices $\mathbf{G}^L$ and $\mathbf{G}^C$ are given, respectively, by

$$\begin{bmatrix} 2aa^* & ab^* - ac^* & ba^* - ca^* & bb^* + cc^* \\ -ab^* + ac^* & 2ad^* & bc^* + cb^* & bd^* - cd^* \\ -ba^* + ca^* & bc^* + cb^* & 2da^* & db^* - dc^* \\ bb^* + cc^* & -bd^* + cd^* & -db^* + dc^* & 2dd^* \end{bmatrix}, \tag{12}$$

$$\begin{bmatrix} 2aa^* & ab^* - ac^* & ba^* - ca^* & -bc^* - cb^* \\ -ab^* + ac^* & 2ad^* & -bb^* - cc^* & bd^* - cd^* \\ -ba^* + ca^* & -bb^* - cc^* & 2da^* & db^* - dc^* \\ -bc^* - cb^* & -bd^* + cd^* & -db^* + dc^* & 2dd^* \end{bmatrix} \tag{13}$$

[see Eq. (11)]. By comparing matrices (12) and (13), we have that

$$\mathbf{G}^C = \begin{bmatrix} G_{11}^L & G_{12}^L & G_{13}^L & -G_{32}^L \\ G_{21}^L & G_{22}^L & -G_{41}^L & G_{24}^L \\ G_{31}^L & -G_{41}^L & G_{33}^L & G_{34}^L \\ -G_{32}^L & G_{42}^L & G_{43}^L & G_{44}^L \end{bmatrix}. \tag{14}$$

This relation is the desired generalization to the vector case of the scalar identity $\gamma^C \equiv \gamma^L$. For macroscopically isotropic media, Eq. (14) becomes simpler as

$$\mathbf{G}^C = \begin{bmatrix} G_{11}^L & 0 & 0 & -G_{32}^L \\ 0 & G_{22}^L & -G_{41}^L & 0 \\ 0 & -G_{41}^L & G_{33}^L & 0 \\ -G_{32}^L & 0 & 0 & G_{44}^L \end{bmatrix}. \tag{15}$$

Other representations of polarization that are often used in the literature are the Stokes vector and the

circular polarization representations. They are defined, respectively, by[13,15]

$$\mathbf{I} = \begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} = \begin{bmatrix} J_{11} + J_{22} \\ J_{11} - J_{22} \\ -iJ_{12} - iJ_{21} \\ -iJ_{12} + iJ_{21} \end{bmatrix}, \tag{16}$$

$$\mathbf{I}_{\mathrm{CP}} = \frac{1}{2} \begin{bmatrix} Q + iU \\ I + V \\ I - V \\ Q - iU \end{bmatrix}. \tag{17}$$

Denoting the corresponding backscattering matrices by $\mathbf{S}$ and $\mathbf{C}$, one can rewrite Eq. (15) in two alternative forms:

$$\mathbf{S}^C = \begin{bmatrix} S_{11}{}^C & S_{12}{}^L & 0 & 0 \\ S_{12}{}^L & S_{22}{}^C & 0 & 0 \\ 0 & 0 & S_{33}{}^C & S_{34}{}^L \\ 0 & 0 & -S_{34}{}^L & S_{44}{}^C \end{bmatrix}, \tag{18}$$

$$\mathbf{C}^C = \begin{bmatrix} C_{11}{}^L & C_{12}{}^L & C_{13}{}^L & C_{32}{}^L \\ C_{21}{}^L & C_{22}{}^L & C_{41}{}^L & C_{24}{}^L \\ C_{31}{}^L & C_{41}{}^L & C_{33}{}^L & C_{34}{}^L \\ C_{32}{}^L & C_{42}{}^L & C_{43}{}^L & C_{44}{}^L \end{bmatrix}, \tag{19}$$

with

$$S_{11}{}^C = \tfrac{1}{2}[S_{11}{}^L + S_{22}{}^L - S_{33}{}^L + S_{44}{}^L], \tag{20}$$

$$S_{22}{}^C = \tfrac{1}{2}[S_{11}{}^L + S_{22}{}^L + S_{33}{}^L - S_{44}{}^L], \tag{21}$$

$$S_{33}{}^C = \tfrac{1}{2}[-S_{11}{}^L + S_{22}{}^L + S_{33}{}^L + S_{44}{}^L], \tag{22}$$

$$S_{44}{}^C = \tfrac{1}{2}[S_{11}{}^L - S_{22}{}^L + S_{33}{}^L + S_{44}{}^L]. \tag{23}$$

