OFIM documentation

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1. Introduction

A 3-dimensional global ocean model (”OFIM”) is under development at NOAA’s Earth System Research Laboratory. The model is destined to become the oceanic counterpart of ESRL’s finite-volume, flow-following, icosahedral atmospheric model FIM. By sharing FIM’s icosahedral mesh, OFIM can be attached to FIM for coupled ocean-atmosphere simulations without incurring the complexities of an interpolating ”flux coupler”.

Two versions of OFIM are undergoing testing at this time. Both treat temperature and salinity as separataly forced and transported prognostic variables.

- One model version, OFIM1, features an adaptive vertical coordinate representing a combination of fixed-depth layers in the upper ocean and constant potential-density layers underneath. This layout is familiar from the ocean model HYCOM (Bleck 2002) and resembles an upside-down rendering of the FIM vertical grid (Bleck et al. 2010) which uses a combination of terrain-following and isentropic coordinates.

- The other model version, OFIM2, only allows the two uppermost coordinate layers to be nonisopycnic. These represent, respectively, a slab mixed layer (Kraus and Turner 1967) and a ”fossil” layer serving as a receptacle for water detrained from the mixed layer. Deviations from assigned target density due to either numerical transport errors or physical processes (“cabbeling”) are suppressed in the isopycnic layers (i.e. layer 3 and beyond) by a special coordinate maintenance algorithm. Depending on the chosen target densities, massless isopycnic layers can occur both on the variable-depth seafloor and on the interface between the nonisopycnic and isopycnic coordinate domains.

The vertical grid in OFIM1 is ”adaptive” in the sense that coordinate layers are not treated rigidly as being either fixed-depth or isopycnic (terrain-following/isentropic in the case of FIM). Any layer, and even regional portions of a layer, can transition from one coordinate mode to the other. The preferred mode for a layer to be in is the isopycnic/isentropic one, but vertical grid spacing constraints override this assignment.

OFIM interacts with FIM by receiving surface fluxes of momentum, heat and freshwater, and providing sea surface temperature and ice coverage information in return. Since FIM and OFIM are sharing the same horizontal grid, information exchange is straightforward and flux interpolation errors are avoided.

FIM treats OFIM as a subroutine. This is to say that only one of the two submodels is running at any time and, while active, is making use of all available processors.

OFIM is designed to capture the gamut of dynamic processes affecting the global SST on weather- and climate-relevant time scales. These processes include sea ice formation, thermally and mechanically forced mixed layer entrainment/detrainment, small-scale diapycnal mixing, and wind- as well as thermohaline-forced large-scale heat transport. Excluded for the time being are dynamic ice spreading, tidal effects, and a wave submodel for surface roughness prediction.

OFIM makes use of the distributed data layout, indirect addressing procedures, and finite-volume numerics developed for FIM and solves a closely related set of prognostic equations. Both models use A-type horizontal staggering of variables. However, their time stepping schemes differ. The 3^{rd} order Adams-Bashforth scheme successfully employed in FIM (Lee et al. 2010) failed in OFIM because it does not permit rigorous enforcement of positive-definiteness in the layer thickness tendency equation. For this reason, OFIM has inherited from HYCOM the traditional leapfrog scheme.

The reason why multistep Adams-Bashforth works in FIM but not in OFIM may be threefold:

1. oceanic orography is steeper than terrestrial orography;
2. the ocean is less stratified and therefore less able to suppress the growth of large-amplitude internal gravity waves;
3. OFIM does not use terrain-following coordinates near the lower boundary as FIM does; hence, massless layers are a standard feature on the sea floor where they pose a particular numerical challenge due to the steepness of bottom slopes.

(We note in passing that eliminating massless bottom layers by adopting terrain-following coordinates is not an option in a ”blue-water” ocean circulation model because the well-known sigma-coordinate pressure gradient error is prohibitively serious near the continental margins and other steep bottom features.)

One difference between OFIM and HYCOM numerics is worth mentioning. HYCOM, like many other ocean models, gains efficiency by separating barotropic gravity waves from other types of fluid motion and by simulating their propagation using a numerically efficient 2-dimensional shallow-water model. We have not yet been able to get this split-explicit mode separation scheme to work in OFIM. The problem appears to be transverse gravitational sloshing in 2-grid cell wide channels, the cause for which we conjecture to be underestimated (or incorrectly coupled) cross-channel pressure gradients on the A grid.

