

Understanding the possible influence of the solar activity on the terrestrial climate: a times series analysis approach

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Outline

- I. Review of the solar-terrestrial problem**
- II. Time Series Analysis and our proposal based on the VAR methodology**
- III. Summary and future work**

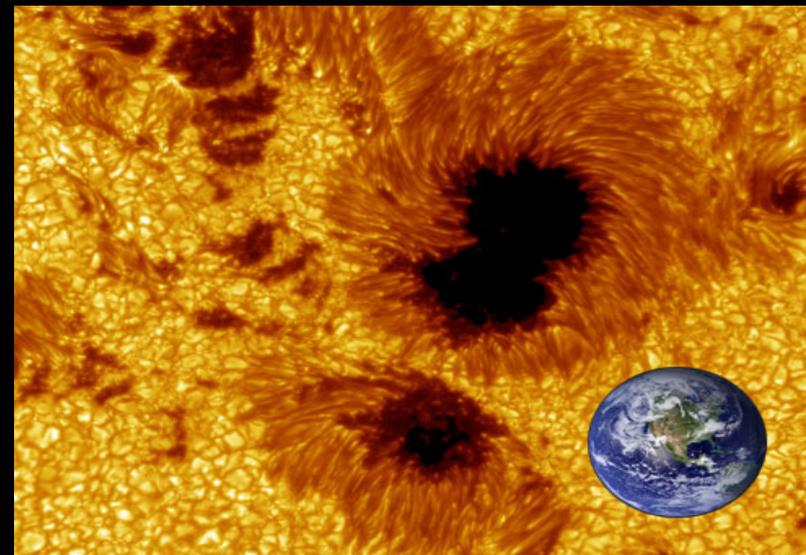
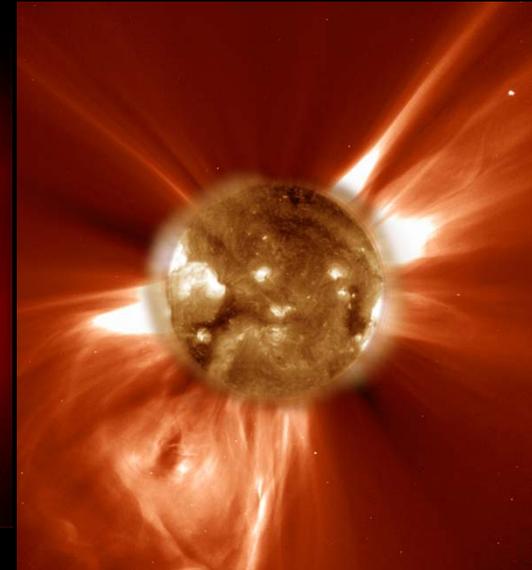
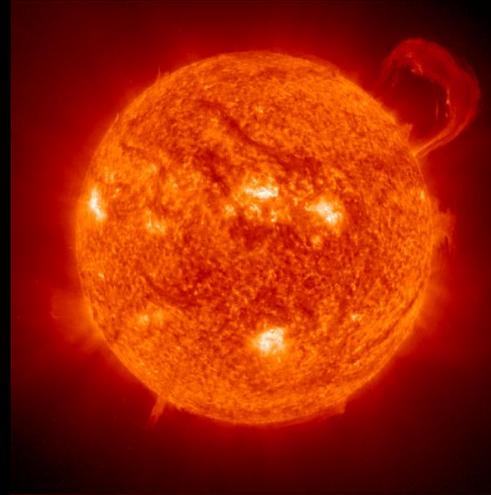
Part I

Review of the solar-terrestrial problem

Quick Facts: Solar activity

- **Solar activity: comprises photospheric and chromospheric phenomena such as sunspots, prominences, and CMEs.**
- **Also refers to the level of solar magnetism (often giving rise to sunspots). It exhibits particular characteristics (preferred scales, orientation and polarity).**
- **Prominences (flares) and CMEs are often associated to sunspots.**
- **Other types of activity involve the appearance of magnetic flux tubes and variations in the global solar magnetic field.**

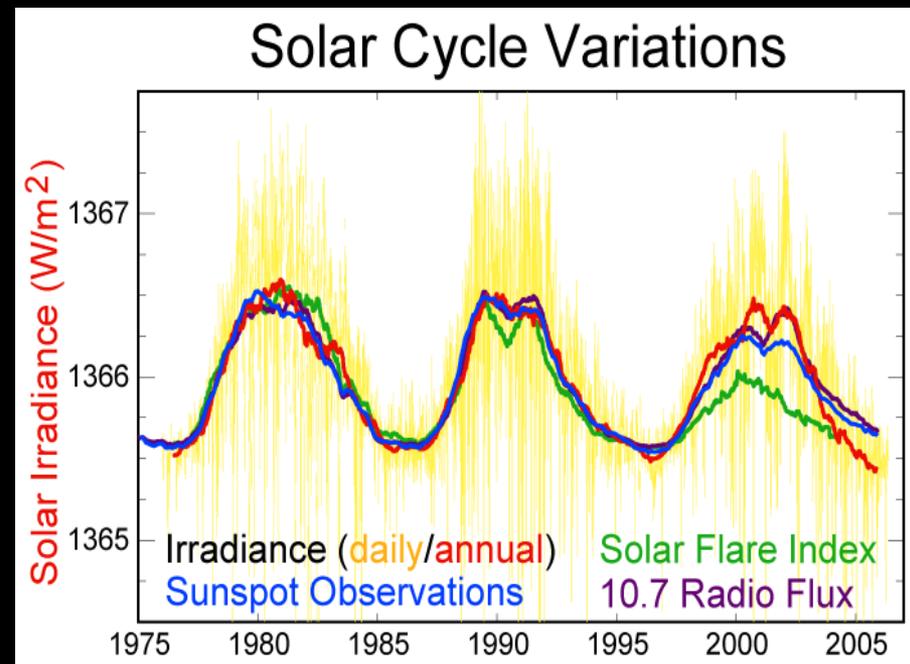
Images courtesy by SOHO (NASA & ESA)



Quick Facts: Solar activity (cont.)

Solar variations

- Defined as the changes in the amount of radiation emitted by the Sun and in its spectral distribution over years to millennia.
- May affect directly or indirectly the Earth's climate:
 1. Eruptional activity (flares, CMEs) may enhance warming through UV induced chemical reactions in the high atmosphere with ozone (Hoyt and Schatten, 1997).
 2. Solar variation, together with volcanic activity probably contributed to climate change (during the Maunder Minimum, 1680-1750). However, changes in solar brightness are too weak to explain recent climate change (Foukal et al. 2006)



Solar cycles

- 11 years (Schwabe cycle or sunspots cycle)
- 22 years: the magnetic field reverses every 11 years
- 70–100 years (Gleissberg cycle). Probably is an amplitude modulation of the 11-year cycle
- Other: 210, 2300, 6000 years!!

Quick Facts: Earth's climate

Factors that can control the climate:

- Volcanic eruptions: lava+ash+tiny particles reach the stratosphere and reflect solar radiation back out to space (cooling ~1 or 2 years).
- Sun: the 11-year solar cycle can cause a small impact on climate. Over thousands of years, changes in the way Earth orbits the Sun can cause large changes in climate.
- Others: greenhouse gases, snow and ice of the cryosphere, aerosols...



Quick Facts: Earth's climate (cont.)

Regional climate: It is the average weather pattern in a place over more than 30 yrs, including the variations in seasons.

Depends on the amount of sunlight it receives, its altitude, topography, and how close it is to oceans

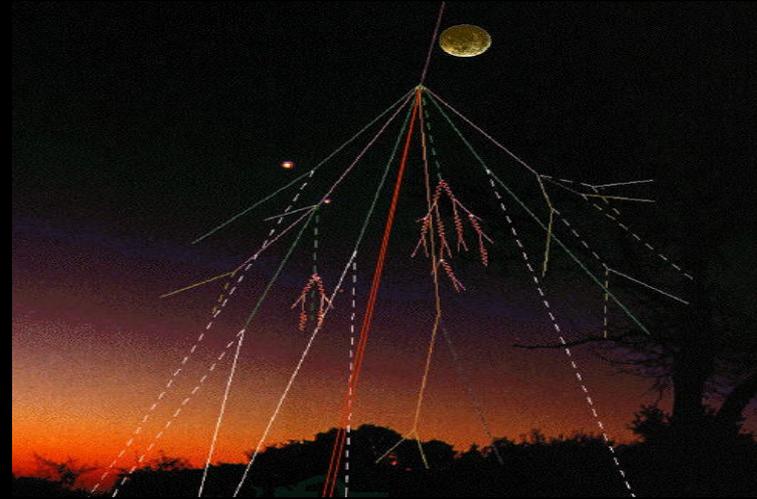
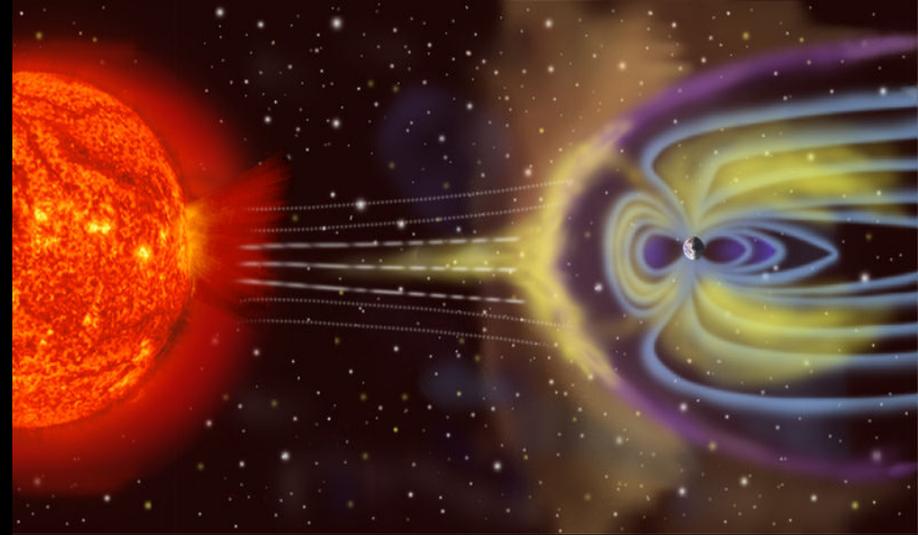
Global climate: is a description of the climate of a planet as a whole, with all the regional variations averaged.