By using Eqs. (15) and (18)–(20), I introduce the following definitions:

$$\begin{aligned} \zeta_{\parallel} &= [G_{11}{}^1 + G_{11}{}^L + G_{11}{}^C]/[G_{11}{}^1 + G_{11}{}^L] \\ &= [G_{11}{}^1 + 2G_{11}{}^L]/[G_{11}{}^1 + G_{11}{}^L], \end{aligned} \tag{24}$$

$$\begin{aligned} \zeta_{\perp} &= [G_{41}{}^1 + G_{41}{}^L + G_{41}{}^C]/[G_{41}{}^1 + G_{41}{}^L] \\ &= [G_{41}{}^1 + G_{41}{}^L - G_{32}{}^L]/[G_{41}{}^1 + G_{41}{}^L], \end{aligned} \tag{25}$$

$$\begin{aligned} \zeta &= [S_{11}{}^1 + S_{11}{}^L + S_{11}{}^C]/[S_{11}{}^1 + S_{11}{}^L] \\ &= [S_{11}{}^1 + S_{11}{}^L + \tfrac{1}{2}(S_{11}{}^L + S_{22}{}^L - S_{33}{}^L \\ &\quad + S_{44}{}^L)]/[S_{11}{}^1 + S_{11}{}^L], \end{aligned} \tag{26}$$

$$\begin{aligned} \zeta_{\mathrm{hp}} &= [C_{22}{}^1 + C_{22}{}^L + C_{22}{}^C]/[C_{22}{}^1 + C_{22}{}^L] \\ &= [C_{22}{}^1 + 2C_{22}{}^L]/[C_{22}{}^1 + C_{22}{}^L], \end{aligned} \tag{27}$$

$$\begin{aligned} \zeta_{\mathrm{oh}} &= [C_{32}{}^1 + C_{32}{}^L + C_{32}{}^C]/[C_{32}{}^1 + C_{32}{}^L] \\ &= [C_{32}{}^1 + C_{32}{}^L + C_{41}{}^L]/[C_{32}{}^1 + C_{32}{}^L]. \end{aligned} \tag{28}$$

Here, $\zeta_{\parallel}$ and $\zeta_{\perp}$ are the copolarized and the depolarized enhancement factors, respectively, that correspond to the case of linearly polarized incident light with the coherency matrix components $J_{11}{}^0 = 1$, $J_{21}{}^0 = J_{12}{}^0 = J_{22}{}^0 = 0$.[8] The enhancement factor ζ corresponds to the case of unpolarized incident light with the Stokes matrix components $I^0 = 1$, $Q^0 = U^0 = V^0 = 0$. The factor ζ_{hp} describes the backscattering enhancement in the helicity-preserving channel when the circularly polarized incident light is given by $[\mathbf{I}_{\mathrm{CP}}{}^0]_2 = 1$, $[\mathbf{I}_{\mathrm{CP}}{}^0]_1 = [\mathbf{I}_{\mathrm{CP}}{}^0]_3 = [\mathbf{I}_{\mathrm{CP}}{}^0]_4 = 0$,[10] while the factor ζ_{oh} describes the backscattering enhancement in the opposite-helicity channel. In terms of the matrices $\mathbf{S}^1$ and $\mathbf{S}^L$, the definitions of Eqs. (24), (25), (27), and (28), respectively, can be rewritten as

$$\zeta_{\parallel} = \frac{S_{11}{}^1 + S_{22}{}^1 + 2S_{11}{}^L + 4S_{12}{}^L + 2S_{22}{}^L}{S_{11}{}^1 + S_{22}{}^1 + S_{11}{}^L + 2S_{12}{}^L + S_{22}{}^L}, \tag{29}$$

$$\zeta_{\perp} = \frac{S_{11}{}^1 - S_{22}{}^1 + S_{11}{}^L - S_{22}{}^L - S_{33}{}^L + S_{44}{}^L}{S_{11}{}^1 - S_{22}{}^1 + S_{11}{}^L - S_{22}{}^L}, \tag{30}$$

$$\zeta_{\mathrm{hp}} = \frac{S_{11}{}^1 + S_{44}{}^1 + 2S_{11}{}^L + 2S_{44}{}^L}{S_{11}{}^1 + S_{44}{}^1 + S_{11}{}^L + S_{44}{}^L}, \tag{31}$$

$$\zeta_{\mathrm{oh}} = \frac{S_{11}{}^1 - S_{44}{}^1 + S_{11}{}^L + S_{22}{}^L - S_{33}{}^L - S_{44}{}^L}{S_{11}{}^1 - S_{44}{}^1 + S_{11}{}^L - S_{44}{}^L}. \tag{32}$$