In the absence of a mode splitting scheme, we are forced to integrate the 3-dimensional momentum and continuity equations – the set responsible for transmitting gravity waves – using a short time step linked to the phase speed of barotropic gravity waves (referred to as the ”barotropic” time step). Tracers can be, and are, advected using a much longer ”baroclinic” time step linked to the internal-wave and the actual flow speed.

The lack of a mode splitting scheme has a silver lining. Such schemes – at least the split-explicit ones – are notoriously unstable (Morel et al. 2008) and thus create hazardous working conditions during model development. Furthermore, the particular modal decomposition developed for HYCOM does not permit changing the bottom pressure in the ”baroclinic” set of equations. Hence, the freshwater flux at the surface must be converted in HYCOM into a virtual salt flux, causing potential problems with negative salinities and long-term freshwater conservation, not to mention the omission in the ocean model of the sizeable return flow of water transported poleward by the atmosphere (Huang 1993). All these difficulties are presently avoided in OFIM.

2. Equation of State

The vertical coordinate in the isopycnic subdomain of OFIM is potential density anomaly referenced to a pressure of 2000 dbar (roughly 2 km), commonly referred to as σ_2. The choice of reference pressure is a compromise minimizing the deviation of coordinate layer slope from the slope of truly neutral surfaces while at the same time rendering today’s global ocean statically stable. (Neither σ_0 nor σ_4 is vertically monotonic everywhere in the world ocean.)

The equation of state (encoded in *sigocn.F90* which is part of *hycom_sigetc.F90*) is taken from Brydon et al. (1999). The thermobaric component of seawater density in the equation of state, whose use would bring the slope of coordinate surfaces closer to that of neutral surfaces over a wide pressure range (Sun et al. 1999), will be added at a future date.

While not precisely buoyancy-neutral at pressures other than 2000 dbar, σ_2 surfaces come a long way toward eliminating the false diapycnal component of numerically induced diffusion of prognostic variables which is an inevitable side effect of solving finite-difference transport equations on constant-depth surfaces. As in the case of FIM, this is the primary motivation for designing a circulation model around an isentropic/isopycnic vertical coordinate.

3. Hydrostatic Equation

Due to its use in a predominantly isopycnic coordinate model, the hydrostatic equation is solved in OFIM in the form

$$\frac{\partial M}{\partial \alpha} = p \tag{1}$$

where $M = gz + p\alpha$ is the Montgomery potential, p is pressure, gz is the geopotential, and α is inverse potential density. The integration over α takes place from the bottom up in *hycom_hystat.F90*. Note that M and α are layer variables while p is carried on layer interfaces.

An initial value of M on the sea floor is computed during model initialization in *hycom_init.F90* by a top-down integration of (1), starting with $z = 0$ and $p = 0$ at the surface. Thereafter, the actual sea floor value of M needed for upward integration of (1) is obtained by correcting the initially computed M for changes in bottom p and α.

4. Momentum Equations

The horizontal momentum equations are solved in the form

$$\left(\frac{\partial u}{\partial t}\right)_s - fv = -\left(\frac{\partial M}{\partial x}\right)_s + p\left(\frac{\partial \alpha}{\partial x}\right)_s + \frac{1}{\Delta p}\nabla_s \cdot (\nu \Delta p \nabla_s u) \tag{2}$$

$$\left(\frac{\partial v}{\partial t}\right)_s + fu = -\left(\frac{\partial M}{\partial y}\right)_s + p\left(\frac{\partial \alpha}{\partial y}\right)_s + \frac{1}{\Delta p}\nabla_s \cdot (\nu \Delta p \nabla_s v) \tag{3}$$

where ν is an eddy viscosity, Δp is the layer thickness, and s is a vertically monotonic but otherwise arbitrary variable denoting the vertical coodinate. Subscript s indicates that partial derivatives are to be evaluated at $s = const$. These equations are similar to the momentum equations in FIM and are solved in *hycom_momtum.F90* in a very similar way. The reasons for adding explicit viscous terms on the r.h.s. of (2) and (3) are twofold:

- Lateral sidewall drag is an important element of western boundary current dynamics;
- Some lateral stirring of momentum is deemed beneficial in coarse-mesh, noneddy-resolving applications.

Even though u, v are the Cartesian velocity components regardless of the definition of the vertical coordinate, one must remember that all horizontal derivatives on the r.h.s. of (2) and (3) are taken along coordinate surfaces. Since α is constant (or nearly so) in interior coordinate layers, the gradient of M dominates the 2-term pressure force expression in those layers. Only in nonisopycnic layers (the mixed layer and fossil mixed layer in OFIM2, and the isobaric subdomain in OFIM1) do both terms play a major role.