Depends on the amount of energy received by the Sun and the amount of energy that is held in the system.

Solar-terrestrial relationships (ST)

BIG QUESTION: how is solar activity felt on the Earth?

The Sun exerts an influence on Earth: light, heat, solar wind, flux of cosmic rays



Solar-terrestrial relationships (ST) (cont.)



As ST influence the amount of sunlight received → temperature

climate

moisture

temperature + moisture + sunlight

all life on Earth

Solar-terrestrial relationships (ST) (cont.)

Involve studies of the **solar spectrum**, the **solar constant**, and the **statistical correlation of solar phenomena** (*such as sunspots, flares and prominences*) **with associated terrestrial activity** (*such as ionospheric disturbances, magnetic variations, aurorae, weather, and variations of atmospheric constituents*).

Briefs on solar-terrestrial climate links

- ❖ Before searching for any statistical relationship: quality of the data is very important. Actually high-density global datasets compiled from surface networks and weather satellite observations are available.
- ❖ **CAUTION!! Avoid circular reasoning: sunspot observations could be affected by atmospheric conditions, the isotope records by climate variations, and the magnetic indices by the geomagnetic field.**
- ❖ Sunspots and weather: First suggested by Galileo, Scheiner and Riccioli (1651). Since then, hundreds of investigations.
Pros: it is the only index that has been recorded for a long time
Cons: No sunspots between 1645 and 1715 (lost records?)
- ❖ Solar cycle and surface temperatures:
Gruithuisen (1826): fine and settled weather followed the decline of sunspot activity.

Köppen (1873): temperature in the tropics peaked around a year before the minimum sunspot activity and cold conditions coincided with sunspot maximum.

Briefs on solar-terrestrial climate links (cont.)

- ❖ Statistical studies that correlate weather and climate with solar activity have been popular for centuries.
- ❖ Laut (2003): found that apparent correlations between solar activity and terrestrial climate do not properly reflect the underlying physical data.
- ❖ A possible mechanism that could link solar activity-weather:

Active Sun

heating causes the upper atmosphere to expand farther into space

increased depth of the atmosphere alters the way in which vertical planetary waves propagate around the globe

this may affect a coupling between the upper and lower atmosphere

possible influence on weather processes

Part II

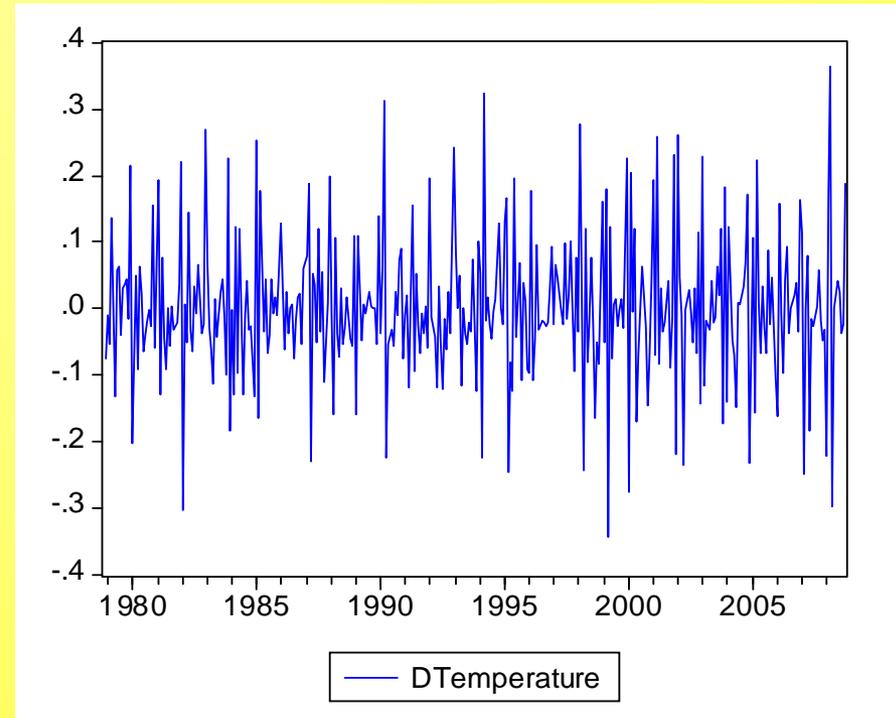
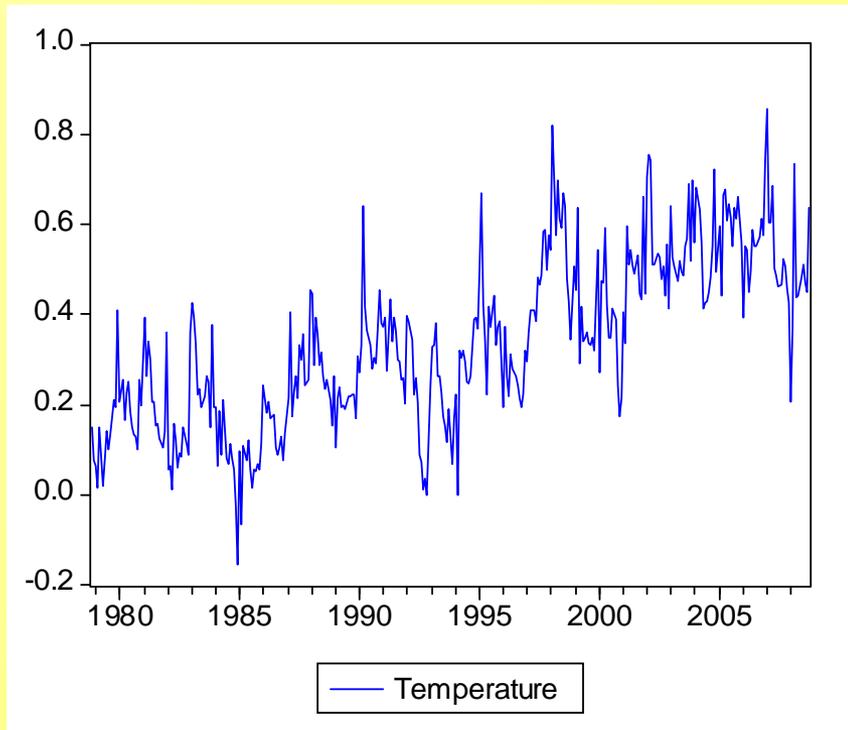
**Time Series Analysis and our
proposal based on the VAR
methodology**

Looking for a “joint” model

- As a first approach, the association between solar and terrestrial environmental parameters is found through some kind of correlation based on multivariate statistics. More sophisticated techniques, like the *wavelet analysis*, are commonly used to find possible periodicities between the solar activity and the climate parameters (for example, Lewis & Freese, 2002).
- In general climate models (global or regional) are complicated systems of differential equations based on the basic laws of physics, fluid motion, and chemistry (Stute et. al., 2001). They are hard to compute and a few of them include the solar influence.
- It is an interesting challenge to find a “joint model”.

Time series: exploration

Time series: check for trend, seasonality, and structural break.