RESULTS AND DISCUSSION

As is well known (see, e.g., Refs. 16 and 17 and references therein), the Bethe–Salpeter equation under the ladder approximation of independent scatterers results in the common vector radiative transfer equation.[18] Therefore in what follows it is assumed that the matrices $\mathbf{S}^1$ and $\mathbf{S}^L$ can be found by solving this equation analytically or numerically.[15,19–23]

For macroscopically isotropic media, the single-scattering Stokes matrix has the form[24,25]

$$\mathbf{F}_S(\theta) = \begin{bmatrix} a_1(\theta) & b_1(\theta) & 0 & 0 \\ b_1(\theta) & a_2(\theta) & 0 & 0 \\ 0 & 0 & a_3(\theta) & b_2(\theta) \\ 0 & 0 & -b_2(\theta) & a_4(\theta) \end{bmatrix}, \tag{33}$$

where θ is the scattering angle. For spherical particles $a_1\ (180°) = a_2\ (180°)$ and $a_1\ (180°) = -a_4\ (180°)$. Therefore

$$S_{11}{}^1 = S_{22}{}^1, \qquad S_{11}{}^1 = -S_{44}{}^1. \tag{34}$$

For randomly oriented nonspherical particles these equalities do not generally hold.

The vector radiative transfer equation together with Eqs. (26), (29)–(32), and (34) can be used to derive some general properties of the backscattering enhancement factors. By inspection, one easily finds that the factors $\zeta_{\parallel}$, ζ, and ζ_{oh} obey the limits

$$\lim_{\mu_0 \to 0} \xi = \lim_{\omega \to 0} \xi = \lim_{\tau \to 0} \xi = 1, \tag{35}$$

where ξ stands for $\zeta_{\parallel}$, ζ, or ζ_{oh}; $\mu_0 = \cos \vartheta_0$ (the z axis of the laboratory coordinate system is assumed to coincide with the inward normal to the upper boundary of the medium), ω is the single-scattering albedo, and τ is the optical thickness of the medium. For spherical particles, the factor $\zeta_{\perp}$ has no such definite limits, while $\zeta_{\mathrm{hp}} \equiv 2$. For randomly oriented nonspherical particles, the factors $\zeta_{\perp}$ and ζ_{hp} obey the limits of Eq. (35).

To verify and illustrate these properties, I performed numerical calculations for several scattering models. In these calculations I used the computational procedures that are extensively described in Refs. 26–29. The results of the calculations are given in Tables 1–5.

Table 1. Backscattering Enhancement Factors for Semi-Infinite Slabs Composed of Spherical Particles with $\mu_0 = 1$ and $m = 1.2$

x	$\zeta_\parallel$	$\zeta_\perp$	ζ	ζ_{hp}	ζ_{oh}
0.9	1.82	1.11	1.56	2.00	1.26
1.4	1.92	1.10	1.59	2.00	1.25
2.5	1.97	1.16	1.59	2.00	1.11
3.7	1.98	1.19	1.60	2.00	1.06
6.5	2.00	1.22	1.61	2.00	1.05
7.2	2.00	1.23	1.62	2.00	1.03

Table 2. Backscattering Enhancement Factors for a Rayleigh-Scattering Slab with $\tau = \infty$ and $\omega = 1$

μ_0	$\zeta_\parallel$	$\zeta_\perp$	ζ	ζ_{hp}	ζ_{oh}
0.0	1.00	1.46	1.00	2.00	1.00
0.05	1.22	1.40	1.19	2.00	1.14
0.1	1.33	1.36	1.29	2.00	1.21
0.5	1.64	1.20	1.52	2.00	1.32
1.0	1.75	1.12	1.54	2.00	1.25

Table 3. Backscattering Enhancement Factors for a Rayleigh-Scattering Slab with $\tau = \infty$ and $\mu_0 = 1$

ω	$\zeta_\parallel$	$\zeta_\perp$	ζ	ζ_{hp}	ζ_{oh}
1.0	1.75	1.12	1.54	2.00	1.25
0.9	1.56	1.37	1.53	2.00	1.37
0.5	1.28	1.75	1.30	2.00	1.23
0.05	1.03	1.98	1.03	2.00	1.02