Lateral drag is communicated only among grid cells belonging to the same coordinate layer. In the interior, this has the effect of rendering momentum exchange via eddy stirring an isopycnal process, as it tends to be in reality.

Sidewall drag is evaluated by depicting the ocean bottom as a assemblage of random-height hexagonal basalt-like blocks and determining the vertical extent to which a given model layer is in contact with one of those blocks. The portion of the layer extending above the block is assumed not to be affected by its presence. Prorating the effect of sidewalls in this fashion avoids temporal discontinuities in sidewall drag in layers subjected to gravity wave sloshing near steep bottom slopes.

One issue regarding the formulation of the horizontal pressure gradient force (PGF) must be mentioned because it remains unresolved at this time. If a layer interface is not a surface $p = const$ or $\phi = const$, the two layers meeting at this interface subject one another to a form drag $p\nabla_s\phi$ (of opposite sign, of course). Without consistent representation of this form drag in the PGF formula, there is no guarantee that an ocean model will simulate the strength of western boundary currents correctly. The reason for this sensitivity is that the form drag – or rather its curl, the pressure torque – controls the gyre-scale spinup of layers beneath the Ekman layer, thereby affecting the amount of water that needs to flow along the western boundary to close the gyre circulation.

The form drag constraint arises in atmospheric modeling as well but is arguably most important in large-scale ocean dynamics (keyword: Sverdrup regime).

In Appendix A of Bleck (2002), a finite-difference expression for the PGF in generalized coordinates is derived that incorporates the above constraint, albeit only for a logically rectangular grid. Attempts to derive analogous PGF expressions appropriate for the unstructured horizontal FIM grid have been unsuccessful so far.

5. Continuity Equation

OFIM, like FIM, is a stacked shallow-water model. The solution procedure for the continuity equation, which largely controls the vertical spacing of layer interfaces, mimics the procedure used in FIM, but allowances must be made, of course, for the presence of coastal boundaries. The coastline in OFIM follows cell edges, which is to say that a given grid cell is either totally land or totally water. This allows us to rigorously apply the kinematic boundary condition stipulating zero flow across coastlines.

The layer-integrated continuity equation is identical to the one solved in FIM:

$$\frac{\partial \Delta p}{\partial t} + \nabla_s \cdot (\mathbf{v}\Delta p) + \left(\dot{s}\frac{\partial p}{\partial s}\right)_2 - \left(\dot{s}\frac{\partial p}{\partial s}\right)_1 = 0. \quad (4)$$

Here, indices 1,2 denote the lower and upper interface, respectively, of the layer under consideration. Note that s always appears in combination with $\partial p/\partial s$ to account for the fact that the dimensions of s are arbitrary and may, in fact, be physically meaningless. As in FIM, the continuity equation is solved using Flux Corrected Transport (Zalesak 1979), with high-order fluxes based on centered 2^{nd} order finite difference expressions.

Due to the small size of baroclinic eddies in the ocean, their influence on the large-scale flow has to be parameterized in coarse-mesh ocean models. Aside from adding explicit lateral "eddy" mixing terms in (2) and (3), the effect of baroclinic instability on the resolved-scale buoyancy field needs to be taken into account. A widely adopted parameterization is that of Gent and McWilliams (1990) which captures the instability-induced slumping of tilted isopycnals by invoking a peristaltic or "bolus" flux transferring mass within isopycnic layers such that available potential energy decreases.

In an isopycnic model, the GM parameterization can be implemented elegantly by smoothing the interface pressure field. The important point to note here is that baroclinic instability is an adiabatic process, implying that interface smoothing may not lead to mass transfer between layers. Instead, vertical displacement of interfaces due to smoothing must be translated into an *intralayer* mass flux. This is accomplished by viewing the smoothing operator $A\nabla^2 p$ (where A is the product of a diffusivity coefficient and the model time step) as the divergence of an "interface pressure flux" $A\nabla p$, i.e., as $\nabla \cdot (A\nabla p)$. The intralayer mass, or layer thickness, flux is then simply the difference of $A\nabla p$ at the upper and lower interface. To make this work in the case of variable bottom topography, $A\nabla p$ must be set to zero wherever an interface coincides with the sea floor. In addition, interface pressure fluxes must be bounded by the amount of mass available in cells shrinking during the smoothing operation.

Interface smoothing is done in OFIM at the end of *hycom_cnuity.F90*. The resulting bolus fluxes are added to the "regular" mass fluxes, which allows them to contribute to lateral tracer transport.