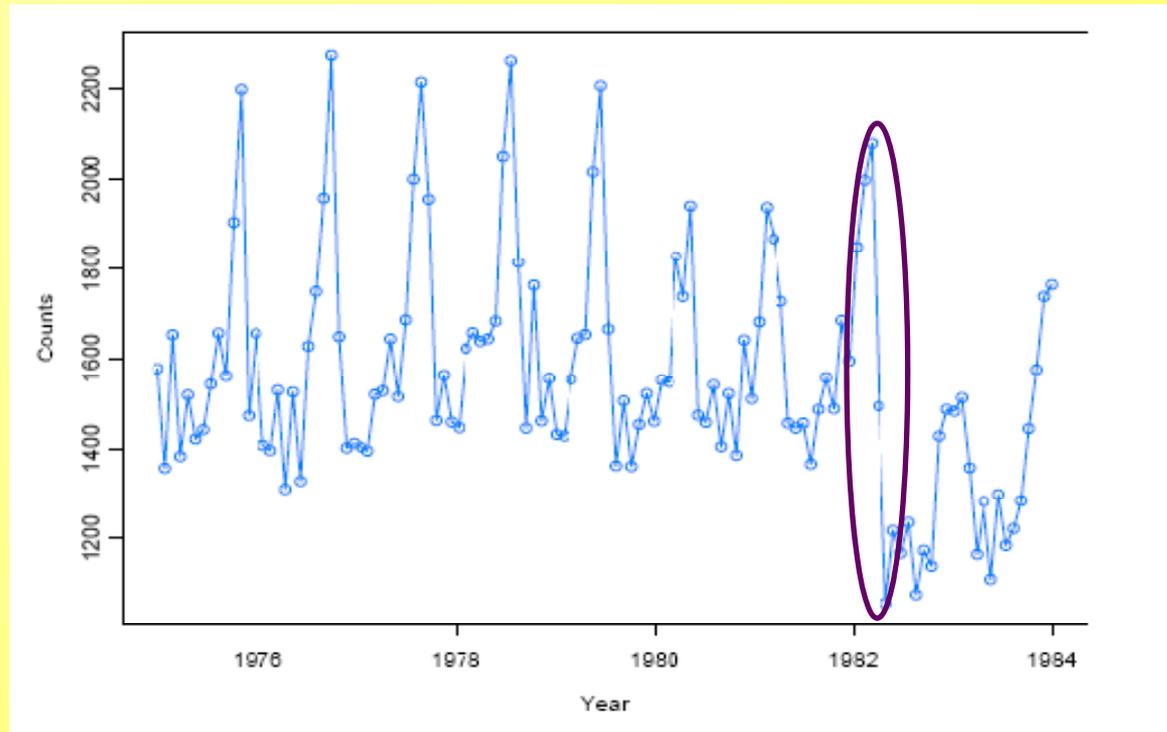
1. Make a series Y stationary (Box-Cox transformation):

$$Y^{(\lambda)} = \begin{cases} Y^\lambda, & \lambda \neq 0 \\ \log(Y), & \lambda = 0 \end{cases}$$

2. Take the difference of the series (unit root tests: Dickey-Fuller).

$$X_t = \nabla Y_t = Y_t - Y_{t-1}$$
$$X_t = \nabla^2 Y_t = Y_t - 2Y_{t-1} + Y_{t-2}$$

Time series: fundamentals



3. If the time series shows a “breakpoint”...apply Chow test to verify it, for example.

VAR Methodology: General ideas

- Do not confuse with VaR (Value at Risk)!!
- VAR means Vector AutoRegressive model. Generalization of AR models (Sims 1972, 1980, 1982).
- Is one the most successful, easy and flexible for the analysis of multivariate time series.
- Extensively used in Econometrics and Finance (Sims 1980).

VAR model describes the evolution of a set of k variables (called *endogenous variables*) over the same sample period ($t = 1, \dots, T$) as a linear function of only their past evolution:

$$y_t = c + \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \dots + \alpha_p y_{t-p} + e_t$$

$$E(e_t) = 0$$

Mean zero

$$E(e_t e_{t-k}') = 0$$

No serial correlation

c is a $k \times 1$ vector of constants (intercept)

α_i is a $k \times k$ matrix (for every $i=1, \dots, p$)

e_t is a $k \times 1$ vector of error terms

p is the number of lags (that is, the number of periods back).

VAR Methodology: General ideas (cont.)

All the variables have to be of the same order of integration.

Cases: 1) All the variables are $I(0)$ (stationary).

2) All the variables are $I(d)$ (non-stationary) with $d > 0$.
In such a case differentiate the series as many times as necessary.

Determination of lag length: is a trade-off between the curse of dimensionality and abbreviate models.

Interpreting VAR models: Allow interpretations about the dynamic relationship between the indicated variables.

Granger causality

Estimation of VAR models: Determine the endogenous variables.
Estimate according to OLS.

Our time series data

- The data are taken from the National Geophysical Data Center (NGDC) and the National Climatic Data Center (NCDC).
- **SUN:** Many indices may be used to represent different aspects of solar activity like the *sunspots number* and the *Total Solar Irradiance (TSI)*. Since THERE ARE NOT DIRECT OBSERVATIONAL data to study the long-term variation we have to rely on **proxy data such as cosmogenic radionuclides**. Thus, we have:

1) *Sunspot number* (yearly averages): 1700-2008

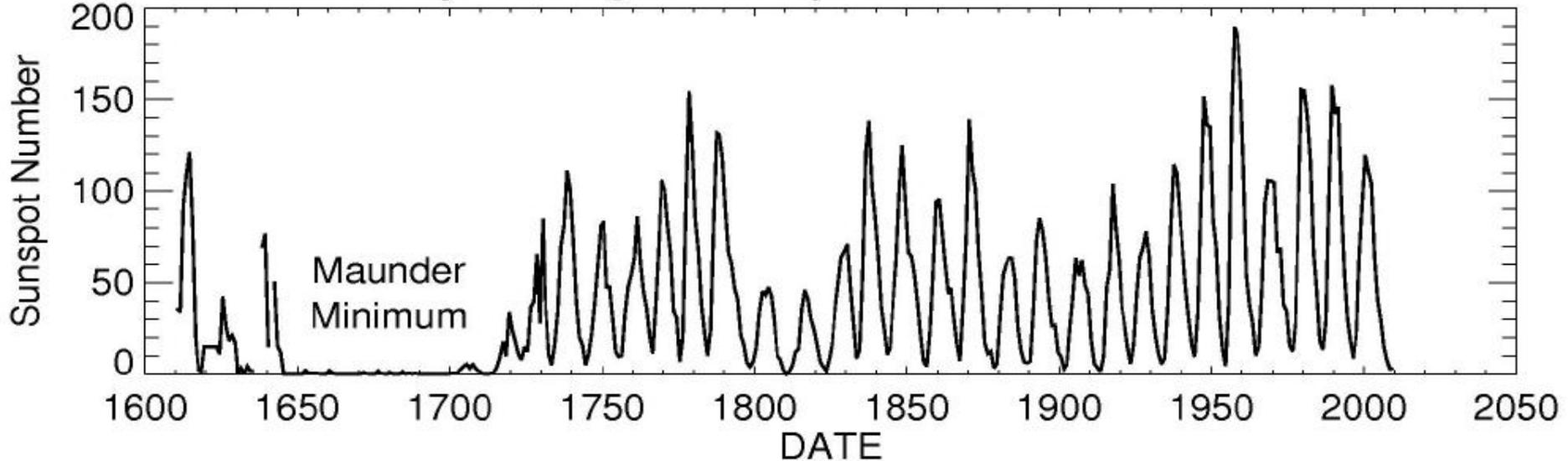
2) *Total Solar Irradiance, TSI* (yearly averages): 1610-2000

3) *^{10}Be concentration in ice cores* (geological measurements): 1424-1985

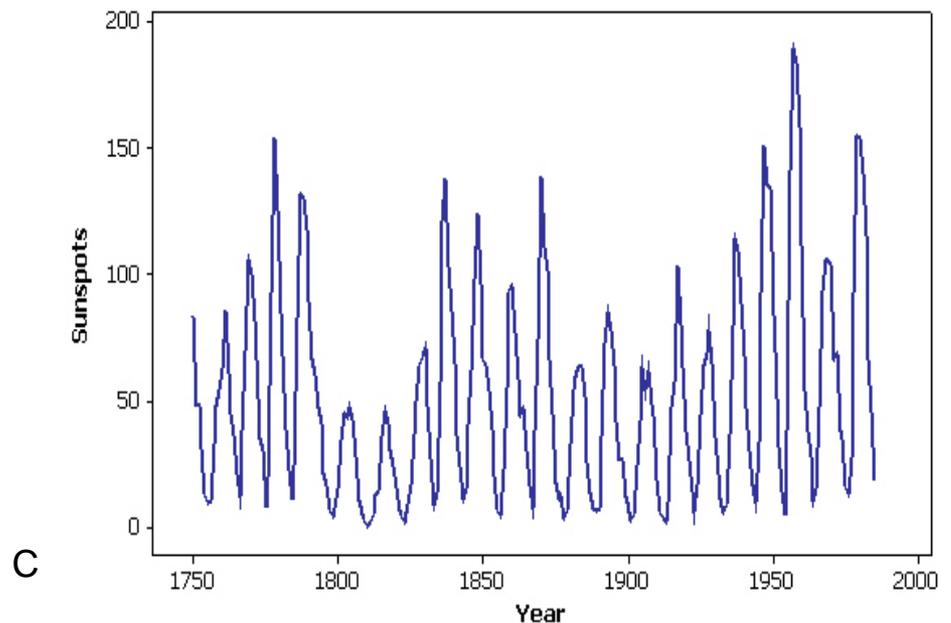
PERIOD OF STUDY: 1700-1985

Sunspots

Yearly Averaged Sunspot Numbers 1610-2009



- Represent the longest recorded aspect of solar variations.
- Probably, the most remarkable feature of solar activity.
- Two major reconstructions: International Sunspot Number (ISN, Wolf number) and the Group Sunspot Number (GSN).