Table 4. Backscattering Enhancement Factors for a Rayleigh-Scattering Slab with $\mu_0 = 1$ and $\omega = 0.99$

τ	$\zeta_\parallel$	$\zeta_\perp$	ζ	ζ_{hp}	ζ_{oh}
∞	1.69	1.19	1.55	2.00	1.31
50	1.69	1.19	1.55	2.00	1.31
5	1.67	1.23	1.55	2.00	1.39
0.5	1.31	1.71	1.35	2.00	1.25
0.05	1.06	1.94	1.08	2.00	1.05

Table 5. Backscattering Enhancement Factors for a Semi-Infinite Slab Composed of Randomly Oriented Oblate Spheroids with $m = 1.5$, $x_{ev} = 4$, and $a/b = 2$

μ_0	$\zeta_\parallel$	$\zeta_\perp$	ζ	ζ_{hp}	ζ_{oh}
0.0	1.00	1.00	1.00	1.00	1.00
0.05	1.56	1.29	1.47	1.60	1.27
0.1	1.72	1.32	1.57	1.76	1.28
0.5	1.95	1.24	1.62	1.96	1.12
1.0	1.98	1.16	1.58	1.99	1.05

In Table 1 the enhancement factors are calculated for semi-infinite slabs composed of homogeneous spherical particles with the index of refraction $m = 1.2$ and different values of the size parameter $x = 2\pi r/\lambda$, where r is the radius and λ is the wavelength. The calculations are reported for $\mu_0 = 1$. The refractive index $m = 1.2$ roughly corresponds to latex in water, and one can verify by inspection that the numbers given are in good agreement with the experimental data of Wolf *et al.*[30] and van Albada *et al.*[31] In accord with the above analysis, the factor ζ_{hp} exactly equals 2. This value was first observed experimentally by Etemad *et al.*[32] and discussed theoretically by MacKintosh and John.[10] The factor $\zeta_\parallel$ is close to 2, especially for larger particles, in agreement with the prediction of the scalar theory. Nevertheless, the scalar theory completely fails to predict correctly the values of the enhancement factor for unpolarized incident light, ζ, which are substantially smaller than 2. The factor ζ_{oh} is smaller for larger particles, while the factor $\zeta_\perp$ seems to be larger. The factor ζ is practically independent of x.

The limits of Eq. (35) are illustrated in Tables 2–4, where the numerical data for semi-infinite and finite slabs composed of Rayleigh scatterers are displayed. Both finite thickness and true absorption terminate the random walk of photons. Thus Tables 3 and 4 evidently demonstrate that the depolarized enhancement factor $\zeta_\perp$ results from lower-order scattering, while the factors $\zeta_\parallel$, ζ, and ζ_{oh} result from all orders of scattering.[30,31]

Finally, in Table 5 the effects of particle nonsphericity are demonstrated. The backscattering enhancement factors are calculated for a semi-infinite slab composed of randomly oriented oblate spheroids with $m = 1.5$, $x_{ev} = 4$, and $a/b = 2$, where $x_{ev} = 2\pi r_{ev}/\lambda$, a/b is the ratio of the semiaxes of the spheroid, and $r_{ev} = (a^2b)^{1/3}$ is the radius of the equal-volume sphere. One sees that, in agreement with the above theoretical analysis, all the enhancement factors tend toward unity with $\mu_0 \to 0$ and that the factor ζ_{hp} substantially deviates from 2.

Note added in proof. Additional general properties of the backscattering enhancement factors can be derived by using the inequalities that must be satisfied by any Stokes transformation matrix of the block-diagonal form given by Eq. (33) [J. W. Hovenier, H. C. van de Hulst, and C. V. M. van der Mee, "Conditions for the elements of the scattering matrix," Astron. Astrophys. **157**, 301–310 (1986)]. By applying these inequalities to the matrix $\mathbf{S}^L$, one has from Eqs. (26) and (29)–(32)

$$1 \leq \zeta_\parallel \leq 2,$$

$$0 \leq \zeta_\perp \leq 2,$$

$$0 \leq \zeta \leq 2,$$

$$1 \leq \zeta_{hp} \leq 2,$$

$$0 \leq \zeta_{oh} \leq 2.$$

One can easily verify that the numbers given in Tables 1–5 satisfy these inequalities.

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