One word of caution: Interface smoothing captures the GM effect only if a layer interface is isopycnic. In OFIM, all interfaces not coinciding with the sea floor are smoothed, but there is no pretense of physical significance in smoothing the bottom of the active and the fossil ML in OFIM2. These two interfaces are smoothed nevertheless, but for reason of numerical smoothness.

A scheme for extending GM to the isobaric coordinate subdomain in OFIM1 (and its parent model HYCOM) is under development. The approach taken is to transform the native grid to a purely isopycnic grid, smooth the resulting interfaces, and transform the mass fluxes inferred from the thickness changes on the transformed grid back to the native grid. Note that this grid transform is a null operation in the isopycnic subdomain.

By transforming tracers to the isopycnic grid and numerically diffusing them on that grid, rather than on the native grid, we also plan to extend isoneutral eddy mixing (Redi 1982) to all parts of the OFIM1 grid domain.

6. Tracer Transport

Because vertical mesh size in OFIM varies in space and time, it is mandatory to solve tracer conservation equations in flux form. Temperature and salinity are transported using the long-time step approach developed for tracer transport in FIM. The salient aspects of this method are laid out here for convenience. For additional details see Bao et al. (2011) and Sun and Bleck (2006).

Let Δt be the time step appropriate for transmitting gravity waves. Solving tracer transport equations on a longer time step $J\Delta t$ ($J > 1$) commensurate with actual flow rather than gravity wave speed, the conservation equation must be based on a rigorously time-integrated form of the mass continuity equation (4),

$$\frac{\Delta p^{n+J} - \Delta p^n}{J\Delta t} + \nabla_s \cdot \overline{\mathbf{v}\Delta p}^J + \left(\overline{\dot{s}\frac{\partial p}{\partial s}}^J\right)_2 - \left(\overline{\dot{s}\frac{\partial p}{\partial s}}^J\right)_1 = 0, \qquad (5)$$

where the overbar denotes integration over J time steps. To assure that the equation is exactly satisfied in the model, the dynamically active fields must already have been stepped forward from time level n to $n+J$. At that instant, both the tendency term and the horizontal flux divergence term in (5) can be determined, the latter by summing up the instantaneous fluxes over the past J time steps. The time-integrated vertical flux terms can then be obtained by vertically summing up (5), using $\dot{s} = 0$ (material surface boundary condition) at the top and bottom of the column.

By combining (5) with the equation $dQ/dt = 0$, expressing conservation of a tracer Q during transport (sources and sinks of Q can be evaluated separately), we arrive at the transport equation

$$\frac{(Q\Delta p)^{n+J} - (Q\Delta p)^n}{J\Delta t} + \nabla_s \cdot (Q\,\overline{\mathbf{v}\Delta p}^J) + \left(\overline{\dot{s}\frac{\partial p}{\partial s}}^J \hat{Q}\right)_2 - \left(\overline{\dot{s}\frac{\partial p}{\partial s}}^J \hat{Q}\right)_1 = 0. \qquad (6)$$

which can be solved for the tracer amount $Q\Delta p$ at time level $n+J$. Here, the caret indicates values interpolated to layer interfaces. For details regarding the conversion of $Q\Delta p$ to Q in massless or near-massless layers, see the FIM documentation.

Eqn. (6) is solved using Flux Corrected Transport where high-order fluxes are again based on centered 2^{nd} order finite differencing.

7. Surface Mixed Layer

The mixed layer (ML) model currently in use in OFIM2 was first formulated by Kraus and Turner (1967) and later refined by Gaspar (1988). The Kraus-Turner (KT) model is based on the turbulence kinetic energy (TKE) equation, vertically integrated over the depth range (the "ML depth") in which surface forcing is able to maintain turbulence against dissipation. The salient closure assumption allowing determination of the ML depth is that turbulence sources and sinks balance in the ML at any time.

TKE is subject to 4 forcing terms:

- wind stirring;
- surface buoyancy flux;
- energy required to lift denser water into the ML;
- dissipation.

The evolution of the ML depends critically on the sign of the buoyancy flux. If heat and/or freshwater is added at the sea surface, the TKE equation boils down to a statement of balance between turbulence-inducing wind stirring and the turbulence-suppressing buoyancy flux. Turbulence is then maintained over a depth range given by the Obukhov length (which is basically the ratio of wind and buoyancy forcing).

If, on the other hand, the buoyancy flux acts to destabilize the ML, the combined TKE input (with some allowance for dissipation) is invested into lifting the center of gravity of the column by entraining denser water into the ML. The rate of ML deepening is set by the TKE input rate and the density contrast across the ML bottom.