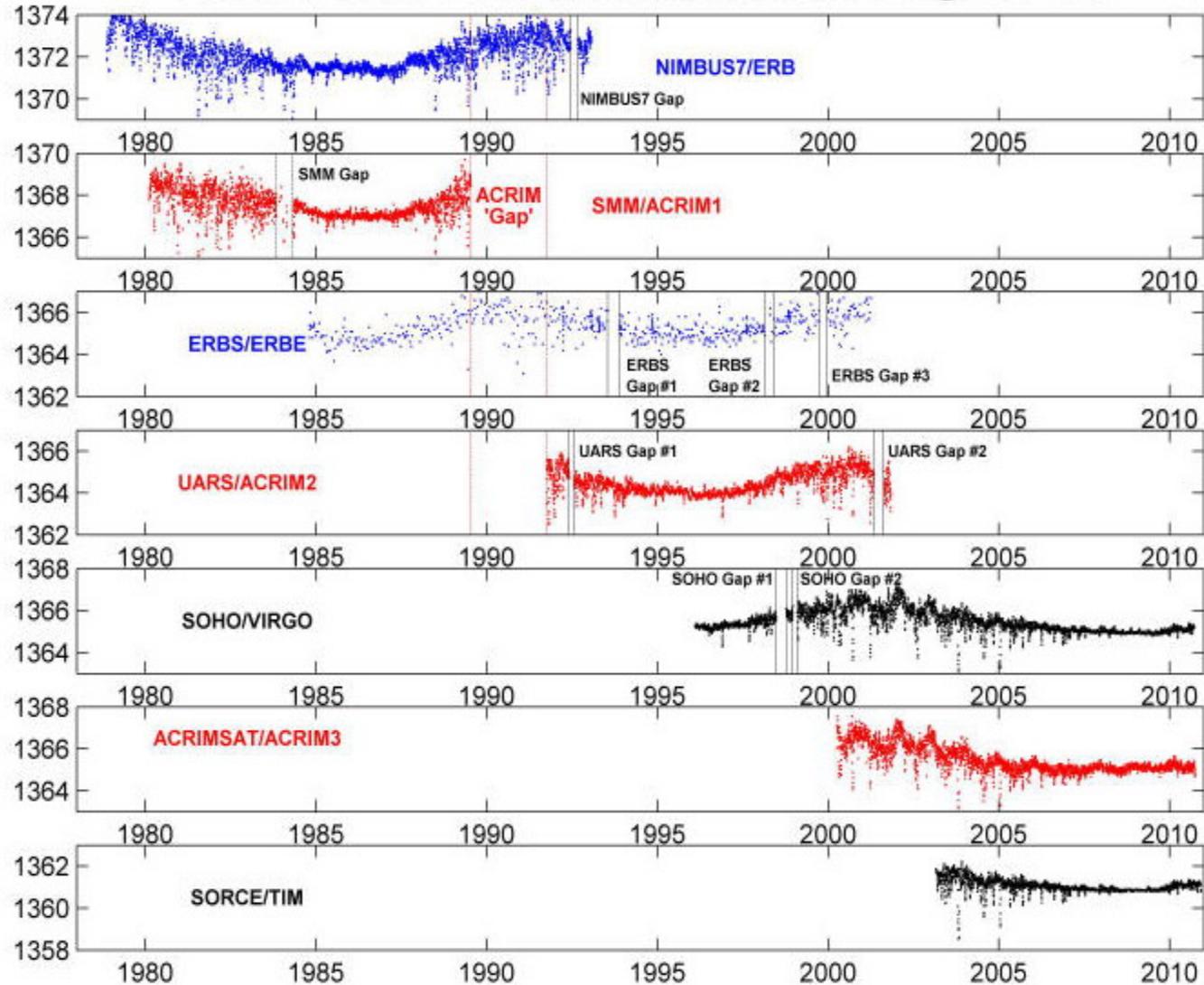


Total Solar Irradiance (TSI)

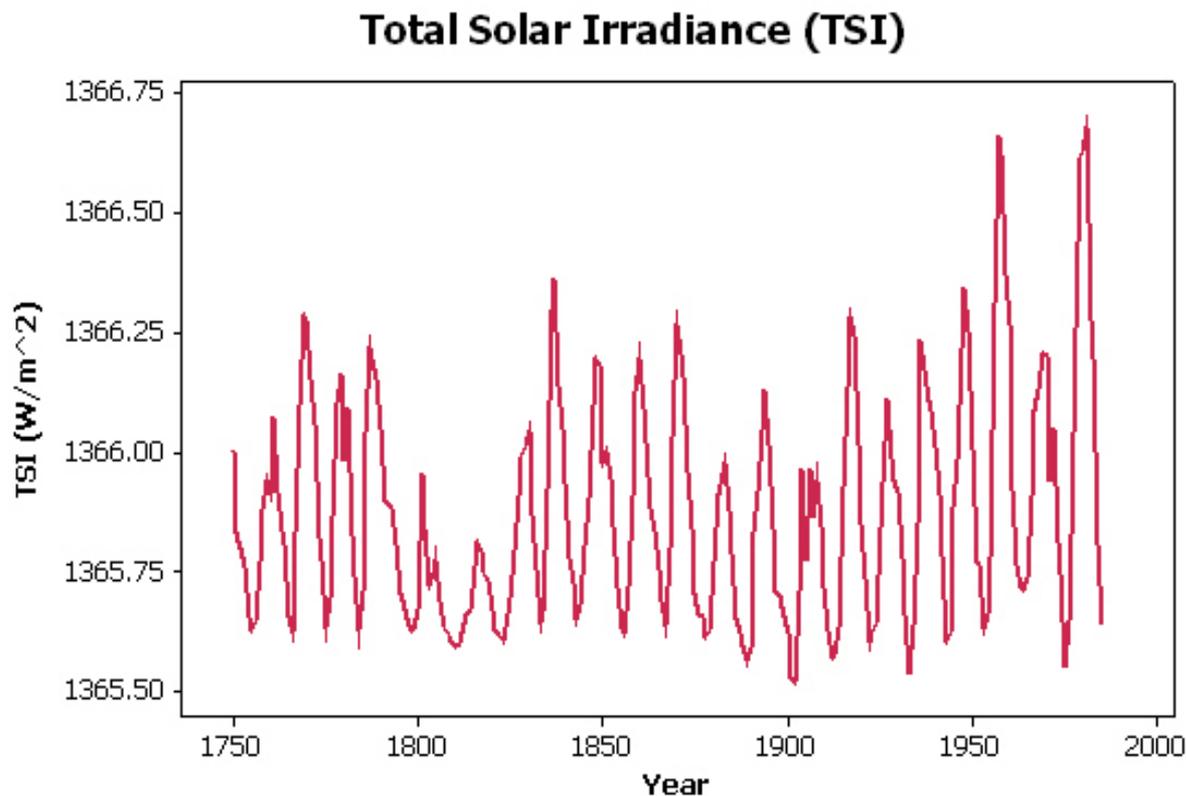
Defined as the amount of radiant energy emitted by the Sun over all wavelengths that fall each second on 1 m² outside the earth's atmosphere.

Taken from ACRIM (<http://www.acrim.com>)

Satellite Total Solar Irradiance Monitoring Results



Total Solar Irradiance (TSI) (cont.)



Data from 1750 to 1978 were taken from the reconstructed time series by Lean et al. (1995)

Cosmogenic isotopes

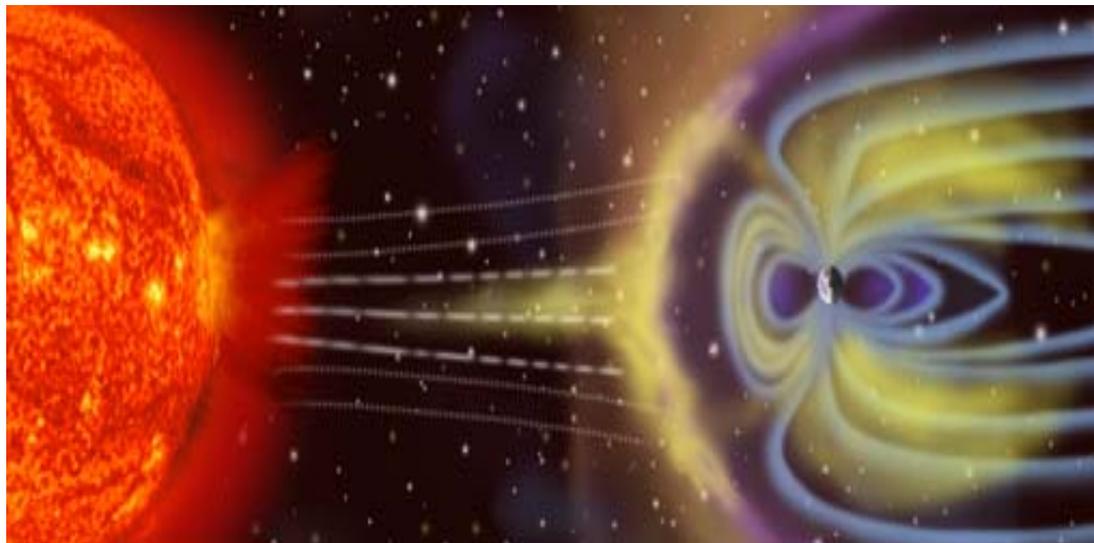
They are produced when cosmic rays collide with atmospheric molecules (upper atmosphere) at high speed.