Note that in the case of ML deepening the KT model diagnoses the *rate of change* of ML depth, not the depth itself as it does for a retreating ML.

When the buoyancy flux changes from being a TKE source to a sink, ML depth can shrink abruptly. The bottom part of the previously deep ML then becomes a "fossil" ML. The active and the fossil ML are represented by coordinate layers 1 and 2, respectively, in OFIM2. Since the active ML is assumed to be fully homogenized at all times, surface fluxes are uniformly distributed over it.

Water accumulating in the fossil ML represents the seasonal thermocline, both in the real world and in the model. In order to allow it to migrate laterally into the permanent thermocline, thereby maintaining its stratification (Luyten et al. 1983) against the perpetual accumulation and upwelling of near-freezing deep water (Stommel and Arons 1960), the fossil ML water must eventually be transferred to isopycnic coordinate layers.

The problem with this transfer is that fossil layer density varies continuously in space and time while the interior layers have prescribed densities. This mismatch presents somewhat of a numerical challenge, but experience shows that the detrainment algorithm described below is sufficiently effective to prevent gradual accumulation of water in layer 2 that has no place to go. Nevertheless, the discrete nature of the interior density field inevitably spawns temporal discontinuities in the discharge of fossil-layer water. As with all truncation errors, increased vertical grid resolution would be the ultimate remedy for this problem.

The general concept of fossil-layer discharge dates back to Bleck et al. (1989) but has evolved somewhat, warranting a full description here.

Let h be the depth of the fossil layer. The first step is to redistribute (”unmix”) T, S within the layer so as to create two sublayers – one sublayer of depth z whose density matches that of the isopycnic layer about to receive the discharged water, and one sublayer of depth h-z whose density matches that of the active ML. In step 2 the lower sublayer is merged with the ”receiving” isopycnic layer while the upper sublayer becomes the new, shallower fossil layer. Unfortunately, matching the densities of two adjacent coordinate layers while conserving T, S during unmixing is not possible in general. A compromise solution has to be devised.

Let indices 1,2,3 refer to the ML, the fossil ML, and the receiving layer, respectively. In order to avoid generating unrealistic water types during unmixing, temperature and salinity in the two sublayers must lie in a ”bounding box” in T, S space spanned by

$$T_{min} = \min(T1, T2, T3) \qquad T_{max} = \max(T1, T2, T3)$$

$$S_{min} = \min(S1, S2, S3) \qquad S_{max} = \max(S1, S2, S3)$$

To be T, S-conservative, unmixing must take place along a straight line through point T_2, S_2. Several situations can occur:

- The point T_2, S_2 lies on the perimeter of the bounding box. In this case, only one of the two variables can be unmixed, depending on which edge of the bounding box the point is located on.
- The point T_2, S_2 lies in the interior of the bounding box. In this case, points T_1, S_1 and T_3, S_3 occupy opposite corners, and both T and S can participate in the unmixing.

We assign T, S conservation the highest priority. Hence, in the general case where the unmixing line does not run through all 3 points, the density of at least one sublayer will deviate from the desired value.

A number of possible unmixing scenarios come to mind, some more physically attractive than others. The ones that have been tested so far are the following.

1. Unmixing takes place along a line running parallel to the diagonal connecting T_1, S_1 and T_3, S_3. The intent here is to make the unmixing line as long as possible, thereby maximizing z, while minimizing density deviations both between the lower sublayer and the receiving layer and between the upper sublayer and the active ML.
2. A point $\hat{T}_1, \hat{S}_1$ close to T_1, S_1 is selected for the upper sublayer to occupy. The intersection of the line through $\hat{T}_1, \hat{S}_1$ and T_2, S_2 with the bounding box perimeter opposite T_1, S_1 then defines the properties of the lower sublayer.

Preliminary testing reveals that scheme 2 yields more consistent results; it is therefore being used in OFIM2 at the time of this writing. To maintain a small density contrast between active and fossil ML, we place the point $\hat{T}_1, \hat{S}_1$ a small distance from T_1, S_1 in the direction of T_2, S_2 by setting $\hat{T}_1 - T_1 = \max(\min(0, T_2 - T_1), -0.1\,\mathrm{C})$ and $\hat{S}_1 - S_1 = \min(\max(0, S_2 - S_1), 0.01\,\mathrm{psu})$

Typically, the receiving layer ends up being too light after merging with the lower sublayer because the bounding box perimeter intercepts the unmixing line before it reaches the target isopycnal of the receiving layer. The resulting discrepancy is subsequently removed by allowing the receiving layer to entrain denser water from one or more layers below. This operation is performed by the coordinate maintenance algorithm described in the following section. Entrainment of denser water into the receiving layer is deemed beneficial because it counteracts the decrease in column potential energy brough about by the artificial unmixing of the fossil layer.

The Kraus-Turner ML model is particularly attractive if the ML bottom can be represented in a numerical model by a coordinate surface, and if vertical grid resolution is too poor to model ML processes as vertical diffusion with a Richardson-number dependent exchange coefficient. (Richardson numbers are notoriously difficult to diagnose in coarse vertical meshes.)

An upgrade in the number of model layers has recently allowed us to shift emphasis in our work to OFIM1 whose vertical resolution is sufficient to permit the use of more modern schemes, such as KPP (Large et al. 1994) and

Canuto et al. (2001).

8. Coordinate Maintenance

All coordinate layers in OFIM2, with the exception of the uppermost two layers, are nominally isopycnic. In the presence of two independently advected and diffused buoyancy-relevant tracers, density in an isopycnic layer will gradually drift away from its assigned target value, both for numerical and physical reasons ("cabbeling"). A special algorithm is therefore needed to keep layer densities close to target. The routine *hycom_maintn.F90* accomplishes this by entraining denser or lighter water from a neighboring layer, depending on the sign of the deviation.

The model computes the amount of water to be entrained from the density offset in the layer under consideration in relation to the densities of the neighboring layers. Due to the nonlinearity (in both T and S) of the equation of state for seawater, the entrained slab of water usually does not precisely restore the desired target density; hence, the maintenance process is iterative but converges quickly.

If a layer is too light but lacks a denser layer underneath, T is redistributed within it to create two sublayers, the lower one matching the desired target density and the upper one matching the target density of the layer above. The upper sublayer is then expelled into the layer above.

The coordinate maintenance routine performs the additional task of consolidating multiple ultra-thin layers that may appear either on the seafloor or along the interface between the fossil layer and the isopycnic interior. Such layers occasionally form due to roundoff errors; they are dynamically inconsequential but can complicate the column physics logic.

In OFIM1, coordinate maintenance follows the strategy outlined in Bleck (2002).

9. Diapycnal Mixing

Both OFIM versions use the McDougall and Dewar (1998) scheme adapted from HYCOM to model small-scale diapycnal mixing in the water column beneath the surface mixed layer. The scheme was specifically developed for isopycnic coordinate models. It diffuses temperature and salinity vertically, but it does so without modifying the density in individual coordinate layers (within the limits of a linearized state equation). However, what does change in the course of the diffusive process is the thickness of layers. Thin layers in the interior of the column often grow at the expense of thick layers while those at the top and bottom gradually vanish. In fact, the scheme predicts an infinite inflation rate for massless layers, requiring an arbitrary bound on interlayer mass transfer.

The algorithm copied from HYCOM is unable to inflate two or more massless layers sandwiched together because simultaneous mass input from both neighbors of a given layer is required to thicken it. This limitation has been removed in OFIM by embedding the HYCOM-based routine *hycom_diapfl.F90* in another one, called *hycom_diamix.F90*, which searches each grid column for sequences of two or more massless layers.

If such a sequence is present, the routine removes these layers from the column but then calls *hycom_diapfl.F90* repeatedly, each time inserting a different one of the massless layers in question. This particular layer, by virtue of it's being solitary, can now be inflated. The time step in this series of calls is reduced appropriately to avoid "overdiffusing" the remaining, mass-containing layers. This leads to "under-inflation" of the massless layers; however, the rate of inflation of such layers is fairly arbitrary, as pointed out earlier.

The requirement to conserve T, S as well as the initially assigned layer densities during the diffusion process means that complete homogenization of a water column is generally not possible. Except under special circumstances, the final state arrived at by the McDougall-Dewar scheme will consist of 2 model layers which, due to their different densities, also differ in their T, S properties.

The original McDougall-Dewar scheme does not distinguish between diffusion coefficients for T and S, and no attempt has been made to generalize the scheme in this direction. The diffusivity in OFIM is set to the larger of two values, $2\times10^{-5}\mathrm{m}^2\mathrm{s}^{-1}$ and $2\times10^{-7}\mathrm{m}^2\mathrm{s}^{-2}/N$, where N is the buoyancy frequency.