The production rate depends on the strength of the cosmic radiation.



which varies with $|B|$ and with the solar activity

Therefore, records of cosmogenic isotope production rates are invaluable for understanding the relation between past climate change, the $|B|$, and variations in the solar activity.



higher solar activity results in stronger shielding and thus lower production of cosmogenic isotopes

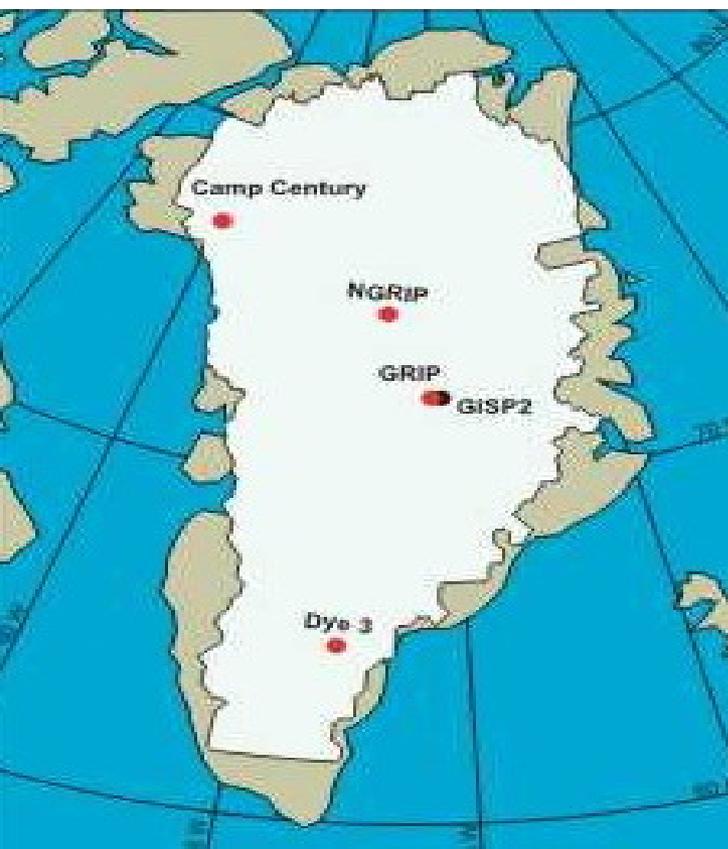
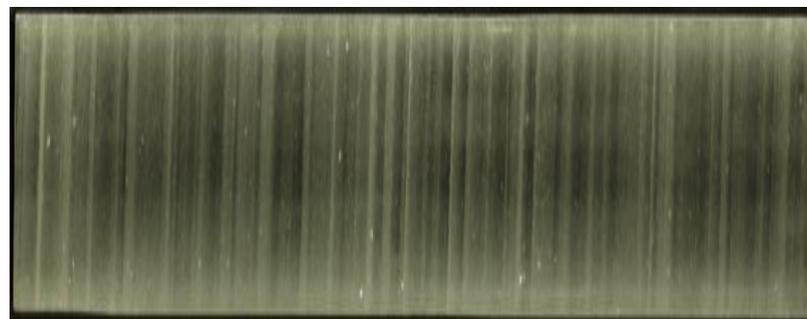
Abundance of cosmogenic isotopes deposited in ice cores

Cosmogenic isotopes (cont.)

Some cosmogenic isotopes:

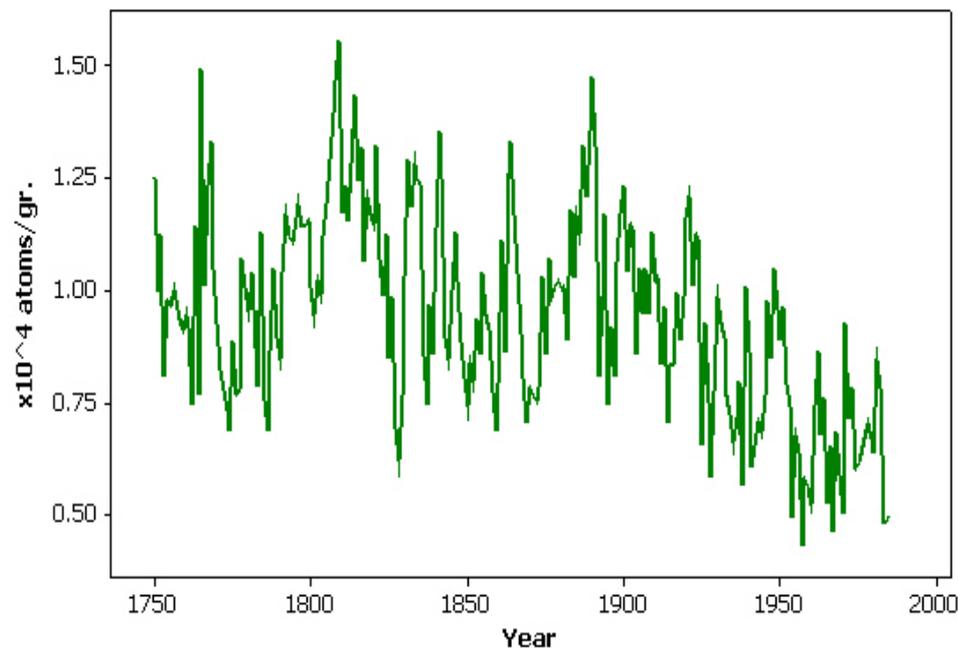
^{14}C (its abundance in ice sheets is very low)

^{10}Be and ^{36}Cl (they can be measured in the ice cores).



Cl

Concentration of Be-10 in ice cores



Our time series data (cont.)

- TERRESTRIAL CLIMATE: We focus our attention in major climate phenomena.
 1. *Global temperature measured in both hemispheres (monthly):* Jan 1850-May 2009
 2. *Multivariate ENSO Index, MEI (monthly):* Jan 1950-Jun 2009
 3. *Northern Atlantic Oscillation, NAO (monthly):* Jul 1821-May 2008
 4. *Pacific Decadal Oscillation, PDO (monthly):* Jan 1900-Jan 2009. Reconstruction using tree rings and other hydrologically sensitive proxies is available since 993-1996.

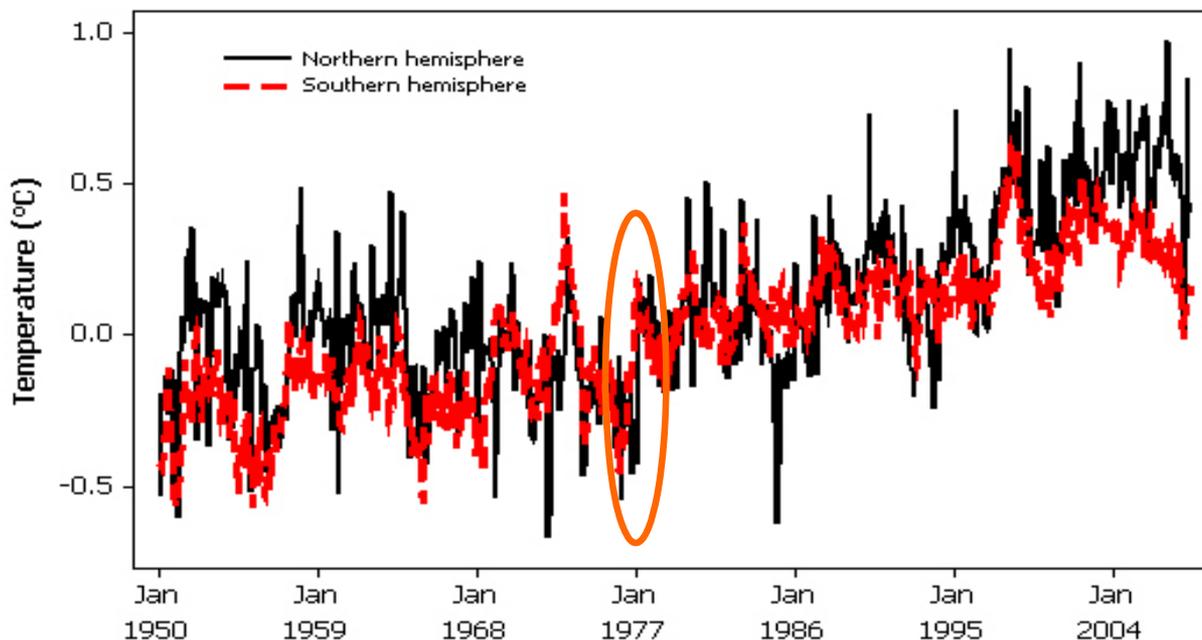
PERIOD OF STUDY: Jan 1950-May 2008

Hemispheric temperature: North and South

Observations indicate a statistical relationship: Northern Hemisphere temperature depends on temperature in the Southern Hemisphere.

Probably caused by the climatic effects of anthropogenic trace gases and tropospheric sulphate aerosols (Kaufmann and Stern, 1997; Gay et. al., 2009).