10. Sea Ice Model

Sea-ice related processes are presently modeled in the simplest possible way with the ”energy loan” model developed for HYCOM. It resembles the single-layer model discussed in the Appendix of Semtner (1976). Freezing takes place whenever latent heat is needed to keep the mixed layer temperature from dropping below the freezing level. When the ocean-ice system is being heated, the incoming energy is used to melt the ice before the water temperature is allowed to rise above the freezing level.

One major task of an ice model is to find the ice surface temperature. In the energy loan model this temperature is obtained iteratively (one iteration per model time step) by stipulating that the vertical heat flux inside the ice agrees with the prescribed heat flux through the ice-air interface. To improve convergence, the effect of the temperature change on the outgoing radiation, and hence on the energy flux computation at the next time step, is incorporated into the iterative scheme.

11. Miscellaneous

11a. *Hydrologic Cycle*

An ocean model will gradually lose freshwater mass and become saltier if precipitation falling onto land is not returned to the ocean. Pending acquisition of a data base cataloguing watersheds worldwide, FIM estimates the direction of river runoff from the terrain slope. To reduce the number of grid cells that are surrounded on all sides by higher terrain and thus cannot be assigned an outflow direction without manual intervention, the topography used by the river runoff scheme is heavily smoothed.

No attempt is presently made to properly account for the time spent by runoff water in the various rivers. Precipitation which falls into a grid cell during an atmospheric time step and is not re-evaporated or used to moisten the soil is simply passed to the neighboring downstream cell, together with water that arrived during the previous time step from adjacent upstream cells. Effectively, then, the river flow speed is uniformly set to 1 grid cell per time step.

The river routing scheme is an adaptation of the one used in climate models at the NASA Goddard Institute for Space Studies.

11b. *Land/Sea Boundary Reconciliation*

Even though OFIM is on the same horizontal grid as FIM, the land mask used in FIM is not necessarily optimal for OFIM where the width of straits and the existence or absence of land bridges is important for the circulation. For this reason, the land mask prepared for OFIM trumps the FIM mask in coupled runs, requiring a reconciliation strategy. In grid cells which FIM thinks are ocean but OFIM says are land, various land surface parameters (vegetation type etc.) need to be set. This is done by nearest-neighbor extrapolation. Grid cells converted from land to ocean require less work but nevertheless need to be properly identified in FIM as water points.

11c. *Finite-difference operations near coastal boundaries*

Despite its regular appearance, the icosahedral grid used in FIM and OFIM (Wang and Lee 2011) is, technically speaking, ”unstructured”. As outlined in Lee and MacDonald (2009) and Bao et al. (2011), finite-volume numerics can be easily implemented on unstructured grids by converting spatial derivative operations (divergence, curl, gradient) into line integrals around individual grid cells. This requires interpolation of variables from cell centers to cell perimeters. (Recall that FIM and OFIM use A grid staggering.)

In FIM, line integral segments along each of the 5 or 6 edges of a grid cell are evaluated using Simpson's rule which requires function values in the center and at each end of the integration interval. Edge *center* values are obtained by averaging the 2 nearest cell center values while *end* values, located at the corners of the pentagons/hexagons defining the grid, are obtained by averaging the 3 nearest cell center values. At present, no allowance is made for distortions in the shape and size of the icosahedral grid cells.

OFIM evaluates line integrals in the same manner, but values needed for the interpolation are not always available because an adjacent cell might be on land or have zero thickness due to sharply rising bottom topography. Variables in these ”ghost” cells are obtained by extrapolation which is carried out in *hycom_edgvar.F90* as part of the general interpolation task. Details are as follows.

1. *momentum:* Let **v** be the velocity vector in the ocean cell adjacent to the ghost cell. The vector in the ghost

cell is then set to either **v** or **-v**, depending on whether free- or no-slip sidewall conditions are specified. In line integrals over u or v, corner values are set to zero, i.e., only center values are used.

2. *interface pressure:* Pressure in the ghost cell is set to the value in the adjacent ocean cell.

3. *pressure gradient force:* the PGF on the r.h.s. of (2) and (3) is based on line integrals over M and α. Ghost values for these 2 variables are set to the value in the adjacent ocean cell.

If neighboring bottom topography only partially covers the lateral face of a grid cell, the weight of the ghost values of u, v, M, α in the interpolation procedure is reduced accordingly. This partial weighting, already mentioned at the end of the momentum equation section, is a feature adopted from HYCOM. The degree of lateral blocking of an ocean cell extending from p_1 to $p_2 (> p_1)$ by neighboring bottom topography is determined by comparing the difference between the neighboring sea floor pressure and p_1 to the difference $p_2 - p_1$. The ghost value and the actual value in the neighboring cell are weighted linearly based on the ratio of the two pressure differences, provided the ratio lies in the interval (0,1).