Temperature of both hemispheres



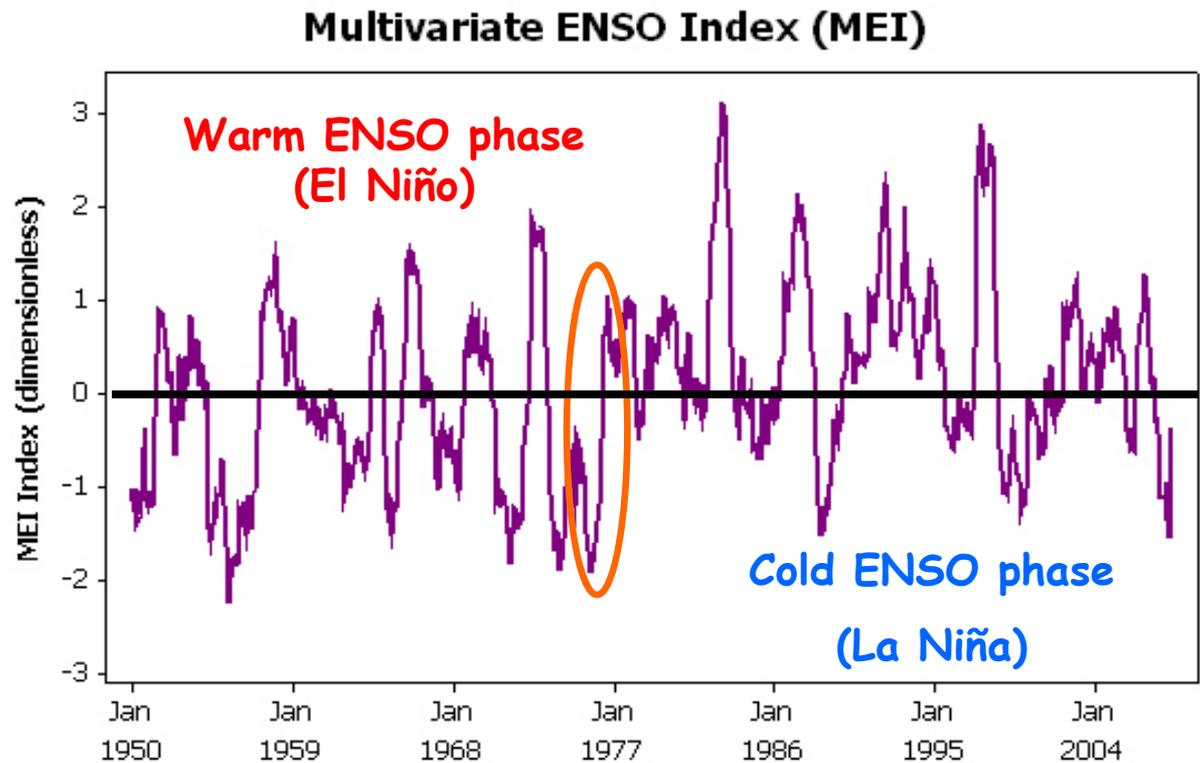
Data taken from the National Climatic Data Center (NCDC). Includes land and marine temperatures.

Multivariate ENSO Index (MEI)

MEI is an index of 6 observed variables over the tropical Pacific:

1. sea-level pressure (P)
2. zonal (U) and
3. meridional (V) components of the surface wind
4. sea surface temperature (S)
5. surface air temperature (A)
6. total cloudiness fraction of the sky (C)

Used to monitor the coupled ocean-atmosphere phenomenon known as the El Niño-Southern Oscillation (ENSO)

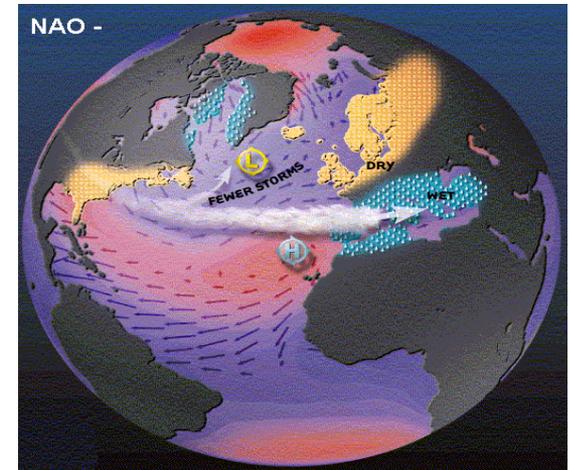
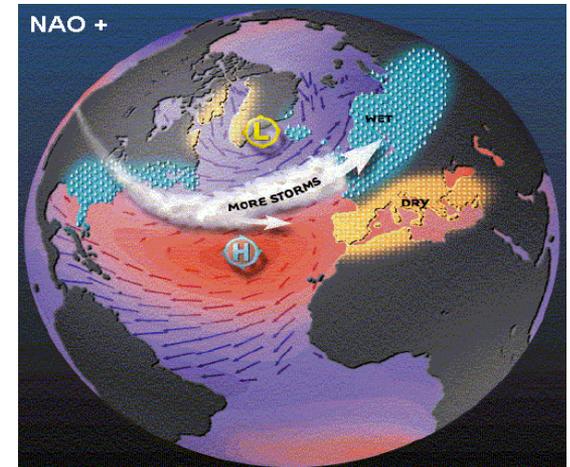
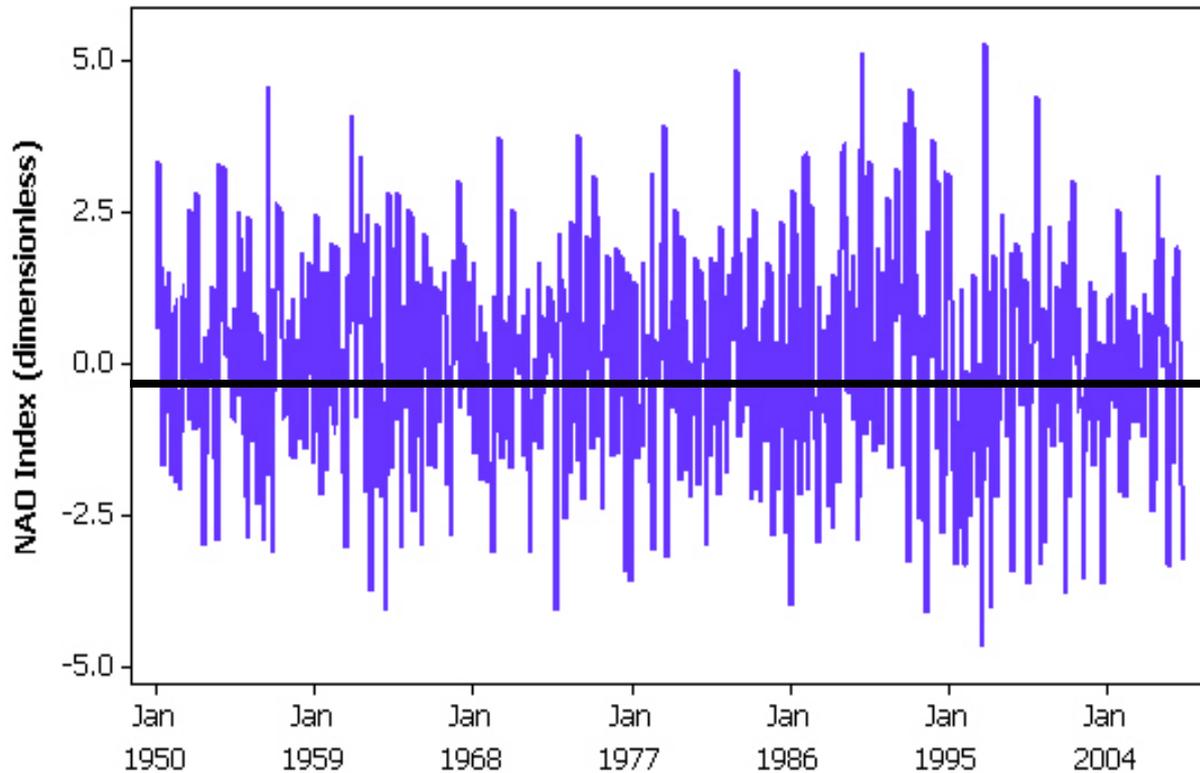


North Atlantic Oscillation (NAO)

NAO is the dominant mode of winter climate variability in the North Atlantic region ranging from central North America to Europe and much into Northern Asia.

It is defined as the anomalous difference between the polar low and the subtropical high during the winter season (December- March).

North Atlantic Oscillation (NAO)



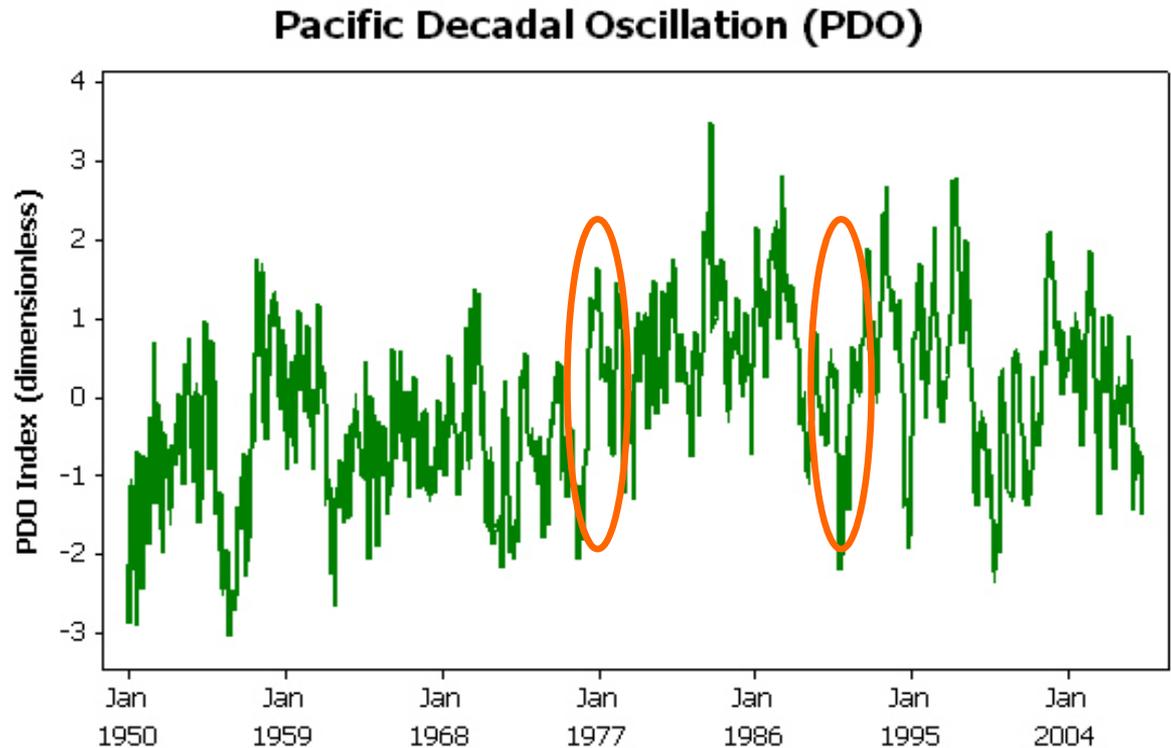
Pacific Decadal Oscillation (PDO)

PDO is a long-lived El Niño-like pattern of Pacific climate variability.

Unlike ENSO, the PDO is not a single physical mode of ocean variability, but rather the sum of several processes with different dynamical origins.

PDO vs ENSO

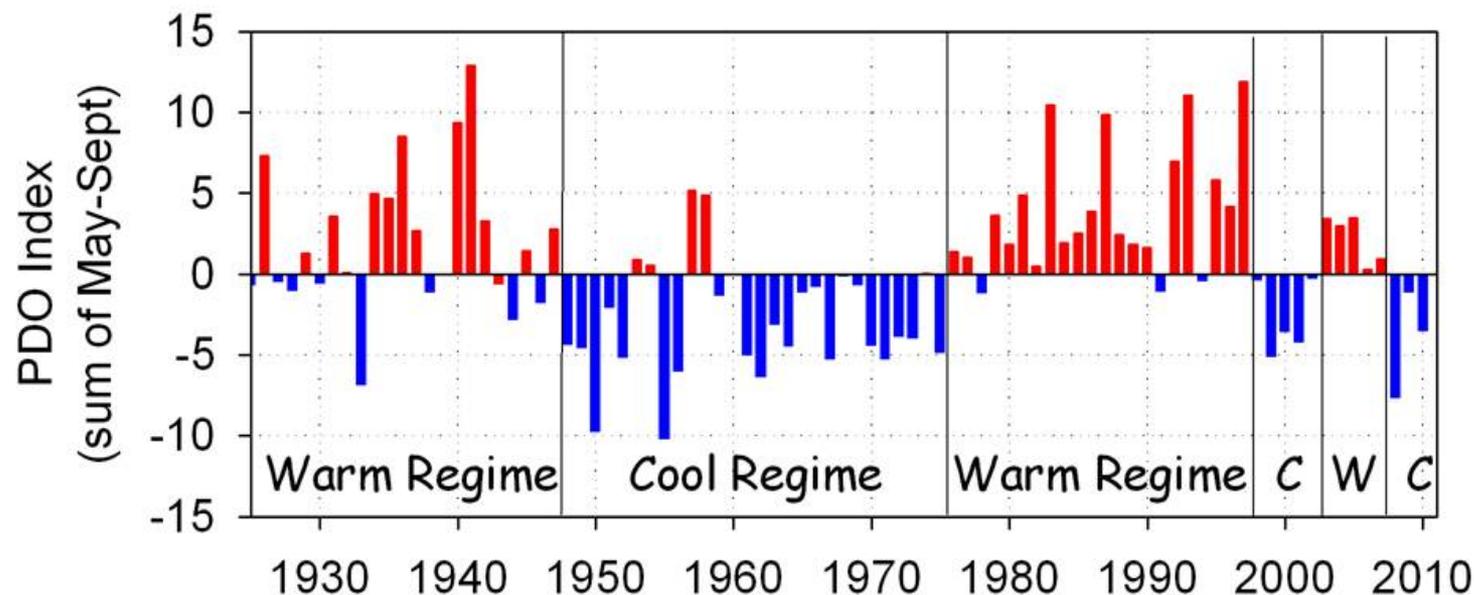
1. **Duration:** PDO (20-30 years), ENSO (6-18 months)
2. **Climatic fingerprints:** PDO (most visible in the North Pacific/North American sector and secondary signatures exist in the tropics), ENSO (the opposite).



Pacific Decadal Oscillation (PDO) (cont.)

PDO is highly correlated with sea surface temperature in the northern California Current (CC) area.

Two phases, of PDO: "warm phase" and a "cool phase"



Taken from NOAA Fisheries Service

Application of the VAR methodology to...

a) *SOLAR-TERRESTRIAL CONNECTION*

- One of the principal issues in quantifying the role of the sun in climate change has been the **absence of long-term measurements of both, the climatic and solar activity phenomena.**
- The time series must be stationary, otherwise apply an adequate transformation and take the first difference (Solar series at first differences and terrestrial series cannot since they show some “breakpoints”).
- We introduce three dummy variables associated to the structural breaks (1976, 1977 and 1990).
- Exogenous variables: the number of sunspots, TSI, d76 (dummy variable for 1976), d77 (dummy variable for 1977), and d90 (dummy variable for 1990). The period of study is January 1950- May 2008.

Application of the VAR methodology to...

- The number of lags is 4.
- The VAR(4) model is formed for 5 equations:

$$\begin{aligned} \text{TN} = & 0.4157793578*\text{TN}(-1) + 0.1914720803*\text{TN}(-2) - 0.02176432137*\text{TN}(-3) + \\ & 0.1112734872*\text{TN}(-4) + 0.09346883089*\text{TS}(-1) + 0.06412269375*\text{TS}(-2) - 0.01354812506*\text{TS}(-3) \\ & + 0.02371462342*\text{TS}(-4) + 0.003679361099*\text{MEI}(-1) + 0.02006879195*\text{MEI}(-2) - \\ & 0.01112715237*\text{MEI}(-3) - 0.007703706486*\text{MEI}(-4) + 0.005446753642*\text{NAO}(-1) + \\ & 0.001971396077*\text{NAO}(-2) - 0.005590666009*\text{NAO}(-3) - 0.006482730746*\text{NAO}(-4) + \\ & 0.002460035967*\text{PDO}(-1) - 0.01060076951*\text{PDO}(-2) + 0.02106321063*\text{PDO}(-3) - \\ & 0.01398173034*\text{PDO}(-4) + 0.01016625849 - 2.48960763e-005*\text{DSUNSPOT} - \\ & 0.006364171471*\text{DTSI} - 0.05588309422*\text{D76} + 0.05363374085*\text{D77} + 0.07231720552*\text{D90} \end{aligned}$$

$$\begin{aligned} \text{TS} = & 0.05844322383*\text{TN}(-1) + 0.005467377033*\text{TN}(-2) - 0.03238839999*\text{TN}(-3) - \\ & 0.0174903937*\text{TN}(-4) + 0.6028320508*\text{TS}(-1) + 0.1587207265*\text{TS}(-2) + 0.015632992*\text{TS}(-3) + \\ & 0.07841729387*\text{TS}(-4) + 0.03266192713*\text{MEI}(-1) - 0.0006861417974*\text{MEI}(-2) - \\ & 0.01926169142*\text{MEI}(-3) - 0.0006299026487*\text{MEI}(-4) + 0.0008818013353*\text{NAO}(-1) - \\ & 0.0002785338341*\text{NAO}(-2) - 0.00257735856*\text{NAO}(-3) + 0.001497635689*\text{NAO}(-4) + \\ & 0.002457879372*\text{PDO}(-1) - 0.003635915427*\text{PDO}(-2) + 0.002887421808*\text{PDO}(-3) - \\ & 0.002927236258*\text{PDO}(-4) - 0.02215541548 - 0.0001618891044*\text{DSUNSPOT} + \\ & 0.002423357328*\text{DTSI} + 0.02985456983*\text{D76} - 0.001380246895*\text{D77} + 0.01920949069*\text{D90} \end{aligned}$$

$$\begin{aligned} \text{MEI} = & 0.148611936*\text{TN}(-1) - 0.09876084322*\text{TN}(-2) - 0.01232381759*\text{TN}(-3) - 0.1469164764*\text{TN}(-4) \\ & + 0.4557789496*\text{TS}(-1) - 0.2878869734*\text{TS}(-2) - 0.02886364156*\text{TS}(-3) - 0.2047235547*\text{TS}(-4) + \\ & 1.194975832*\text{MEI}(-1) - 0.2925613105*\text{MEI}(-2) + 0.2008458593*\text{MEI}(-3) - 0.1918578255*\text{MEI}(-4) + \\ & 0.005278023673*\text{NAO}(-1) + 0.01521172767*\text{NAO}(-2) + 0.0006703998107*\text{NAO}(-3) + \\ & 0.007221400673*\text{NAO}(-4) + 0.005026392626*\text{PDO}(-1) + 0.01217966 \end{aligned}$$