References

Bao, J.-W., S. Benjamin, R. Bleck, J. Brown, J. Lee, A. E. MacDonald, J. Middlecoff, and N. Wang, 2011: *Manuscript description of FIM model*. `http://fim.noaa.gov`.

Bleck, R., 2002: An oceanic general circulation model framed in hybrid isopycnic-cartesian coordinates. *Ocean Modelling*, **4**, 55–88.

Bleck, R., S. Benjamin, J. Lee, and A. E. MacDonald, 2010: On the use of an adaptive, hybrid-isentropic vertical coordinate in global atmospheric modeling. *Mon. Wea. Rev.*, **138**, 2188–2210.

Bleck, R., H. P. Hanson, D. Hu, and E. B. Kraus, 1989: Mixed layer-thermocline interaction in a three-dimensional isopycnic coordinate model. *J. Phys. Oceanogr.*, **19**, 1417–1439.

Brydon, D., S. Sun, and R. Bleck, 1999: A new approximation of the equation of state for sea water, suitable for numerical ocean models. *J. Geophys. Res.*, **104**, 1537–1540.

Canuto, V. M., A. Howard, Y. Cheng, and M. S. Dubovikov, 2001: Ocean turbulence. Part I: one-point closure model - momentum and heat vertical diffusivities. *J. Phys. Oceanogr.*, **31**, 1413–1426.

Gaspar, P., 1988: Modeling the seasonal cycle of the upper ocean. *J. Phys. Oceanogr.*, **18**, 161–180.

Gent, P. R. and J. C. McWilliams, 1990: Isopycnal mixing in ocean circulation models. *J. Phys. Oceanogr.*, **20**, 150–155.

Huang, R. X., 1993: Real freshwater flux as a natural boundary condition for the salinity balance and thermohaline circulation forced by evaporation and precipitation. *J. Phys. Oceanogr.*, **23**, 2428–2446.

Kraus, E. B. and J. S. Turner, 1967: A one-dimensional model of the seasonal thermocline. Part II: the general theory and its consequences. *Tellus*, **19**, 1909–1924.

Large, W. G., J. C. Williamns, and S. C. Doney, 1994: Oceanic vertical mixing: a review and a model with a nonlocal boundary layer parameterization. *Rev. Geophys.*, **32**, 363–403.

Lee, J.-L., R. Bleck, and A. E. MacDonald, 2010: A multistep flux-corrected transport scheme. *J. Comput. Phys.*, **229**, 9284–9298.

Lee, J.-L. and A. E. MacDonald, 2009: A finite-volumne icosahedral shallow water model on a local coordinate. *Mon. Wea. Rev.*, **137**, 1422–1437.

Luyten, J. R., J. Pedlosky, and H. Stommel, 1983: The ventilated thermocline. *J. Phys. Oceanogr.*, **13**, 292–309.

McDougall, T. J. and W. K. Dewar, 1998: Vertical mixing and cabbeling in layered models. *J.Phys.Oceanogr.*, **28**, 1458–1480.

Morel, Y., R. Baraille, and A. Pichon, 2008: Time splitting and linear stability of the slow part of the barotropic component. *Ocean Modelling*, **23**, 73–81.

Redi, M. H., 1982: Oceanic isopycnal mixing by coordinate rotation. *J. Phys. Oceanogr.*, **12**, 11541158.

Semtner, A. J., 1976: A model for the thermodynamic growth of sea ice in numerical investigations of climate. *J. Phys. Oceanogr.*, **6**, 379–389.

Stommel, H. and A. Arons, 1960: On the abyssal circulation of the world ocean-I. Stationary planetary flow patterns on a sphere. *Deep-Sea Res.*, **6**, 140–154.

Sun, S. and R. Bleck, 2006: Geographic distribution of the diapycnal component of thermohaline circulations in coupled climate models. *Ocean Modelling*, **15**, 177–199.

Sun, S., R. Bleck, C. Rooth, J. Dukovicz, E. Chassignet, and P. Killworth, 1999: Inclusion of thermobaricity in isopycnic-coordinate ocean models. *J. Phys. Oceanogr.*, **29**, 2719–2729.

Wang, N. and J.-L. Lee, 2011: Geometric properties of the icosahedral-hexagonal grid on the two-sphere. *SIAM J. Sci, Comput.*, in press.

Zalesak, S., 1979: Fully multidimensional flux-corrected transport algorithms for fluids. *J. Comput. Phys.*, **31**, 335–362.