$$\begin{aligned} \text{NAO} = & - 0.217302536*\text{TN}(-1) + 0.5030269876*\text{TN}(-2) - 0.03308796698*\text{TN}(-3) - 0.7432585038*\text{TN}(-4) \\ & - 2.101948525*\text{TS}(-1) + 1.277353892*\text{TS}(-2) + 1.705056102*\text{TS}(-3) - 1.468874636*\text{TS}(-4) - \\ & 0.1144750504*\text{MEI}(-1) + 0.2703982426*\text{MEI}(-2) - 0.01395242706*\text{MEI}(-3) - 0.002887021035*\text{MEI}(-4) \\ & + 0.06455876044*\text{NAO}(-1) - 0.00584958774*\text{NAO}(-2) - 0.1152592998*\text{NAO}(-3) - \\ & 0.02172776389*\text{NAO}(-4) + 0.09743199273*\text{PDO}(-1) - 0.2378575433*\text{PDO}(-2) + \\ & 0.04866845998*\text{PDO}(-3) - 0.04233465648*\text{PDO}(-4) - 0.1752534208 - 0.005778786704*\text{DSUNSPOT} - \\ & 0.1117309069*\text{DTSI} - 0.1261699177*\text{D76} + 0.4753017255*\text{D77} + 0.1568757931*\text{D90} \end{aligned}$$

$$\begin{aligned} \text{PDO} = & - 0.2197953404*\text{TN}(-1) - 0.03820588422*\text{TN}(-2) + 0.1015417463*\text{TN}(-3) + \\ & 0.03589452339*\text{TN}(-4) + 0.4491622947*\text{TS}(-1) + 0.1314749038*\text{TS}(-2) - 0.3954144948*\text{TS}(-3) - \\ & 0.2195387272*\text{TS}(-4) + 0.1048383082*\text{MEI}(-1) + 0.1386835067*\text{MEI}(-2) - 0.127237357*\text{MEI}(-3) + \\ & 0.05241936305*\text{MEI}(-4) - 0.008485193832*\text{NAO}(-1) + 0.019590763*\text{NAO}(-2) - \\ & 0.004348721662*\text{NAO}(-3) + 0.008112569379*\text{NAO}(-4) + 0.6694522229*\text{PDO}(-1) + \\ & 0.02275402482*\text{PDO}(-2) - 0.02192080133*\text{PDO}(-3) + 0.03078448077*\text{PDO}(-4) - 0.154393354 + \\ & 0.0006912452539*\text{DSUNSPOT} + 0.0752161097*\text{DTSI} + 0.3254285306*\text{D76} - 0.03724350584*\text{D77} - \\ & 0.07817683806*\text{D90} \end{aligned}$$

Statistical validation of VAR(4)

- ✓ **Significance:** All the variables are statistically significant in each equation -except for NAO- which means that each one contributes to explain the solar-terrestrial connection for climate.
- ✓ **Stability:** VAR(4) satisfies the stability condition.
- ✓ **No serial correlation:** According to the Lagrange Multiplier (LM) test we conclude that the VAR model has no serial correlation.
- ✓ **Normality of the residuals:** Just the residuals in Eq. 1 are NOT normally distributed.
- ✓ **Homoskedasticity:** The residuals of the VAR model are heteroskedastic.
- ✓ **Granger Causality:** It seems that **TSI does not affect the major climate phenomena represented by NAO, PDO and MEI.**

Application of the VAR methodology to...

b) THE SOLAR ACTIVITY

- **Goal: analyze the possible interactions among the variables that describe the solar activity.**
- **The series has been differentiated one time. The period of study is January 1950- May 2008.**
- **The number of lags is 8.**
- **The VAR(8) model is formed by 3 equations:**

$$\begin{aligned} \text{DSUNSPOT} = & 0.1113600845 * \text{DSUNSPOT}(-1) - 0.2973416184 * \text{DSUNSPOT}(-2) - \\ & 0.419163106 * \text{DSUNSPOT}(-3) - 0.1743530743 * \text{DSUNSPOT}(-4) - 0.3284029764 * \text{DSUNSPOT}(-5) - \\ & 0.2767894141 * \text{DSUNSPOT}(-6) - 0.2225192541 * \text{DSUNSPOT}(-7) - 0.1833680442 * \text{DSUNSPOT}(-8) \\ & + 22.55944433 * \text{DTSI}(-1) + 13.37746552 * \text{DTSI}(-2) + 11.1889712 * \text{DTSI}(-3) - 1.654012908 * \text{DTSI}(-4) + \\ & 3.145680852 * \text{DTSI}(-5) + 14.26928039 * \text{DTSI}(-6) - 2.580070161 * \text{DTSI}(-7) - 6.46161074 * \text{DTSI}(-8) + \\ & 4.688740208 * \text{DBE10}(-1) - 2.57496354 * \text{DBE10}(-2) - 16.77615067 * \text{DBE10}(-3) - \\ & 10.56086565 * \text{DBE10}(-4) - 10.70199143 * \text{DBE10}(-5) + 1.185318124 * \text{DBE10}(-6) - \\ & 3.707387411 * \text{DBE10}(-7) - 8.164767435 * \text{DBE10}(-8) + 0.3784042278 \end{aligned}$$

$$\begin{aligned}
\text{DTSI} = & 0.002596846289 * \text{DSUNSPOT} (-1) - 0.0003193589421 * \text{DSUNSPOT} (-2) + \\
& 0.0004323392493 * \text{DSUNSPOT} (-3) + 0.001042917745 * \text{DSUNSPOT} (-4) - 5.617649703e- \\
& 005 * \text{DSUNSPOT} (-5) + 0.0008160838282 * \text{DSUNSPOT} (-6) + 0.001087692558 * \text{DSUNSPOT} (-7) + \\
& 0.0001936949776 * \text{DSUNSPOT} (-8) - 0.1634240965 * \text{DTSI}(-1) - 0.3611082388 * \text{DTSI}(-2) - \\
& 0.4313562073 * \text{DTSI}(-3) - 0.3693256228 * \text{DTSI}(-4) - 0.3689870796 * \text{DTSI}(-5) - 0.4125681239 * \text{DTSI}(-6) \\
& - 0.4094719685 * \text{DTSI}(-7) - 0.3871268466 * \text{DTSI}(-8) + 0.04579883398 * \text{DBE10}(-1) - \\
& 0.04767212105 * \text{DBE10}(-2) - 0.1119724557 * \text{DBE10}(-3) - 0.07598088491 * \text{DBE10}(-4) - \\
& 0.07729673409 * \text{DBE10}(-5) - 0.003144076057 * \text{DBE10}(-6) - 0.05272420326 * \text{DBE10}(-7) - \\
& 0.004955332335 * \text{DBE10}(-8) + 0.003739446497
\end{aligned}$$

$$\begin{aligned}
\text{DBE10} = & - 0.0005611436929 * \text{DSUNSPOT} (-1) + 0.0009545110434 * \text{DSUNSPOT} (-2) - \\
& 0.001894489107 * \text{DSUNSPOT} (-3) + 0.000919045962 * \text{DSUNSPOT} (-4) + \\
& 0.0001236158032 * \text{DSUNSPOT} (-5) + 0.001990310211 * \text{DSUNSPOT} (-6) - \\
& 0.000334006528 * \text{DSUNSPOT} (-7) - 0.0006160778476 * \text{DSUNSPOT} (-8) - 0.08978848028 * \text{DTSI}(-1) - \\
& 0.1070456224 * \text{DTSI}(-2) + 0.1340781048 * \text{DTSI}(-3) - 0.05160025291 * \text{DTSI}(-4) - 0.1434707316 * \text{DTSI}(- \\
& 5) - 0.2158860878 * \text{DTSI}(-6) - 0.04758360722 * \text{DTSI}(-7) - 0.07348690467 * \text{DTSI}(-8) - \\
& 0.5079414973 * \text{DBE10}(-1) - 0.2291050495 * \text{DBE10}(-2) - 0.1240066709 * \text{DBE10}(-3) - \\
& 0.2254312002 * \text{DBE10}(-4) - 0.2959934302 * \text{DBE10}(-5) - 0.2942766266 * \text{DBE10}(-6) - \\
& 0.15978805 * \text{DBE10}(-7) - 0.1914517183 * \text{DBE10}(-8) - 0.004090470262
\end{aligned}$$

Statistical validation of VAR(8)

- ✓ **Significance:** All the variables are statistically significant which means that each one contributes to explain the solar activity.
- ✓ **Stability:** VAR satisfies the stability condition.
- ✓ **No serial correlation:** According to the Lagrange Multiplier (LM) test we conclude that the VAR model has no serial correlation until lag 7.
- ✓ **Normality of the residuals:** NO
- ✓ **Homoskedasticity:** The residuals of the VAR model are heteroskedastic.
- ✓ **Granger Causality:** **The concentration of Be¹⁰ in the ice cores does not affect the existence of sunspots and the magnitude of the TSI.** Hence we can remove this variable from the VAR model

Part III

Summary and future work

Summary

- Many attempts to find a theoretical model (solar activity-climate) have been done, but most of them search for periodicities or correlations among the set of variables.
- The “wavelet analysis” on the time series data can give misleading periodicities.
- We propose a model based on the VAR methodology. This consists in two parts: a) a model for the solar activity and b) the model for the solar-terrestrial connection.
- For the first one we find a VAR(8) and apparently the ^{10}Be concentration does not play a fundamental role in the solar activity.
- For the second one we find a VAR(4) where the solar variables are exogenous. It seems that the sun (described only for the number of sunspots and the TSI) has a **weak connection to Earth**, at least for the major climate phenomena.

Some ideas for future work

- **Search for another proxy variables that describe the solar activity. Improve the VAR model for interpretation.**
- **Analyze solar data by cycles.**
- **We need to improve the technique to estimate the structural breaks in the temperature data, MEI, PDO.**
- **Introduce variables related to regional climate (for example: precipitation, pressure, temperature) to obtain a VAR regional model.**
- **Solve the multi-scale and reconstruction problems in some of our time series.**

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